MAA FOCUS

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MAA FOCUS Staff
Editor: Jacqueline Jensen-Vallin, *maafocus@maa.org*
Managing Editor & Art Director: Beverly Joy Ruedi, *bruedi@maa.org*

Letters to the editor should be addressed to Jacqueline Jensen-Vallin, MAA, 1529 18th Street, NW, Washington, DC 20036, or, by email, to *maafocus@maa.org*.

Subscription and membership questions should be directed to the MAA Customer Service Center, 800-331-1622; email: *maaservice@maa.org*;
301-617-7800 (outside U.S. and Canada);
fax: 301-206-9789.
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Advertising inquiries: *advertising@maa.org*

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From the Editor

I'm very excited about this issue of *MAA FOCUS*. Preparing for this issue, I realized that I had a number of submissions about issues of inclusivity in our community. This is one of MAA's core values—advocating for and celebrating diversity by promoting mathematics for all and broadening access through initiatives to engage diverse audiences. I hope that you will see these themes throughout this issue.

While compiling this issue, I wanted to give voice to groups of people who have been historically silenced or heavily edited. To this end, I have tried to only lightly edit most of the articles appearing in this issue. My goal was for people to be heard as *they* intended. Therefore, the opinions in the articles belong to the authors and not necessarily to the MAA as an organization. I have not managed to hear all voices, but my goal is to continue to listen and give access to groups historically under-represented in mathematics. We hope that sharing these opinions leads to open communication and a conversation about ideas.

Thanks to all of you who have contributed to this issue. I am grateful for those who lent their voices, thoughts, opinions, and ideas.

Jacqueline Jensen-Vallin

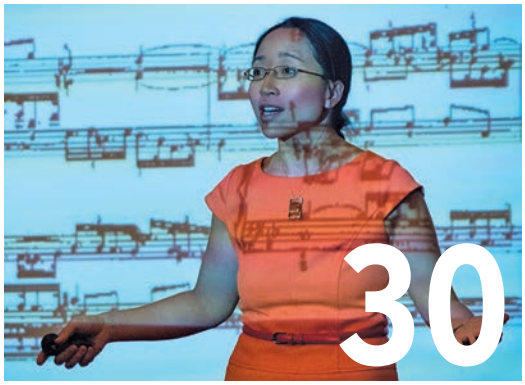
About the Cover
The image is courtesy of Frank Farris. Look on p. 5 of this issue for a note on the mathematics and philosophy behind the image.

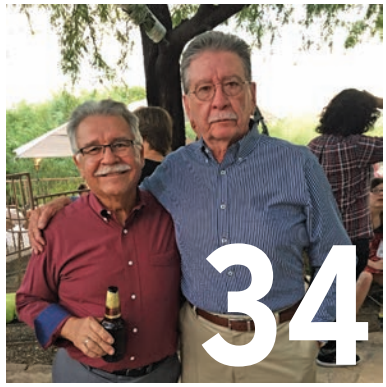
Photo Credit p. 24: PeopleImages, E+

MAA FOCUS

Mathematical Association of America

Vol. 39, No. 1 | February/March 2019

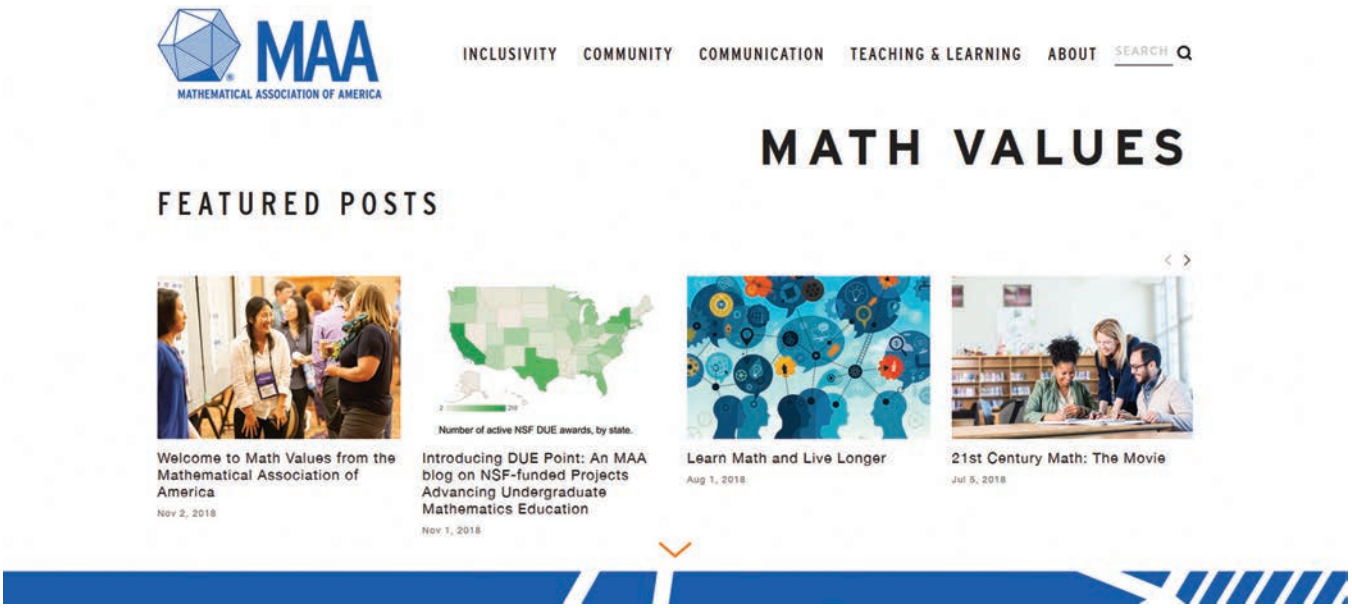




in this ISSUE

news

departments



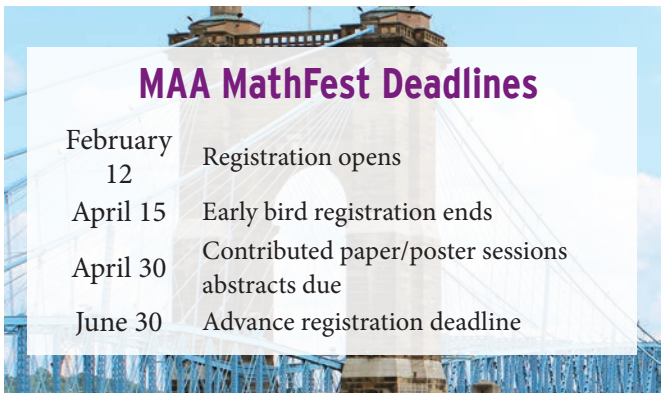
MAA Launches New Blog Platform

I am pleased to announce the launch of MathValues.org, the MAA's new blog platform. We're excited to begin this new journey to share fresh perspectives on issues important to our community.

The MAA has long provided a variety of mechanisms for the mathematical sciences community to share ideas, to learn from each other, and to enrich our professional lives. New technologies are driving cultural shifts in the ways in which we communicate with each other, and we're adapting to these changes. MAA's recently-adopted core values—community, inclusivity, communication, and teaching & learning—will serve as focal points for MathValues.org, We also expect to use the new platform to explore how mathematics supports new discoveries in fields as varied as oceanography, physics, and data science. MathValues.org will have some familiar faces such as Keith Devlin and David Bressoud plus many new ones as this blog explores the diverse voices of mathematics and discusses topics related to and affected by mathematics. Thanks to funding from NSF, we'll offer overviews of mathematics projects funded by the NSF Division of Undergraduate Education (DUE Point). And we'll look for opportunities to share views from colleagues across disciplines to illustrate the importance of mathematics in STEM fields and beyond.

I hope you'll find the new site attractive, and visit often to see what's new. And be on the lookout for other new initiatives in the coming months as we continue to embrace our mission to advance the understanding of mathematics and its impact on our world.

—Michael Pearson



MAA Project NExT
NEW EXPERIENCES IN TEACHING

MAA Project NExT is introducing new application cycle!

The first round of applications for the 2019 cohort of MAA Project NExT was due on October 15. New(ish) faculty who are already in full-time teaching positions should have used that deadline. Decisions were made by December 1. Those accepting positions during this academic year (to start Fall 2019) will have a second application deadline of April 15, 2019. For more information, see the MAA Project NExT website: projectnext.maa.org.

A Book Celebrating 20 Years of the EDGE Program

As part of Springer's AWM Book Series, in December 2019, *A Celebration of the EDGE Program's Impact on the Mathematics Community and Beyond* will be available—look for it at the 2020 JMM! Edited by Susan D'Agostino, Johns Hopkins University; Sarah Bryant, Dickinson College; Amy Buchmann, University of San Diego; Michelle Craddock Guinn, Belmont University; and Leona Harris, University of the District of Columbia, this book is a tribute to a program providing support and success for women in math graduate programs and future careers. Four of the five editors are former EDGE participants and one is a former EDGE instructor, all of whom earned PhDs in the mathematical sciences. D'Agostino, a member of the first EDGE cohort in 1998, was also the first EDGE participant to earn a PhD in mathematics (Dartmouth College, 2003), though there have been many more since.

