# Curriculum Burst 144: Ending with 23

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How many distinct four-digit numbers are divisible by $3$ and have $23$ as their last two digits?


## QUICK STATS:

### MAA AMC GRADE LEVEL

This question is appropriate for the lower high-school grades.

### MATHEMATICAL TOPICS

Number sense: Divisibility rules

### COMMON CORE STATE STANDARDS

**A-APR.1 (Tangentially)** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

### MATHEMATICAL PRACTICE STANDARDS

**MP1** Make sense of problems and persevere in solving them.
**MP2** Reason abstractly and quantitatively.
**MP3** Construct viable arguments and critique the reasoning of others.
**MP7** Look for and make use of structure.

### PROBLEM SOLVING STRATEGY

ESSAY 7: **Perseverance is key.**

**SOURCE:** This is question # 25 from the 2003 MAA AMC 10B Competition.

## THE PROBLEM-SOLVING PROCESS:

The best, and most appropriate, first step is always …

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| **STEP 1:** Read the question, have an emotional reaction to it, take a deep breath, and then reread the question. |

I feel like I get this question! We are looking for four-digit numbers of the form $ab23$ (here $a$ and $b$ are single digits) that are divisible by three. (Oh. And $a$ cannot be zero!) And I remember a divisibility rule for the number three:

> *A number is divisible by three precisely when its digits sum to a multiple of three.*

(See http://www.jamestanton.com/?p=1287 for a video on this and other divisibility rules.)

So all we need do is count the number of possible values for $a$ and $b$ so that $a+b+5$ is a multiple of three. Let's just work our way through it!

Case $a+b+5=6$: So $a+b=1$ and we have just the number $1023$. (One answer.)

Case $a+b+5=9$: So $a+b=4$ and we have the numbers $4023$, $3123$, $2223$, $1323$. (Four answers.)

Case $a+b+5=12$: So $a+b=7$ and we have $7023$, $6123$, …, $1723$. (Seven answers.)

Case $a+b+5=15$: So $a+b=10$ and we have $9023$, $8123$, …, $1823$. (Nine answers.)

Case $a+b+5=18$: So $a+b=13$ and we have $9423$, $8523$, …, $4923$. (Six answers.)

Case $a+b+5=21$: So $a+b=16$ and we have $9723$, $8823$, and $7923$. (Three answers.)

Case $a+b+5=24$: So $a+b=19$. This won't happen with single digits for $a$ and $b$.

Alright, we have $1+4+7+9+6+3=30$ four-digit numbers that fit the bill!

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| **Extension:** There are $9\times10=90$ possible values for $a$ and $b$ as a pair and $30$ of those pairs gave a four digit number that is divisible by three. Is it a coincidence that $30$ is exactly one third of $90$? |

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