

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 116: Three Special Factors

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How many integers between 1000 and 2000 have all three of the numbers 15, 20 and 25 as factors?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Number Sense: Greatest Common Factors, Prime Factorizations.

COMMON CORE STATE STANDARDS

6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 2: [DO SOMETHING](#)

SOURCE: This is question # 19 from the 2003 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

I am not quite sure how to think about this question.
I could list all the multiples of 15 between 1000 and 2000, and all the multiples of 20, and all the multiples of 25, and then see how many numbers are common to all three lists.

The multiples of 20 : 1000, 1020, 1040, ..., 2000.
The multiples of 25 : 1000, 1025, 1050, ..., 2000.
The multiples of 15 are hard!

Hmm.

What does it mean for a number to be a multiple of 15?

Well, it better be divisible by both 3 and 5. So I want the numbers between 1000 and 2000 divisible by both 3 and 5, and by 20 and by 25.

Oh! For a number to be divisible by 20 it needs to be a multiple of both 4 and 5, and to be divisible by 25, by 5 and by 5 again.

Summarizing: We're looking for numbers between 1000 and 2000 that are divisible by three, by four, and by five twice. These numbers are the ones divisible by $3 \times 4 \times 5 \times 5 = 3 \times 100 = 300$.

So what are the multiples of 300 in the range we're considering? 1200, 1500, and 1800. There are just THREE such integers!

Extension: How many integers between one million and two million are simultaneously multiples of 24, 30, 75, 80, and 128?

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