From the book's proposal: "The EDGE Program (Enhancing Diversity in Graduate Education) began 20 years ago to provide support for women entering doctoral programs in the mathematical sciences. With a steadfast commitment to diversity among participants, faculty, and staff, EDGE initially alternated between Bryn Mawr and Spelman Colleges. In later years, EDGE has been hosted on campuses around the nation and expanded to offer support for women throughout their graduate school and professional careers. The refereed papers in *A Celebration of the EDGE Program's Impact on the Mathematics Community and Beyond* range from short memoirs, to pedagogical studies, to current mathematics research. All papers are written by former EDGE participants, mentors, instructors, directors, and others connected to EDGE. Together, these papers offer compelling testimony that EDGE has produced a diverse new generation of leaders in the mathematics community. This volume contains technical and non-technical works, and it is intended for a far-reaching audience, including mathematicians, mathematics teachers, diversity officers, university administrators, government employees writing educational or science policy, and mathematics students at the high school, college, and graduate levels. By highlighting the scope of the work done by those supported by EDGE, the volume offers strong evidence of the American Mathematical Society's recognition that EDGE is a 'Program that Makes a Difference'."

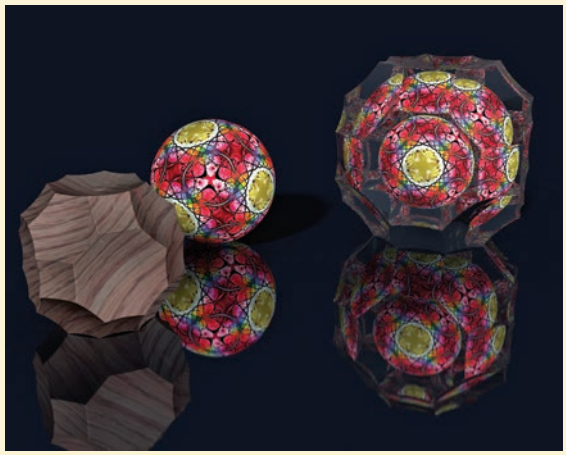
You may find information on the EDGE Program, which celebrated its 20th anniversary in 2018, here: www.edgeforwomen.org/.

About the Cover
Celebrating the Imperfect

Frank A. Farris

A perfect rhombicubeoctahedron has 26 regular faces: six octagons, eight hexagons,and twelve squares. That perfection is misleading. For instance, square faces might make it appear that we can rotate the solid through 90 degrees about an axis perpendicular to a face center and preserve the whole solid; no, the solid is only invariant under 180 degree rotations about such an axis.

In creating this work, I started with a process for creating geometric meshes from points placed at what would become the centers of polygonal faces. When I scaled the points slightly incorrectly, I defined a solid where the 12 squares turned out to be not quite square; the eight hexagons not quite hexagonal. At first, it bothered me that I had missed the perfection of the famous Archimedean solid. Now I see that it's something to celebrate: The slight imperfections can teach us about the symmetry group of the solid. (Thanks to members of our Bay Area Art and Math group for telling me not to reject this piece.)



The piece was staged in Photoshop using two spheres, two copies of the geometric mesh for the polyhedron, and a reflective floor. I colored both spheres with a pattern computed from a floral collage and a function of cube/octahedral symmetry, using techniques from my book, *Creating Symmetry, The Artful Mathematics of Wallpaper Patterns.* I colored one of the polyhedra with a wood grain, and defined the other to be made of crystal glass. About 24 hours of computer time produced the reflections and refractions. For me, the result illustrates how mathematical perfection shines through the imperfect lenses that we humans provide.

MAA Congratulates the National Association of Mathematicians on its 50th Anniversary

The National Association of Mathematicians (NAM) was founded in 1969. With membership open to all, NAM promotes excellence in the mathematical sciences while serving as a voice for underrepresented Americans in the mathematical sciences community.

The MAA is proud to join NAM to celebrate its 50th Anniversary Year throughout 2019.

The MAA and NAM have long shared the goal of broadening participation in mathematics, and we look forward to continuing to work with our friends and colleagues in NAM to continue this important work.

As usual, MAA MathFest 2019 will feature the annual David Blackwell Lecture. We're already planning to co-host additional events to mark NAM's Golden Anniversary. We hope you'll join us in Cincinnati on July 31–August 3 as we mark this milestone for NAM, celebrate its achievements, and join in discussions of how you can contribute to the critical work that still remains.

The NAM Board of Directors has initiated a Golden Anniversary Campaign to seek to endow some of the long-standing programs and activities that NAM sponsors annually. These programs and activities have encouraged the mathematical development of underrepresented American students and professionals, and enriched the entire mathematical sciences community. For more information, visit NAM's website, www.nam-math.org.

Joint Mathematics Meetings 2020 Proposal Deadlines

March 15, 2019	Invited Paper Session proposals due
April 15, 2019	Workshop, Panel, Poster, Town Hall, SIGMAA, and Other Mathematical Session proposals due

Check out your Member Library

Login in at maa.org, click on "My Profile," and then click on "Member Library." The library has recent MAA Notes volumes available for free download with more coming soon.

Warren Page Retires from CMJ Media Highlights

The November 2018 issue was the 200th consecutive issue of *The College Mathematics Journal* that Warren Page helped craft. He initiated the Classroom Capsules section in 1979 and served as *CMJ* editor 1984–1988. There were two significant changes in the January 1984 issue besides the editorial rotation: The journal took on its current name and, more relevant here, the first Media Highlights column appeared. Page has assembled summaries from his media correspondents with the assistance of many section co-editors ever since. He is stepping down from this position and Tanya Leise (who began as a correspondent in 2005 and has been a co-editor since 2011) took the lead starting with the January 2019 issue, the first in Dominic Klyve's term as *CMJ* editor.

To honor Page for his many contributions to the Association and especially to the *CMJ*, the MAA Board of Directors named him Editor Emeritus of the *CMJ*. Page responded "I am proud to be a member of an organization such as MAA that takes the time to honor the contributions of its members."

SCUDEM—SIMIODE Challenge Using Differential Equations Modeling

SCUDEM IV 2019 is a three member team event for high school and undergraduate students taking place over the period 1–9 November 2019 with Challenge Saturday being 9 November 2019 held at local sites in the United States and beyond. Registration runs from 1 September–25 October 2019. Learn more about SCUDEM at simiode.org/scudem, including all past problems, student submissions, and problem author comments; fun MathBowl questions (and answers!); and Faculty Development program. SCUDEM is sponsored by SIMIODE—Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations. All modeling scenario and materials for teaching and a rich community of support are FREE in SIMIODE. SIMIODE is supported by the National Science Foundation. Currently we are seeking local site host coordinators for SCUDEM IV 2019 and invite faculty to include their school as a local site. There is a host stipend and complete information can be found at simiode.org/scudem/2019/hosting.

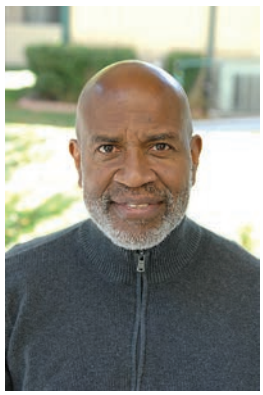
Candidates for MAA President and Vice President

The MAA will be holding elections for President and Vice President from March 21 to May 2. Voting will be all electronic. All members will receive an email with insructions (if they have an email address on file with the MAA). Members without an email on file will receive instructions by postal mail. Full candidate statements can be found in your member Video Library—log in at maa.org, click on "My Profile," and then Video Library.

Candidates for President

Matt Boelkins

In all of my work I would uphold the vision that mathematics is a vital human endeavor, that everyone benefits from doing mathematics, and that the MAA is making the world a better place through mathematics.



Lloyd E. Douglas

My priority would be on expanding our efforts that foster the connections with all those potential participants in our mission. I believe that by doing so, we can enhance the experience for those who are already engaged in our activities and broaden our reach to have impact on others and society.

Jennifer J. Quinn

MAA's strength comes from its members. I love the MAA—a "big tent" for mathematicians whether learners, teachers, users, or creators of mathematics. In working together, we shall advance the understanding of mathematics and its impact on the world, and add value to the lives of our diverse members.

Candidates for Vice President

Audrey Malagon

I'll use my experience, leadership, and strong community connections to address issues facing the MAA, including a changing landscape of employment and finding new ways to engage all members and broaden our support and impact. Building on past success, I will ensure the MAA is the organization where every mathematician has a place.

Tommy Ratliff

One of our major challenges is to provide opportunities for all members to meaningfully participate in our community as colleges depend more on non-tenure track faculty and offer less financial support for professional activities. Initiatives should be intentionally designed to expand our existing efforts to increase access and advance equity within the mathematical community.

April D. Ström

In my current role as AMATYC's Southwest Vice President, I see many opportunities to cultivate stronger collaborations among the MAA and community colleges that extend our efforts to build inclusive teaching and learning environments. I am eager to play a role in advancing the mission of the organization so that we continue to cultivate a sense of family, inclusion, faculty growth, love of mathematics among our students, and love of teaching among our colleagues.

News

MAA's Mission for the Second Century

Michael Pearson

Since our Centennial in 2015, MAA leaders, together with staff, have spent a lot of time thinking about how to sustain MAA's vital leadership role within the mathematical sciences. All of us recognize that the world we now inhabit is much different than existed when we were established. Mathematics was just as critical to society then, as it is now. So to think about the future, we recognize that it's important to reflect on the past.

