

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 113: Sitting and Standing

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Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Number Sense: Fractions and Proportions

COMMON CORE STATE STANDARDS

7.NS.1d Apply properties of operations as strategies to add and subtract rational numbers.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: [PERSEVERANCE IS KEY](#)

SOURCE: This is question # 20 from the 2004 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This is a wordy problem with fractions in it. It feels a bit “tangled” and hard to sort through.

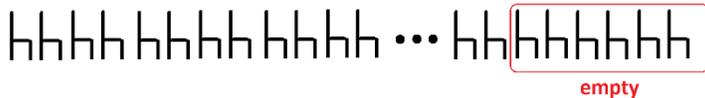
Deep breath. I’ll take it slowly.

Two-thirds of the people in a room are seated in three-fourths of the chairs. The rest of the people are standing. If there are 6 empty chairs, how many people are in the room?

I read the numbers “two-thirds,” “three-quarters,” and “six,” but only six is an actual count of something: it is the number of empty chairs. I don’t know the number of people (that’s what we’re being asked to find out) and I don’t know the number of chairs. Hmm.

Since the question mentions two things about chairs, let me focus on them.

Here are some chairs:



Six of them are empty.

Two-thirds of the people in a room are seated in three-fourths of the chairs.

So this means that three-fourths of the chairs are full.

Oh! The six empty chairs represent one quarter of all the chairs and so there are $4 \times 6 = 24$ chairs in all: 18 full and 6 empty.

Two-thirds of the people in a room are seated in three-fourths of the chairs.

So two-thirds of the people sit in 18 chairs.

Hmm.

So one-third of the people sit in 9 chairs.

That’s it! One-third of the people in the room is 9 people, so this means there are 27 people in the room (and of these 18 are sitting and 9 are standing). Great. We got it!

Extension: a) Four-fifths of the people in a room sit in three-quarters of the chairs. The rest of the people are standing. If there are 8 empty chairs, how many people are in the room?

b) How easy is it to create puzzles like these? Can any pair of fractions be used? For example, could a puzzle start: *Three-sevenths of the people in a room sit in five-ninths of the chairs?*

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