Isosceles right triangle \( ABC \) encloses a semicircle of area \( 2\pi \). The circle has its center \( O \) on hypotenuse \( AB \) and is tangent to sides \( AC \) and \( BC \). What is the area of triangle \( ABC \)?

QUICK STATS:

**MAA AMC GRADE LEVEL**
This question is appropriate for the middle-school grade levels.

**MATHEMATICAL TOPICS**
Geometry

**COMMON CORE STATE STANDARDS**

6.G.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

7.G.4 Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

**MATHEMATICAL PRACTICE STANDARDS**

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP7 Look for and make use of structure.

**PROBLEM SOLVING STRATEGY**

ESSAY 3: **ENGAGE IN WISHFUL THINKING**

**SOURCE:** This is question # 23 from the 2005 MAA AMC 8 Competition.
THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Okay. I can think of something relevant I can say right away.

The area of a circle is $\pi r^2$, so the area of a semicircle is $\frac{1}{2} \pi r^2$. We need:

$$\frac{1}{2} \pi r^2 = 2\pi .$$

This means $\pi r^2 = 4\pi$ and so $r = 2$. The radius of the circle is 2.

Oh. It looks like triangle $ABC$ is made of a square (of area 4) and two triangles that, together, make another copy of that square. If this is true, then the area of $\triangle ABC$ is $4 + 4 = 8$ and we’re done!

**Extension 1:** Is any of this wishful thinking true? Is the blue region actually a square? Is each small triangle indeed half that square? What results from geometry class do we need to – hopefully – prove all that we wish for is true?

**Extension 2:** When Angie answered this question she reflected the figure across the diagonal line.

Does this give another way to think about the question?

**Extension 3:** An equilateral triangle encloses a semicircle as shown.

If the area of the semicircle is $2\pi$, what is the area of the triangle?

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