In 1915, H.E. Slaught, at the time the editor of the *American Mathematical Monthly*, circulated a letter to ascertain interest in establishing a new society with the following main functions:

(1) To provide organized activity in the large field between the fields of secondary school mathematics and the field of pure research.

(2) To form a medium of communication and a forum for exchange of ideas between teachers and others interested in collegiate mathematics.

(3) To furnish a place for publication of scientific articles and papers adapted to this intermediate field.

(4) To publish historical articles, book reviews, notes and news, and indeed any matters of interest to the great body of men and women related to this field.

In December 1915, the inaugural meeting of the new Mathematical Association of America was held in Columbus, OH. The MAA's first century was certainly guided by these interests, recognizing that indeed it is rather a "large field" that existed, and still exists, between high school and research mathematics.

When the MAA was founded, what constituted mathematical literacy, and mathematical readiness for college or career, was much different than now. Much of this is documented in *The Mathematical Association of America: Its First Fifty Years.* Still, the efforts of the Association throughout its existence have reflected the concern with developing mathematical capacity in the broadest possible sense, and the high-quality exposition of mathematics that we still view is necessary for this purpose.

The mission of the Association remains consistent with our earliest years, but the world we inhabit has changed dramatically, and continues to do so. The expansion of the MAA in the second half of the 20th Century, following the Second World War, mirrors the tremendous growth in participation in postsecondary education during that period, as well as the increasingly-important role mathematics has played across STEM fields as well as other disciplines and professions.

Today, in fact, we are hard-pressed to identify any field in which the importance of mathematics is not felt, while new fields such as artificial intelligence/machine learning, built on mathematical foundations, are having dramatic impacts on massive scales in the lives of all citizens.

In such a mathematics-enriched world, the MAA has adopted a revised mission statement "to advance the understanding of mathematics and its impact on our world."

The adoption of the new MAA mission statement was part of a multi-year process of reflection on the role of mathematics, and especially the role of the Association, in our increasingly technology-soaked, data-driven, always-connected world. Through extensive internal study and conversations across our community, we came to realize that the **MAA must be more proactive and outward-facing**. We recognize how critical mathematics is to all sectors of society, but the role of mathematics is not nearly so obvious to all. And, unfortunately, access to opportunities to develop quantitative skills is not equitably distributed to all. The mission statement is meant to reflect our resolve to both better-communicate the role and importance of mathematics, and to embrace the shared commitment of all people towards stewardship of our world.

Our mission goes hand-in-hand with our newly developed core values: community, inclusivity, communication, and teaching & learning.

Community is reflected throughout MAA programs: meetings; SIGMAAs; the work of CUPM and other key MAA projects to develop guidelines and recommendations for our profession; outreach efforts; and partnerships with other organizations, including but going beyond the mathematical sciences. Our history is grounded in grass-roots work of our members. Section meetings and MAA MathFest, reflect the special character of our Association. As we continue to explore new ways to adapt to new professional challenges, we will look for new avenues to support our community.

Inclusivity has long been important to the MAA. While we can rightfully be proud of our substantial efforts to include women and other underrepresented populations in our discipline, and in Association activities, recent attention to sexual discrimination and barriers across society to welcoming all citizens have made it painfully obvious that there is much more to be done. We must reflect on structures and habits within our own departments and institutions, and ultimately become advocates for equity of opportunity for our fellow citizens regardless of gender, ethnicity, race, or sexual identity.

The role of communication to the MAA is long standing, and continues to be reflected through our publications

program. We believe that the quality of exposition in MAA journals is the best in the world, and the move to partnering with Taylor & Francis to market, produce, and distribute our journals was made with the full intent of improving access to our existing audience, while reaching new audiences internationally. This is fully in keeping with our refreshed mission, recognizing that our world is shrinking and the challenges we face as a society require us to develop and maintain dialogue with colleagues and partners around the world.

Recent publication of the *MAA Instructional Practices Guide* continues our long-standing commitment to improving teaching & learning of mathematics. In keeping with our values, the authors note that our work is ultimately a force for social change and, using evidence-based research on how we learn, calls on all of us to "help our colleagues improve and to collectively succeed at teaching mathematics to all students so that our discipline realizes its full potential as a subject of beauty, of truth, and of empowerment for all." Coming on the heels of the *2015 MAA CUPM Curriculum Guide*, the new *MAA Instructional Practices Guide* is a part of the rich fabric of our efforts to work collectively to improve our profession and build mathematical capacity.

Throughout the discussions we had to shape MAA's response to the rapidly-changing world in which our work is situated, we returned again and again to the ways in which mathematics shapes and enhances our own lives. Francis Su articulated this notion in his retiring presidential address at the Joint Meetings in Atlanta in 2017. The realization of how we experience mathematics, coupled with the ways in which mathematics's role in our world has evolved and grown, led to our adoption of our new vision statement, which was approved in final form in October 2018:

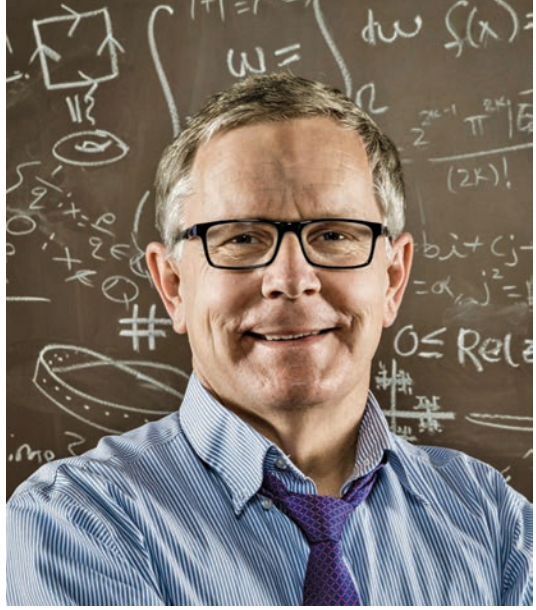
We envision a society that values the power and beauty of mathematics and fully realizes its potential to promote human flourishing.

We hope you'll join us as we seek to fulfill our mission, carry out our work in ways consistent with our values, work towards our vision, and be part of our efforts to ensure that MAA, too, flourishes for many years to come. ■

News

Michael Dorff, MAA President

—Jacqueline Jensen-Vallin



We welcome Michael Dorff (Brigham Young University) as the 58th president of the MAA. Dorff has long been involved in the BIG (Business, Industry, and Government) community of mathematicians, including being a co-founder of PIC Math. He is also well-known for supporting undergraduate research in mathematics, and being a founder of CURM (Council on Undergraduate Research in Mathematics).

Personally, I knew Dorff by reputation because of his support of undergraduate research in mathematics, and was excited to get to know more about him when we shared a lunch table at a conference a couple of years ago. As with many of the MAA leaders, whose reputations precede them, Dorff was approachable, interesting, and open to sharing thoughts and ideas about teaching, learning, and research.

We welcome Michael Dorff as the new president of MAA, and hope to give you an opportunity to get to know him through the following interview.

What was your path to mathematics?

Here is a surprise—it was through recreational math and math puzzles! When I was growing up, I was good at the math taught in school but it never really excited me. As an undergraduate student, I took more humanities and literature classes than math classes. Eventually, I needed to declare a major. A math degree required fewer classes than a humanities degree and I was good at math and I had some experience teaching, so I chose to be a math education major. When I graduated, I taught high school math in Los Angeles and then in Germany. I really liked teaching high school. I would give my students math puzzles to solve for extra credit. There was one puzzle that none of the students could solve. I did not know how to solve it either, so I spent one day working on it for several hours. I finally solved it by using some ideas from abstract algebra I had learned as an undergraduate. To see theoretical math applied to the math puzzle was great. That was the moment that I began to fall in love with mathematics. After teaching high school for several years, I began graduate school studying math.

What motivates you to serve the MAA?

Few people will remember me for my math paper on convolutions of planar harmonic mappings or for the Dorff Theorem (which does not yet exist). Instead, I will be remembered for my relationships with people. Now, I am by every online test a definite introvert. You may not think that if you see me give a talk or see me teach. I had to learn how to captivate students with math, how to give exciting talks, how to meet and talk with people I do not know. The MAA has helped me learn these things. I am a better teacher because of the MAA. I am a better presenter of math talks because of the MAA. I am a better organizer of programs that help faculty such as CURM and PIC Math because of the MAA. I want to have other people, especially people who might not have as many advantages as the rest of us have had, benefit from the MAA as I have. That is what motivates me to serve in the MAA.

Tell us about PIC Math and your experience introducing students to the BIG community.

Many math majors work in BIG (business, industry, and government) after graduating, and there are lots of jobs available for them if they have the right background and training. However, math faculty do not always have experience interacting and working in BIG. PIC Math was created to address these issues. It prepares math and statistics students for careers in BIG by engaging them in BIG research problems by increasing awareness among math and statistics faculty and undergraduate students about career options, providing research experience working on real problems from industry, and preparing students for BIG careers. The PIC Math program includes

- a three-day faculty summer workshop that provides faculty with information and training to teach a class that prepares students for industrial careers,
- a spring semester course during which students work on a research problem from industry and communicate their work while also learning skills to prepare for a career in industry, and
- a student recognition conference.

PIC Math is a program of the MAA and is funded by grants from NSF and NSA. In the first three years of the program,

PIC Math courses have been taken by over 1300 undergraduates at over 100 US universities and colleges.

