

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 123: At Least as Many Heads

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Harold tosses a nickel four times.
The probability that he gets at least as many heads as tails is ...

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Statistics and Probability

COMMON CORE STATE STANDARDS

8.SP.7a Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
- MP2** Reason abstractly and quantitatively.
- MP3** Construct viable arguments and critique the reasoning of others.
- MP7** Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 7: [PERSEVERANCE IS KEY](#)

SOURCE: This is question # 21 from the 2002 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

Ooh! This is an innocent looking question, short and sweet. But because it is so short and I am feeling nervous about it!

Okay. Harold tosses a coin four times. We want to know the chances that he gets at least as many heads as tails. (So, for example, something like HHTH is good for Harold, but TTTH is not.)

Hmm.

Well the chance of tossing a head is $\frac{1}{2}$, and to get at least as many heads as tails, he better have either two, three, or four heads.

There is one way to get four heads: HHHH.

How many ways are there to get three heads? How many ways are there to get two heads? They seem hard to answer.

Hmmm.

Maybe I could just list all the possible outcomes Harold could see. Here goes:

HHHH	HHHT	HHTT	TTTH	TTTT
	HHTH	HTHT	TTHT	
	HTHH	HTTH	THTT	
	THHH	THHT	HTTT	
		THTH		
		TTHH		

(I think I have them all. There is a total count of 16 of them and that seems right.)

Well, I see that there are $1 + 4 + 6 = 11$ of the sixteen that have at least as many heads as tails:

HHHH	HHHT	HHTT	TTTH	TTTT
	HHTH	HTHT	TTHT	
	HTHH	HTTH	THTT	
	THHH	THHT	HTTT	
		THTH		
		TTHH		

Oh! That's it! Harold will get one of these 16 results and 11 of them satisfy what he wants. The chances of Harold seeing at least as many heads as tails is thus $\frac{11}{16}$.

Extension: Harold tosses a nickel 5723 times in a row. What is the probability that he will see at least as many heads as tails? (How could we think about this question and avoid doing work?!)

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