

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 117: Hands of a Clock

By Dr. James Tanton, MAA Mathematician in Residence

What is the measure of the acute angle formed by the hands of a clock at 4:20 a.m.?

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Geometry: Angle measurements; Rates of Rotation.

COMMON CORE STATE STANDARDS

Connected to: **7.G.5** Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 9: [AVOID HARD WORK](#)

SOURCE: This is question # 20 from the 2003 MAA AMC 8 Competition.



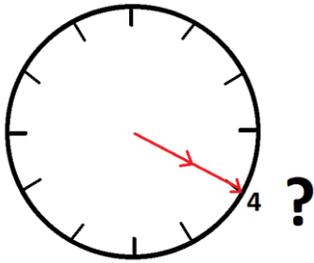
THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question looks innocent but I have a feeling it is complicated!

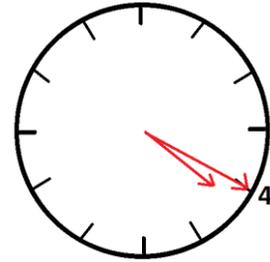
At the time 4:20, the minute hand is point at the twenty minute position, which is the location of the number 4 . And where is the hour hand is pointing at 4 :20? At the number 4 as well? (It the fourth hour.) The angle between the two hands is zero?



This can't be right!

Oh! The hour hand points at the number 4 only right at the hour of four-o'clock. It then moves slowly from 4 to 5 as the hour progresses.

The picture is more like this and there is a positive angle between the hands of the clock!



What angle is that?

Well ... the hour hand sweeps between the numbers 4 and 5 over an hour. So at 4 : 20 is has moved a third of the way through this arc. Aah! The arc between 4 and 5 represents one-twelfth of a full 360° , which is 30° , and we want one third of this. The angle between the two hands must be 10° . Nice!

Extension 1: What is the first time after midday that the hour and minute hands make a perfect right angle?

Extension 2: If I pick a time of day at random, what are the chances that the smallest angle made by the hour and the minute hands of a clock at that time is acute?

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