My math background is in theoretical math (I don't like the term "pure" math). But around 2005 I realized there was a need for students to be better aware of career opportunities in BIG. I first became involved in connecting with BIG when I started the BYU Careers in Math speakers series (see article in MAA Focus, bit.ly/2Ff0jS7). In 2014, Suzy Weekes and I founded the PIC Math program. As a result of this I have talked with recruiters and employees at over 100 companies that hire math majors. I still love theoretical math, but I also find it fascinating how math is or can be used by Google, Nike, Walmart, FedEx, the Orlando Magic NBA team, the Field Museum in Chicago, Jane Street Capital, Habitat for Humanity, Youngstown Police Department, Chick-fil-a, Massachusetts General Hospital, and Disney.

Do you have plans to continue these interactions as President of the MAA?

Definitely. I will continue co-directing PIC Math while serving as president of the MAA, but I am not going to stop there. There are so many exciting things happening in mathematics right now that I am thrilled to promote as MAA president. Another one is data science. In comparison to faculty in other disciplines, mathematics faculty have been slow to become knowledgeable and involved with data science. There is a benefit for mathematics and statistics faculty and students to have a better understanding of data science. In June 2019, we will offer a 4-day data science training workshop for faculty (go to the MAA PIC Math website for more information, maa.org/picmath), and a group at the MAA are crafting a proposal for a series of data science faculty workshops addressing issues from incorporating data science examples in standard mathematics and statistics courses to creating data science minors/majors in mathematics and statistics departments. In addition to data science, there is a need for the mathematics community to emerge from our silos and interact with those in other communities, such as non-mathematicians and mathematicians in other countries.

In the past few years there have been too many important national STEM committees and organizations that do not include mathematicians, and this hurts the mathematics community (I was at a large STEM conference where someone remarked that STEM stood for science, technology, engineering, and medicine—math had been omitted!). During the past four years I have had the chance to travel to many developing countries from Cambodia to Rwanda giving math talks. Mathematicians in developing countries are eager for us to interact with them, and this is a valuable thing to do.

Finally, the mathematics community of the 20th century is beginning to reach out beyond its traditional borders to accept and include all groups of mathematicians. We are not anywhere near there yet. I am shocked by the stories I have

Michael Dorff with his five daughters.

been told about how female mathematicians, Latino mathematicians, and LGBTQI+ mathematicians have been treated. We are all mathematicians and even more importantly, people. As MAA president, I will continue the emphasis of Francis Su and Deanna Haunsperger of striving to make a community in which all mathematicians are treated with respect and given opportunities to grow.

What recent positive changes have you seen in the MAA?

Recently, the MAA has instituted some major changes. The governance structure has changed from a large Board of Governors to a small Board of Directors and a Congress; production and marketing services for MAA journals have been consolidated with Taylor & Francis; MAA books have been outsourced to the AMS; the AMS and the MAA have announced that they will no longer co-manage the Joint Mathematics Meetings starting with the 2022 JMM; etc. Some of these changes are really good. Some are necessary and the best possible change at this time. Some will need to be changed again in the future. This is how improvement happens—it is not stagnant or linear.

But changes are hard. Human nature likes things to stay the same. When significant changes are proposed, we sometimes think that we know better than the leaders who are proposing the changes. As department chair of a very large department, I have a few faculty members in my own department who are confident that they know how to best deal with a non-mathematical administrative problem and they are eager to tell me what I should do. Now, sometimes they do have good ideas which is why it is important to listen to people. But often they do not know all the background details. The changes the MAA has made were not hastily done. I know they were carefully researched and discussed over years (not hours or days). Personally, I think it is great that the MAA is moving forward in order to make a better MAA even though it will not always be easy. I am eager to see what great new ideas and programs that we as MAA members will come up with. I am excited about the future of the MAA. ■

News

Workshop Builds Diversity of Math Teachers' Circles

—Brianna Donaldson

In June 2018, teams of mathematics professionals from K–12 schools and higher-education institutions all over the country gathered at the MAA's Carriage House for a week-long Math Teachers' Circle (MTC) training workshop. MTCs, which began at the American Institute of Mathematics (AIM) in 2006, are communities of K–12 teachers and mathematicians who meet regularly to work on rich mathematics problems. The goals are to support teachers as mathematicians, to connect mathematics professors with K–12 education, and to build a K–20 community of mathematics professionals dedicated to improving mathematics education for all students. A growing body of research suggests that MTC participation increases teachers' mathematical knowledge for teaching, professional engagement, and use of high-leverage classroom practices that promote student learning. Teachers develop their identities as mathematicians, and at the same time, mathematicians develop their identities as teachers.

The MAA had previously worked with AIM to host similar MTC training workshops—seven of them from 2008 to 2014. These workshops, along with eight others hosted by AIM in California, helped launch dozens of MTCs across the country, forming the backbone of a strong national network that has continued to grow through mentoring, seed grants, and the coordinated efforts of regional leaders.

Building Capacity for Diversity

Last summer's workshop focused explicitly on building MTCs that are inviting, equitable mathematical spaces for teachers who work primarily with underrepresented students. The workshop had been months in the making, but in many ways it was years overdue. The growth and successes of the MTC Network to date had resulted from merely responding to demand for our program, rather than proactively seeking opportunities to increase access. So, when Michael Pearson approached AIM with the news that a generous donor to the MAA wanted to support increased access to mathematics for students from underrepresented groups, we were thrilled and eager to collaborate on a MTC training workshop that would support this goal.

For several months, we worked closely with the MAA and an incredibly dedicated and talented group of facilitators to advertise, recruit, and plan the workshop. Leadership teams

All of the photos are courtesy of Bob Klein.

came from cities all over the country: Merced, CA; Savannah, GA; New Orleans, LA; and Dayton, OH. A group from the highly successful Navajo Math Circles project wanted to build their long-term capacity to serve local teachers. The State Department of Education in Oklahoma sent a team to plan a statewide network of MTCs, with the full backing of the state superintendent and major public institutions of higher education.

The week's activities consisted of MTC math sessions, structured team planning activities, and group discussions and guest speakers focused on diversity, inclusion, and equity. Mathematical highlights included the following:

- Joshua Zucker (co-founder of the MTC Network) facilitated an introductory session on compositions. The compositions of an integer n are all the ways of writing n as a sum of strictly positive integers. Unlike partitions, order matters in compositions, which significantly increases their accessibility as a topic for investigation in a MTC setting.
- Jana Talley (Associate Professor of Mathematics at Jackson State University and co-founder of the Capital City Circle) led a session on the mathematics of the card game *Set*, focusing on a physical model of the game's geometric structure.
- Bob Klein (Interim Dean of Ohio University East, co-founder of the Southeast Ohio MTC, and Executive Director of the Alliance of Indigenous Math Circles) delighted the participants with Liar's Bingo, in which an intriguing mind-reading parlor trick motivates the use of a variety of problem-solving strategies. This allowed participants to practice organization, persistence, and making and breaking conjectures about the meaning of patterns.

- Pedro Morales-Almazán (currently at the University of California, Santa Cruz, and formerly a Lecturer at The University of Texas at Austin and organizer of the MTC there) led participants through several explorations that revealed structure in the randomness of the digits of π.
- David Scott (Emeritus Professor of Mathematics at the University of Puget Sound and co-founder of the South Sound MTC) led a session on gerrymandering, directly tying together mathematics and issues of social justice.
- Joshua Zucker closed out the mathematical portion of the week with Conway's Rational Tangles. It turns out that any rational number can be represented by completing a particular series of dance moves with a pair of ropes, and any fraction thus represented can be untangled using an algorithm involving the same dance moves—no matter how "knotty"!

Focusing on Facilitation and Equity

Throughout the week, Donna Fernandez (IB Coordinator for Navajo Preparatory School and Regional Coordinator for the Alliance of Indigenous Math Circles) expertly led participants through debriefing discussions after each math session, examining in detail how various pedagogical moves and interactions during each session contributed to—or detracted from—participants' overall mathematical experiences. Each facilitator brought their own perspective to developing norms of interaction and a sense of mathematical community among the workshop participants. During Zucker's introductory session, he created a list of problem-solving strategies that should be in everyone's mathematical toolbox ("Do something!" is a perennial favorite). Talley kicked off her session by having participants recite the Math Circle Pledge, which the Merced team printed on a banner when they got home: "I solemnly swear that if I already know the answer to the math circle problem, I will not yell it out loud but instead will challenge myself to take a different approach to the problem or think more generally about the problem so our whole Circle can experience the joy of learning." During Morales-Almazán's session, he introduced us to several principles of improv theater that also apply surprisingly well to creating a supportive culture for doing mathematics. For example, recognizing and building on what others say and do—using a "yes, and" frame—was the starting point for a hilarious trust- and communication-building exercise in which we co-constructed mini-stories a sentence at a time.

Building on the mathematics, a series of special events and guest speakers challenged us to take our thinking and planning to another level. We were treated to a special evening screening of "Navajo Math Circles," the moving documentary film by George Csicsery, enhanced by comments from several individuals who were at the workshop and also part of the work depicted by the film. Toya Frank (George Mason University) shared her research on factors that contribute to African-American math teachers staying in the profession, noting that belonging to a community and having opportunities to focus on mathematics are major needs expressed by the teachers she has worked with. Michael Young (Iowa State University) and Aris Winger (Georgia Gwinnett College), co-founders of the Mathematicians of Color Alliance, led an extremely personal discussion in which we collectively considered which students we were excluding from our mathematics classrooms.

