Curriculum Burst 117: Hands of a Clock
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What is the measure of the acute angle formed by the hands of a clock at 4:20 a.m.?

QUICK STATS:

MAA AMC GRADE LEVEL
This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS
Geometry: Angle measurements; Rates of Rotation.

COMMON CORE STATE STANDARDS
Connected to: 7.G.5 Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

MATHEMATICAL PRACTICE STANDARDS
MP1 Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY
ESSAY 9: AVOID HARD WORK

SOURCE: This is question # 20 from the 2003 MAA AMC 8 Competition.
THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

**STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question looks innocent but I have a feeling it is complicated!

At the time 4:20, the minute hand is point at the twenty minute position, which is the location of the number 4. And where is the hour hand is pointing at 4:20? At the number 4 as well? (It the fourth hour.) The angle between the two hands is zero?

This can’t be right!

Oh! The hour hand points at the number 4 only right at the hour of four-o-clock. It then moves slowly from 4 to 5 as the hour progresses.

The picture is more like this and there is a positive angle between the hands of the clock!

What angle is that?

Well ... the hour hand sweeps between the numbers 4 and 5 over an hour. So at 4:20 is has moved a third of the way through this arc. Aah! The arc between 4 and 5 represents one-twelfth of a full 360°, which is 30°, and we want one third of this. The angle between the two hands must be 10°. Nice!

**Extension 1:** What is the first time after midday that the hour and minute hands make a perfect right angle?

**Extension 2:** If I pick a time of day at random, what are the chances that the smallest angle made by the hour and the minute hands of a clock at that time is acute?
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