

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 104: Divisible by 13

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How many three-digit numbers are divisible by thirteen?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Number Sense: Multi-digit addition and multiplication.

COMMON CORE STATE STANDARDS

5.NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 1: [ENGAGE IN SUCCESSFUL FLAILING](#)

SOURCE: This is question # 18 from the 2005 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Oh heavens! How can you tell when a number is divisible by 13? That's hard!

Rather than panic –deep breath – can I start by thinking just of an example of a three-digit number that is divisible by 13? Well, 130 is an obvious one: ten times 13.

Oh! Subtract 13 from this and we get that 117 is also a multiple of 13. And so too is $117 - 13 = 104$. (This must be eight times 13.)

Alright, this is something. We see that 104 is the smallest three-digit number that is a multiple of 13. What is the largest? Hmm.

Well ... Twenty times 13 is double ten times 13. So
 $20 \times 13 = 2 \times 130 = 260$

Doubling again:

$$40 \times 13 = 260 + 260 = 520$$

Add another ten 13s:

$$50 \times 13 = 520 + 130 = 650$$

Keep going:

$$60 \times 13 = 650 + 130 = 780$$

$$70 \times 13 = 780 + 130 = 910$$

We've got a "space of 90" left before we hit the four-digit number of one-thousand. How many more 13s can we sneak in?

Well $7 \times 13 = 91$ is too big, but $6 \times 13 = 78$ is okay. We have that $76 \times 13 = 910 + 78 = 988$ is the largest three-digit multiple of 13.

Okay we have $8 \times 13 = 104$ up to $76 \times 13 = 988$ as the three-digit multiples of 13. How many of them does that make?

I am tempted to say that there are $76 - 8 = 68$ of these numbers. But let me write it out.

8×13 is the first.

9×13 is the second.

10×13 is the 3rd.

11×13 is the 4th.

...

76×13 is the 69th.

There are 69 three-digit multiples of 13.

Extension: Here's a strange divisibility rule for 13.

To tell if a number is divisible by 13, delete its last digit and add four times that deleted digit to what remains. The original number is a multiple of 13 only if the new number is. (And you can repeat this procedure as many times as you wish until you obtain a result that obviously is or is not a multiple of 13.)

EXAMPLE: Testing whether or not 13403 a multiple of 13:

$$1340\cancel{3} \rightarrow + \begin{array}{r} 1340 \\ 12 \\ \hline 1352 \end{array} \rightarrow + \begin{array}{r} 135 \\ 8 \\ \hline 143 \end{array} \rightarrow + \begin{array}{r} 14 \\ 12 \\ \hline 26 \end{array} \checkmark$$

26 is a multiple of 13. This means that 13403 is too.

Why does this strange divisibility test work?

(See <http://www.jamestanton.com/?p=1287> for a whole slew of divisibility tests like this one.)

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