The most exciting and energizing event of the week was the talk on "Dehumanizing Mathematics" by well-known mathematics educator Rochelle Gutiérrez in the MAA Distinguished Lecture Series. Speaking to a standing-room-only crowd, she focused on some of the myriad ways in which students can be systematically excluded from participating fully in mathematics. It was shocking, sobering, and ultimately inspiring. Participants also greatly enjoyed the opportunity to interact with Gutiérrez the next day, and she even joined us in the rope dance exploring Conway's Rational Tangles!

With the help of structured discussion guides, consultation from facilitators, and a variety of organizational tools and resources developed by the MTC Network, teams worked diligently on plans for their Circle's vision and goals, recruitment, meeting logistics, and budget and fundraising. Presentations about related programs, including student Math Circles, Julia Robinson Mathematics Festivals, the American Mathematics Competitions, and the MAA's Tensor-SUMMA grants program, provided additional food for thought regarding how teams might connect their work to students as well as teachers.

Following Up

The week concluded with presentations from each of the teams about their plans for beginning a MTC when they returned home. Overall, the level of enthusiasm was high. "This workshop not only gave our team the tools and resources to get our circle off the ground, it challenged my thinking about *what* mathematics is, *who* mathematics is for, and *why* we study it," wrote Scott Mitter (Math Teacher and Chair, Kettering City Schools, Dayton, OH).

As we were leaving, one participant told me that partway through the week, she finally felt comfortable enough to ask a teammate all her "stupid white people questions" about his experiences as an indigenous person, opening the door to the first of many deep conversations. This anecdote, and the week as a whole, truly brought home how superficial and pro forma our conversations about diversity tend to be, when in fact, just like mathematics itself, diversity is a fundamental part of being human that deserves to be shared and celebrated. Acknowledging and valuing our differences built the trust we needed to interact on an equal mathematical footing. A participant remarked that at the end of the week, she had no idea who was a "teacher" or a "mathematician"; we were all math professionals together.

I hope and believe that this workshop was just the beginning of our work together in developing a more inclusive and diverse community of MTCs nationwide. All the teams have already begun taking initial steps toward this goal, supported in part by seed grants made possible by the generous donation that funded the training workshop. The Oklahoma team even successfully applied for and received a major grant from the U.S. Department of Education to support their work. According to Julian Guerrero, Jr., who is Comanche and Kiowa and serves as the Executive Director of American Indian Education for the State of Oklahoma, "I did not realize how committed AIM and the MAA were to challenging the discipline of mathematics by demanding more equity and the reversal of dehumanizing people of color. I came skeptical, but am leaving energized to bring a set of tools to support, strengthen, and heal indigenous students in K–12 public education. This work must continue because it has to." ■

Brianna Donaldson has served as Director of Special Projects at the American Institute of Mathematics and Director of the Math Teachers' Circle Network since 2008.

EiM

Ethics in Mathematics

—Catherine Buell and Victor Piercey

Editor's note: *The pronouns ze and zir are used in place of gendered pronouns throughout this article.*

The use and misuse of data has been a topic of political discussion for the last decade or more, from Snowden's revelations about the U.S. National Security Agency to the Cambridge Analytica scandal. These scandals directly involve mathematicians and computer scientists, and raise ethical issues for mathematicians. In April 2018, two Cambridge University academics, Maurice Chiodo and Piers Bursill-Hall, and their students organized the first Ethics in Mathematics (EiM) conference. Participants included mathematicians, legal scholars, computer scientists, and philosophers. In their ranks were Turing Award winners, whistleblowers, scholars, and leaders in their field from the UK and Australia, and the US. The conference was recorded, so you can visit bit.ly/EiMRecordings to see the presentations.

Some readers might be surprised that ethics has a place in mathematics, or they may feel that their work in pure mathematics is too far removed from applications to have to worry about ethics. While we see an abundance of ethics courses in business, finance, media, criminal justice, biology, environmental science, psychology, philosophy, and computer science, they are often lacking in mathematics. Many STEM-oriented ethics courses use case studies to touch upon ethical issues in academia (publications, intellectual property) and industry (monetary questions or whistleblowing). However, they rely heavily on the fields intersecting mathematics. Why is a mathematical ethics course so rare? Are we all good, wholesome, naturally ethical people? Is mathematics in its purest form unconnected to ethics? The answers: we don't know, probably not, and a resounding no. One of the presenters at EiM, Anna Alexandrova, stated "No research, no matter how pure, is immune from social responsibility."

The authors came away from the EiM conference convinced that we need to begin a conversation about the role ethics plays in our profession and how to teach ethics to our students.

Ethics and the Profession

There are obvious issues concerning mathematics research and publication, such as plagiarism and self-plagiarism. Beyond these publication processes, there are serious issues to consider surrounding the substance of the work done in research.

The clearer cases for ethical issues in mathematical research lie in applied mathematical areas. Statisticians have to address what informed consent means for a group when conducting clustered trials. Part of the cause of the financial crisis of 2008 involved abuse of the Black-Scholes model for pricing financial options. As financial products become more sophisticated, should one who conducts research in financial mathematics model new and potentially dangerous products?

Recent concerns about ethical behavior address the use and abuse of data. With modern computing power, countless dimensions of our lives are reduced to bits of data. Some of the ethical questions have to do with viruses and malware. Vint Cerf, one of the founders of the internet, recounted at EiM that in the early days the goal was to have internet speeds as fast as terabytes per second, but should we proceed with development before minimizing risk? Besides concerns about viruses, there are important questions about our data. Who should have access to that data? What is the appropriate level of privacy one should expect for their data? How should that data be used? Recent scandals involving the U.S. National Security Agency, the U.K. Government Communications Headquarters, and Cambridge Analytica strike at the heart of these questions.

The issues raised above could be encountered in academic research as well as in work in industry. When working in a business environment, the nature of the relationship between employer and the researcher raises ethical issues dealt with across legal, medical, and mathematical professions. A mathematician can create a product that can be used unethically. Consider the following examples:

- If there is an algorithm that makes it easier for police to racially profile, should the mathematician who authors the algorithm consider this application in zir work?
- Suppose the results of a mathematician's work risk losing a client whose business keeps a significant portion of the firm's workforce employed. The pressure from clients may tempt a mathematician into falsifying data. What should the mathematician do?
- If a mathematician sees zir employer misusing data, should ze publicize zir observations?

So what about pure mathematicians and their research? Clearly, research in number theory has applications in cryptography. These applications raise ethical questions. Howev-

Buell speaking at the EiM conference at Cambridge.

er, to what extent is the pure mathematician responsible to society-at-large for the applications of their research? Whitfield Diffie raised this question in the context of applications which aren't conceived until 150 years after the conclusion of the original work. Mathematicians must nevertheless ask how pure one's work really is. Perhaps even more critical is to ask whose agenda one's work (no matter how pure) really serves, especially when grant funding is involved.

Ethics in Teaching

In academia, our teaching work also raises questions about ethical obligations attached to those duties. As with those teaching other disciplines, mathematicians must teach well and in an equitable fashion, and write letters of recommendations for students or advisees who are applying to work for employers whose ethical reputation is dubious.

However, mathematics plays a unique role in education that raises particular ethical issues. Often mathematics is used as a filtration device, separating students who can proceed to professions with high earning potential and those who cannot (see also the article by Bill Velez in this issue). This inappropriate use of the teaching of mathematics has profound implications for income inequality. Michael Harris and Reuben Hersch go even further, arguing that the culture of mathematics is one of elitism which turns the mathematics classroom into part of a system that replicates existing power structures. If this is the case, our profession must reflect on the ethical implications and the need for change.

Overall, there are questions that all members of the mathematical profession should ask:

- Whose interest is our work serving?
- How does our work impact existing power structures?
- Is a particular problem, no matter how interesting, pure, or abstract, worth dedicating our intellectual capital to as opposed to more pressing societal problems?

In his presentation at EiM, Whitfield Diffie challenged us to consider what is distinct about mathematics and the consequences of those distinctions for the application of ethics.

What is needed is an "ethical consciousness" among the mathematical profession. This frame of mind regularly grapples with these problems, including promoting thorough public discussion.

Teaching ethics to the next generation

James Franklin (University of New South Wales), a speaker at the EiM conference, wrote "How I taught the world's only ethics in mathematics course" (bit.ly/2Q8R0vD). A policy, instituted in the mid 1990's, required all undergraduates to take a course in "Professional Issues and Ethics" in their major. The class ended in 2012, but Franklin continued his work and created the Ethics in Mathematics Wikipedia page (bit.ly/2xdFh0P).

A stand-alone course at the undergraduate or doctoral level in mathematics could inform professional and academic mathematicians. Discussion of the ethics of the mathematics we create or use in both pure and applied mathematics serves to both ethically self-regulate and create a culture of discipline regulation. However, if ethical thinking in mathematics is deemed important, then having one course, just a checkbox on a course list, may not create the desired cultural shift. Much like reading and writing, ethics and social responsibility in mathematics would benefit from cross-mathematical-curriculum integration.

Pre-calculus, calculus, discrete math, linear algebra, computational mathematics, statistics … show us your ethics! Small, yet meaningful discussions or activities highlighting ethics historically or case studies (environmental justice in calculus or decision theory and algorithms) could support these efforts. A mathematics curriculum can be both rigorous and socially responsible.

