

Curriculum Inspirations

Inspiring students with rich content from the
MAA American Mathematics Competitions



Curriculum Burst 124: False Relation

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Let a , b , and c be numbers with $0 < a < b < c$. Which of the following is impossible?

- (A) $a + c < b$ (B) $a * b < c$ (C) $a + b < c$ (D) $a * c < b$ (E) $\frac{b}{c} = a$

QUICK STATS:

MAA AMC GRADE LEVEL

This question is appropriate for the middle-school grade levels.

MATHEMATICAL TOPICS

Algebra and Number Sense

COMMON CORE STATE STANDARDS

6.EE.5 Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

MATHEMATICAL PRACTICE STANDARDS

- MP1** Make sense of problems and persevere in solving them.
MP2 Reason abstractly and quantitatively.
MP3 Construct viable arguments and critique the reasoning of others.
MP7 Look for and make use of structure.

PROBLEM SOLVING STRATEGY

ESSAY 10: [GO TO EXTREMES](#)

SOURCE: This is question #15 from the 2007 MAA AMC 8 Competition.



THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always ...

STEP 1: Read the question, have an emotional reaction to it, take a deep breath, and then reread the question.

This question looks a little scary because of all the algebraic expressions. But let me just take a deep breath and take in slowly what the question is actually asking.

Let a , b , and c be numbers with $0 < a < b < c$.

Okay. We have three numbers, just numbers, all bigger than zero, with a the smallest, c the biggest, and b in between.

Which of the following is impossible?

We're being asked to identify something that can't be so.

Let's just try an example of a set of numbers. But just to really test possibilities I am going to choose numbers that are extreme. Let's try:

$a = 0.00000001$, or any positive number really close to zero.

$c = 1000000000000$, or any ridiculously large number.

$b =$ something in between.

Here are the options we need to consider:

(A) $a + c < b$ (B) $a * b < c$ (C) $a + b < c$
(D) $a * c < b$ (E) $\frac{b}{c} = a$

With my examples (A) isn't working. (B) is working, (C) is working, (D) could work (for example with $a = 0.01$, $c = 100$ and $b = 5$), and (E) could work (for example, $\frac{1}{100} = 0.01$). So the answer must be (A) – it is the only option left that could be impossible!

Extension 1: What is the mathematical reason that makes option (A) impossible? If given three positive numbers a , b , and c , with $a < b < c$, why must $a + c$ be larger than b ?

Extension 2: Given two positive numbers a and b , must $a^2 + b^2$ always be greater than ab ? Must $\sqrt{a + b}$ always be smaller than $\sqrt{a} + \sqrt{b}$?

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