In each case below, provide an appropriate example and explain your reasoning. If no such example exists, briefly explain why.

- Provide an example of a function f such that $f(a+b) = f(a) + f(b)$ for all real numbers a and b. Find all such examples.
- Suppose $\lim_{x\to 3} f(x) = 0$ and $\lim_{x\to 3} g(x) = 0$.
  (a) Provide an example of function f and g such that then $\lim_{x\to 3} \frac{f(x)}{g(x)}$ is equal to 5.
  (b) Provide an example of function f and g such that then $\lim_{x\to 3} \frac{f(x)}{g(x)}$ does not exist.
- Provide an example of a rational function $r(x) = \frac{f(x)}{g(x)}$ (where $f(x)$ and $g(x)$ are polynomials) such that $r'(x)$ is not a rational function.
- Provide an example of a function f such that $f'(3)$ exists but f is not continuous at 3.
- Provide examples of differentiable functions f and g such that $\frac{d}{dx}[f(g(x))] = f'(g'(x))$. Find all such examples.
- Provide an example of a function f and a real number c such that f has a tangent line at c but $f'(c)$ does not exist.
- Provide an example of a function f such that $\lim_{x\to 0} f(x) = f(0)$, but $f'(0)$ does not exist.
- Provide an example of a function f that is increasing and differentiable on (a, b) but $f'(x) \neq 0$ for all x in (a, b).
- Provide an example of a function g such that $g''(0) = 0$, but g does not have an inflection point when $x = 0$.
- Provide an example of a definite integral that cannot be evaluated using the Fundamental Theorem of Calculus.
- Provide an example of a function f, an interval $[a, b]$, and a real number $c \in [a, b]$ such that the Mean Value Theorem does not apply yet $f'(c) = \frac{f(b) - f(a)}{b - a}$.
- Provide an example of each of the following.
  (a) A nonconstant function f and an interval $[a, b]$ such that the corresponding definite integral equals 0.
  (b) A nonconstant function f and an interval $[a, b]$ such that the corresponding definite integral equals 2.
  (c) A nonconstant function f and an interval $[a, b]$ such that the corresponding definite integral equals -1.
  (d) A function f and an interval $[a, b]$ such that the corresponding definite integral does not exist.
  (e) A function f and an interval $[a, b]$ such that the corresponding definite integral equals infinity.
- Provide examples of functions f and g such that $\int f(x)g(x)\,dx = \int f(x)\,dx \cdot \int g(x)\,dx$.