We find the K–12 curriculum in the US modeling this integration of ethical conversations into mathematics to support learning, interest, and transference of knowledge. Examples can be found at the Chicago-based Math For a Change (bit.ly/2FPvSFD) and Radical Math (bit.ly/2rgv4Qd). Another EiM speaker, Paul Ernest, recently came under criticism for his analysis of K–16 education in a new textbook chapter *The Ethics of Mathematics: Is Mathematics Harmful?*, where he provided his "recommendation for the inclusion of the

philosophy and ethics of mathematics alongside its teaching all stages from school to university, to attempt to reduce or obviate the harm caused; the collateral damage of learning mathematics."

Colleges and universities do seem to be engaging students in ethics and social justice in mathematics classes aimed towards non-STEM students. Courses like *Mathematics in Society, Liberal Arts Mathematics, Quantitative Reasoning, Ethnomathematics* or social justice-themed courses like *Statistics* or a *First-Year Seminar* may provide students access to mathematics intertwined with social responsibility and citizenship. Why are these courses created this way? Here are some ideas.

- Desire for mathematics to be accessible and contextualized.
- Creating a society of critical thinking citizens with mathematical literacy.
- Personal engagement with material improves student buy-in.
- Culturally-inclusive content and pedagogy helps to engage a wider audience who haven't experienced success in mathematics classrooms.
- Mathematics is not necessarily rigorous, but still builds problem-solving skills and necessary math skills.

There are likely many more reasons; however, we'd argue that those mentioned above should be of importance to all STEM and mathematics students. So why aren't these courses part of their experiences? Typically, discussions surround the last item, where faculty suggest that a mathematics curriculum cannot be both mathematically rigorous and socially-responsible. This is a challenge of content and pedagogy, but also mathematical culture.

Piercey speaking at the EiM conference at Cambridge.

Challenges and an Invitation

Perhaps the first challenge is convincing the community that mathematics is not neutral. The world not being divided into "good and ethical people" and "evil and unethical people" and the gray areas and ambiguity may be difficult in mathematical culture. Both users and innovators of mathematics need to be aware of the equity issues encountered through mathematics research and teaching. Others may suggest that incorporating ethics or social justice will water down the mathematics, i.e., "this isn't a social science" course. We first must acknowledge the harm we may be causing math research and math education based on who or what we value to include and exclude.

We would like to see mathematics professional societies critically evaluate and expand their statements on ethics into a more comprehensive set of guidelines. This might be something worth exploring jointly. The ASA has guidelines (bit.ly/2zzllaV) that could serve as a model.

In Francis Su's January 2017 address on the Mathematics for Human Flourishing he says,

> I want us as a mathematical community to move forward in a different way. It may require us to change our view of who should be doing mathematics, and how we should teach it. But this way will be no less rigorous and no less demanding of our students. And yet it will draw more people into mathematics because they will see how mathematics connects to their deepest human desires.

If we do not include humanity, equity, and the desire to create a just world into a mathematics curriculum or mathematics class, then we are likely unable to achieve the goal of changing the culture or mathematics to one of inclusiveness. Truly, it is only one piece of the puzzle and it will take a village. Therefore, we invite others to join the community dedicated to ethics in mathematics and building a just world through mathematics. The authors are proposing a Special Issue of PRIMUS on Mathematics and Ethics and seeking articles and experiences on ethics courses for majors, general education, and undergraduate research. Talks and presentations at a future Joint Mathematics Meetings and MAA MathFest are in preparation, and of course EiM2. ■

Catherine Buell is an associate professor at Fitchburg State University where she conducts research across mathematics and mathematics education and teaches with a passion for social justice and equity. In addition to her duties at Fitchburg, she teaches in various Massachusetts Correctional Institutes. Victor Piercey is an associate professor at Ferris State University where he is the Director of General Education. Before becoming a mathematician, he was an attorney, and is dedicated to using his life experience to enhance the teaching of underprepared undergraduates with math anxiety.

News

Pursuing Gender Equity in Mathematics Competitions

A Case of Mathematical Freedom

—Jo Boaler, Montse Cordero, and Jack Dieckmann

There are two versions of mathematics. The version that most people know is one we think of as performance mathematics. It is a subject of short, closed questions with a set of methods students memorize to answer them. These are questions that fill school students' lives for many years yet they are questions that no student has ever asked. In the performance culture, students rate themselves according to how quickly and accurately they can answer the short questions. This then defines the kind of learner they are—or even the kind of person they are. When students are inside the performance culture they are unable to appreciate the beauty or meaning of mathematics. This performance culture is brought about by the testing, over-crowded standards and pacing guides imposed upon teachers and learners. Administrators keep the mathematics performance culture alive because, sadly, that is the only mathematics many of them know. They often cannot imagine anything else.

But there is another version of mathematics that can be made available to everyone—and once they experience it, they never see mathematics in the same way again. We think of this version as mathematical freedom. In contrast to the culture of performance, when students are in a culture of freedom they do not try to recall methods from memory; instead they think deeply about ideas and connections, they create mathematical pathways, and they communicate with others, reasoning about mathematical choices. Instead of just answering questions with a memorized method, students think: What ideas and connections can usefully be applied here? and how can I extend this thinking to new problems and questions that, to date, have never been asked? Mathematical freedom creates a desire to stretch further, towards more understanding, and towards new mathematical ideas and connections.

Most students do not experience mathematical freedom. Even students who are successful in mathematics and become mathematics majors may go through their whole mathematical career excelling at performance mathematics, for which they are celebrated. They never experience the freedom that comes from working on deep, visual and creative problems. In Jo Boaler's Stanford class called "How to Learn Mathematics" we meet undergraduates who are highly successful mathematics performers who are then invited to experience mathematics differently, engaging with mathematical freedom in a supportive atmosphere. The students collaborate, think visually, create new mathematical pathways, and connect ideas. This experience is transformational for them.

Knowing the value of engaging students in a culture of mathematical freedom, we were intrigued when mathematician Sol Garfunkel contacted us and asked us to study the modeling competition, run by the Consortium for Mathematics and its Applications (COMAP), he has led for many years. The MCM/ICM competition was taken last year by approximately 1400 students in the US, 60,000 in China and 300 from other countries. Sol had a particular question: he knew that mathematics competitions have not, in the past, been appealing to girls or women. As an example of this, every year a team of the most successful students who take part in the US Math Olympiads are invited to the International Math Olympiads. In the last 11 years, the US did not send a single female student. The Putnam exam does not keep gender records but a conservative estimate would suggest that 95% of participants and winners are men. The exclusion of women (deliberate or not) from mathematics competitions is a non-trivial issue, as engaging students in mathematics competitions could be an important way of attracting people to a career in mathematics.

Some people have thought that the low number of girls and women in mathematics competitions reflects the fact that women cannot or do not want to compete. The results of the COMAP modeling competition suggest something different: when students are offered the opportunity to collaborate, to experience multi-dimensional mathematics, and to engage in mathematical freedom, women enroll and women are successful. In the COMAP mathematics modeling competition, 43% of participants are women and 43% of winners are women. Together we posed the question—what is it about the competition that creates a considerably more gender equitable experience?

The Stanford Study

To study this question a team of researchers at Stanford conducted a multi-method study that included the following:

- Over 42 hours of observations of the competition in action in two universities
- Interviews with eight mathematics faculty in participating institutions
- Interviews with two participating teams of students
- A survey of 59 mathematics faculty
- A survey of 1327 competitors in ten countries.

Unlike traditional mathematics competitions, the Mathematical Contest in Modeling (MCM) and the Interdisciplinary Contest in Modeling (ICM), held simultaneously, focus on mathematical modeling. Students choose one problem from

either contest to work on. The MCM problems are primarily based on continuous mathematics, discrete mathematics, and data analysis. The ICM problems are based on environmental science, network science and policy issues. While both deal with modeling of real phenomena, the ICM problems require considerably more knowledge of non-mathematical disciplines (bit.ly/2zZ1dRg). Contestants work in groups of up to three students to solve a single problem over four days. The first task is to select the problem they would work on, out of the six options mentioned above. Examples of the problems offered to students included an analysis of different states' renewable energies, the cost of privacy, and trends in global languages. Students are allowed to access any (non-human) sources of information with citations. In order to justify their conclusion along with a one-page summary intended for a relevant audience, participants produce an extended document that showcases their mathematical work from the four days.

Analysis of the student data

As part of the interviews, participants were asked, "Why did you enter the competition?" The students talked about the different strengths they felt they could bring to the problem solving. They felt that the competition allowed them to bring their full selves to the table, and not be judged only upon narrow mathematics knowledge. A word cloud of the students' responses illustrates the different ways they anticipated contributing to their team.

The research team reviewed the interview and survey results and looked for patterns and themes within and across the different sources. The themes were then clustered to see how they related to each other, and from that analysis, a framework was produced. Analysis of the reasons high numbers of women participate in MCM/ICM drew from observations, interviews, and survey results. This analysis produced three themes that were central to the equitable outcomes:

- the collaborative nature of the competition,
- multidimensional mathematics and modeling,
- and freedom to create.

Each of the themes is described below.

Collaboration

In surveys and interviews with participants the opportunity to collaborate was a central theme. As one student shared:

> It involves a lot of collaborative work, allowing multiple people to have input in the solution to a problem. I'm a firm believer that better solutions are created when people work together.

Students reflected that collaboration was sometimes difficult but they knew that working to understand the thinking of their peers, and to express their own mathematical reasoning, and to build on each other's ideas was extremely productive.

Multi-dimensional Mathematics and Modeling

Many of the students talked about valuing the multi-dimensionality of the modeling they experienced. Students described being able to be creative, to use different mathematical ideas, and to apply their knowledge to interesting and real problems of the world. As students reflected:

> We were using knowledge from a bunch of different classes. And math classes overlap a little bit, but you don't really see the actual overlapping until you have to apply something like that.

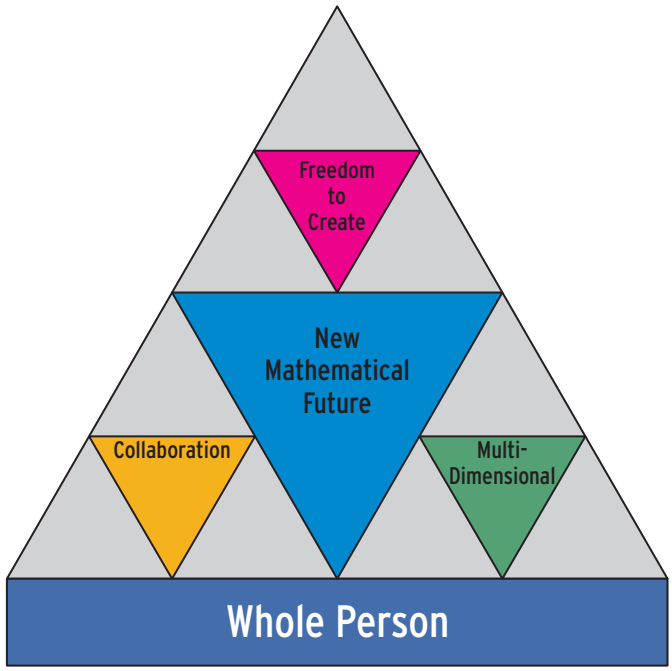
Another student reflected:

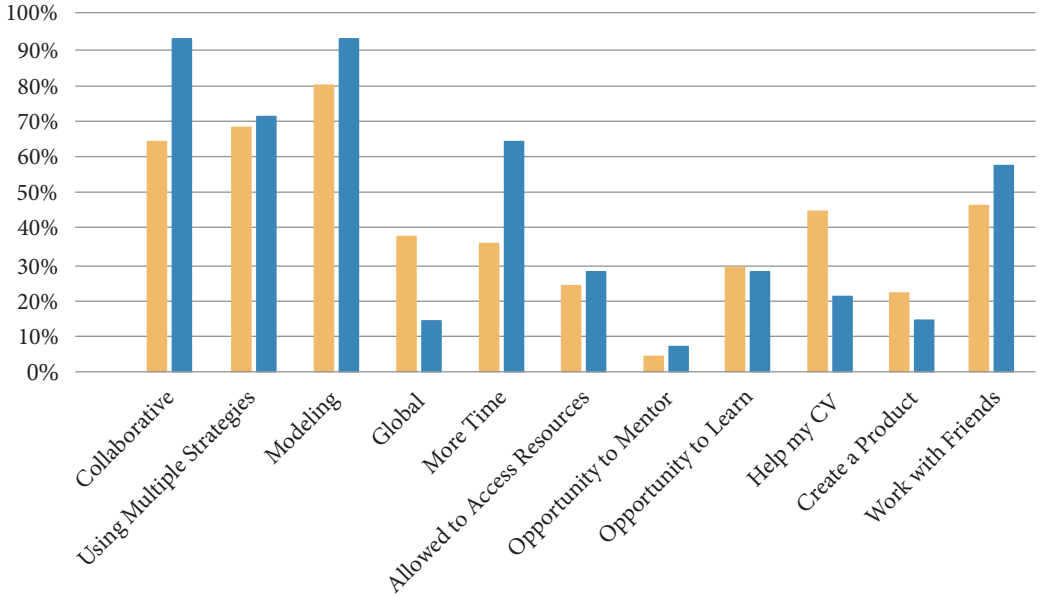
> I got an unforgettable memory during those four days. As a student who majored in math, it's the first time I have applied the knowledge to solving problems that are so close to real life which makes me very excited!

The students highlighted the opportunity to use ideas from across the domain of mathematics, something that had been missing from their previous experiences in mathematics classes.

Freedom to create

Even though students were working in a competition, many of them talked about the freedom they experienced to use mathematical ideas, and create, contrasting this with their





Answers to the question which aspects of the competition did you most value, by gender (Blue is female, yellow is male).

prior experiences of graded, performance mathematics:

> My favorite thing was definitely the feeling of using all of our resources that we've learned in our school for something that's not a test and not a graded project. But it was really interesting because we're really testing our skills that we know and that we've learned without being in that testing schedule. And that was a huge thing for me 'cause I've never done anything like that. And that was definitely, by far, the best part for me.

Whether the students were successful in the competition or not, they felt proud of creating a mathematical product that combined their thinking and problem solving skills, with sufficient time to feel the freedom to think deeply and creatively.

The students described how the three themes above gave them the opportunity to see mathematics differently, and to see a future for themselves in mathematics. From the 1327 students who completed the survey, 67% reported that their participation in MCM/ICM would "change their future pathways," as one student reflected:

> That was awesome … where I'm at now, it's just, I'm on the right track now, and I know that. It's no longer questioning: "What am I gonna do after college?" I have an idea, I know that's what I wanna do, and I feel good about it.

The three themes, resting upon the foundation of the students being able to contribute to their teams as whole people, contributed to a gender-equitable mathematics environment.

When students were asked which aspects of the competition they most valued, women were more positive about modeling, having time, and working with friends, and they were statistically significantly more likely to value collaboration ($p = 0.04$, $n = 1327$).

The finding that women particularly enjoyed working in a collaborative environment, with an opportunity to solve more open problems, and use multiple strategies, confirms previous research on gender equitable school environments (see, for example, Boaler (2002), Paying the Price for "Sugar and Spice": Shifting the Analytical Lens in Equity Research. *Mathematical Thinking and Learning*, 4(2&3), 127–144.)

In Conclusion

The United States has a serious math problem. Mathematics anxiety is rampant across the nation and mathematics classrooms in schools are often filled with uninterested students, who have turned away from mathematics. Inequities are more severe in mathematics than any other subject and the numbers of students qualifying with mathematical degrees is insufficient for the quantitative needs of the 21st century (Boaler (2016), *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching*. Jossey-Bass/Wiley: Chappaqua, NY). All of these problems come about, we would argue, because of the dominance of the performance culture in the teaching of mathematics. This culture is not only unappealing to most people but it systematically excludes women.

Mathematics competitions could be an opportunity for a different kind of engagement with mathematics. The Putnam exam and the Mathematics Olympiads both include beautiful questions with opportunities for exploration, and the requirement to give extended proofs and arguments. But the culture surrounding the examinations of a high pressure, individual, timed competition is unappealing to most women.

When students are offered the opportunity to challenge themselves through collaborative experiences where they address multi-dimensional mathematical problems, and engage in mathematical freedom, women enroll and women are successful. Importantly, 67% of the participants report that the competition changes their future pathways. One of the mathematics faculty members we interviewed reflected that: "MCM/ICM is a different kind of experience compared to math competitions like the Putnam. In my opinion, it is a more accurate reflection of what professional and academic mathematicians do (reading, writing, working in a team,

exchanging mathematical ideas, attacking problems that are not initially well-defined, spending time on a problem instead of having a shorter time period)." She added that she invites students into the competition to give them a taste of mathematics research. The students in the COMAP modeling competition were given the opportunity to engage in 21st century mathematics and for many of the students it changed their views of mathematics and their own sense of belonging in the discipline. In the mathematics competition landscape of harsh inequalities this case serves as a shining light that we hope others will appreciate and follow. ■

Jo Boaler is professor of mathematics education at Stanford University, co-founder of youcubed.org *and author of* Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching. *Jack Dieckmann is the Director of Research at Youcubed at Stanford. Montserrat Cordero supports curriculum, research, and technology initiatives at Youcubed at Stanford.*

Spring MAA Section Meetings

ALLEGHENY MOUNTAIN
April 5–6, Shepherd University, Shepherdstown

EASTERN PA & DELAWARE
March 23, King's College, Wilkes-Barre

FLORIDA
February 15–16, Polk State College, Lakeland

GOLDEN
February 23, American Institute of Mathematics

ILLINOIS
March 29–30, Southern Illinois University-Carbondale

INDIANA
April 5–6, University of Indiana, Indianapolis

KANSAS
March 29–30, Pittsburg State University, Pittsburg

KENTUCKY
March 29–30, Centre College, Danville

LOUISIANA-MISSISSIPPI
February 21–23, Mississippi, Clinton

MD-DC-VA
April 12–13, Hood College and Frederick Community College, Frederick

MICHIGAN
April 5–6, University of Detroit Mercy

MISSOURI
April 4–6, Lindenwood University, St. Charles

NEBRASKA - SOUTHEAST SOUTH DAKOTA
April 5–6, College of Saint Mary - Omaha

NEW JERSEY
April 13, 2019, Raritan Valley Community College

NORTH CENTRAL
April 5–6, 2019, Augsburg University, Minneapolis

OHIO
April 5–6, University of Akron, Akron

OKLAHOMA-ARKANSAS
March 28–30, Northeastern State University, Tahlequah

PACIFIC NORTHWEST
April 12–13, University of Portland, Portland

ROCKY MOUNTAIN
April 5–6, Fort Lewis College, Durango

SEAWAY
April 5–6, St. John Fisher College, Rochester

SOUTHEASTERN
March 7–9, Lee University

SOUTHERN CALIFORNIA - NEVADA
April 6, California State University - Channel Islands

SOUTHWESTERN
April 12–13, Western New Mexico University, Silver City

TEXAS
March 28–30, Tarleton State University, Stephenville

WISCONSIN
April 12–13, Carthage College, Kenosha

For the most up-to-date information on your section's activities go to maa.org/sections and click on the link for your section.

News

Mini Conference for National Research Experience for Undergraduates Program (NREUP) Participants

Last fall the MAA hosted a mini conference—Undergraduate Program & Diversity Focus Day—at the MAA Carriage House. Participants included 30 undergraduate students, faculty members, and STEM enthusiasts from Howard University (organizer: Dr. Dennis Davenport), Morgan State University (organizer: Dr. Leon Woodson), American University, and the University of Maryland. The mini conference was a new and exciting opportunity for participants in NREUP to share their research and experiences.

Participants engaged in thought-provoking activities designed to expand their understanding of mathematics and its impact on our world.

- Dr. Rachel Levy led a mini math modeling workshop that highlighted the importance of using mathematicsl approaches to solving real world problems.
- Dr. Emille Davie Lawrence (University of San Francisco) led a career Q&A session. She discussed her experiences as an Associate Professor and her work advocating broader participation in the mathematical sciences through outreach and mentoring.
- A group math challenge, "Switching Light Bulbs" led by Carrie Winterer, encouraged participants to employ creative and collaborative problem-solving methods to identify patterns in the light bulbs that are left on in various scenarios.
- Two MAA NREUP student research groups presented their research. The intimate atmosphere of the mini conference provided a low-stress environment in which students could strengthen their presentation skills.
- Focus group discussions brought to light several recurring themes centering around the topics of MAA programs, outreach, and diversity.

The MAA hopes this conference can be a model for future collaborations and gatherings. If your section would like to host a NREUP mini conference in the future, please contact programs@maa.org.

Top: Group photo of conference participants. Above: Shakuan Frankson and Myka Terry presenting "Investigating First Returns: The Effect of Multicolored Vectors." Bottom leftt: Dr. Rachel Levy during the Mini Math Modeling Workshop. Bottom right: Ethan Laurenceau, Ashley Oaks, and Jamen Rollins presenting "Number of Trees with n Boundary Edges."

Departments

TOOLKIT

Making Uncomfortable Conversations Productive

—Dave Kung

The College President's email takes you by surprise. Maybe it's about an overtly racist incident on campus, or homophobic graffiti splashed across a bathroom, or a sexual assault that happened at a big campus event. You finish the note, written carefully in the language of administrator-ese, and turn to prepare for your classes. What if students want to talk about the incident in class? What if most don't—but a few are clearly upset? How should you deal with what might be a very uncomfortable conversation—for both you and your students?

Issues of race, gender, sexuality, and disability status (to name just a few) arise constantly in our culture, including on your campus and in your classroom. Occasionally, the worst of human prejudices bubble to the surface, prompting administrators and diversity coordinators to mobilize. More frequently the subtle offenses slide by without ruffling too many feathers: a calculus study group talks over their lone female member; a teacher's instruction to "talk to a neighbor" leaves a transgendered student further isolated; a #BlackLivesMatter shirt draws annoyed looks and whispers about how #AllLivesMatter.

If we want to move forward, toward a world with greater understanding and inclusion, we have to be willing to facilitate hard conversation—especially when they are uncomfortable. Here's some tips for doing so.

Don't shy away

At a recent inclusion workshop, a math professor responded to a scenario about race by saying that he'd prefer to focus on the math and not get political—he'd guess that most of the class would prefer that too. Even if a few people weren't ready to move on, that's what he'd want to do. The people of color in the room read the subtext: he was willing to move on if all the white students were with him. That's not equitable—a point he eventually agreed with. In our MAA Project NExT sessions we pitch it this way: those moments are already political. Shying away from a hard conversation *is* a political choice. You don't have the option to avoid the controversy, but you can do your best to make sure everyone has an opportunity to learn.

Set the stage

Talking about issues like race and gender are uncomfortable in part because they are not usually discussed in a math class. Some instructors disrupt that expectation by addressing those issues on the first day of class—I intentionally take the edge off by talking about my own biases. The first day, my students hand me an index card containing their name (and other info). I talk briefly about how research shows that math teachers call on men more than women, that they ask men more theoretical questions—patterns that hold for female teachers as well. To overcome my own implicit biases, I typically ask a question of the whole class, then rifle through my stack of cards and address the question to a randomly chosen student. I acknowledge that my "cold calling" can make some students uncomfortable and let them "pass" if they need to, but when my students understand that it makes my actions more equitable, they appreciate it.

In short, **my biases are not my fault, but they are my responsibility**. In a highly interactive class, where student-student interactions are more likely to be laced with unintentional slights, it's even more complex: **My students' biases are not my fault, but they are my responsibility**.

Acknowledge the discomfort

If a particular topic makes you uncomfortable, your students are probably in the same boat. You can sand down the hard edge by simply acknowledging everyone's discomfort. "We're going to talk about what happened on campus yesterday, and that makes me feel pretty uncomfortable. I'd rather just do math. But I know some might not be able to concentrate on the math until we clear the air. It takes time and effort on everyone's part to understand how to think about such incidents, and I hope you'll join me in having an open, respectful conversation." And if you want to better understand the origins of your discomfort, Robin DiAngelo's work on *white fragility* would be a good place to start.

Take a minute

If something boils up in your classroom and tension mounts, it can be really challenging (and sometimes counterproduc-

tive) to start a conversation. You can cool things down (and buy much-needed time to think through your next steps) by asking everyone to take a minute to write down their reactions—both thoughts and feelings—to what just happened. While they write, you can make a plan, phone-a-friend, or seek out another lifeline.

Set the ground rules

If your class includes lots of dialogue, you've probably already established discussion rules. It's easier to get everyone to "listen to all voices" and "assume positive intentions" if those are social norms that have been in place from the beginning. When launching into a difficult conversation, it's useful to remind everyone of the rules—or lay them out for the first time. When something particularly incendiary comes up, I like to borrow a trick from the therapy and conflict management world: "Please try to use I-statements (as in 'I felt offended when…') instead of you-statements ('you offended me')."

Validate, validate, validate

When talking about strong feelings (let's be honest, not a strength for us mathematicians), people will open up more if they feel listened to. Your response can give them that, even if you disagree with what they're saying. "It sounds like you're really angry, and that what she said really upset you. Thanks for sharing. I don't think she meant that white lives don't matter. Let's try to clarify."

Get help

Many difficult conversations bring up deep-seated issues, ones that you may not feel equipped to handle. Your institution's counseling center or diversity office should be willing to help, as are many colleagues across campus with more experience facilitating difficult discussions.

Thankfully, dealing with touchy issues does get easier with practice. Maybe if we all practice facilitating such conversations, we can move to a place where they aren't so desperately needed. ■

Dave Kung is a professor of mathematics at St. Mary's College of Maryland and serves as Director of MAA Project NExT.

Note for some readers

Most of what I've written above comes from a straight, white/Asian, cis-gendered (male) perspective. That's not surprising, given my own identity. Did you notice? I'm guessing most of the women and minorities reading this did. If you were able to read this whole essay thinking that it was written from your perspective, that's another form of privilege you have that others might not. Even when we discuss prejudice, we do so within the biased confines of our culture. Discussions about identity issues can play out very differently depending on your own identity. Confronting a male student who talks over his female classmates will be different for faculty of different gender identities. That student is likely to respond differently to a female professor—for exactly the same reason his actions are problematic in the first place. He (like most of us) has learned social norms that are highly gendered. Thinking that it's his right to interrupt, like thinking that this essay is written from a perspective similar to your own, is a privilege that's not granted equally. To change the culture and even the playing field, it requires all of us—especially those in positions of privilege—to facilitate difficult conversations.

Recommendations for MAA Committee Members!

It's that time of year again, time to recommend a colleague (or yourself!) to serve on one of our many MAA committees in 2020. The form for making such recommendations is easy to complete. Just visit

www.surveymonkey.com/r/MAAcommitteesuggestions

to make a suggestion!

More information about the various MAA committees and the overall governance structure of the Association appears at www.maa.org/about-maa/governance. There are many ways to serve—committees exist with a focus on prizes, special lectures, meetings, publications, student activities, and continuing attention to the profession and all aspects of the curriculum and our community. So please take a moment to identify a niche and make a recommendation!

Math Tidbit

Given that 2017 was the last prime year and 2027 is the next prime year, 2019 = 3 × 673 is not a prime year. However, 1/1/2019 (112019) is a prime day. And, there are six other prime days during the month of January, 2019. The other days are:

1/3/2019 (132019)
1/4/2019 (142019)
1/8/2019 (182019)
1/22/2019 (1222019)
1/30/2019 (1302019)
1/31/2019 (1312019).

Thus, there are 7 prime days in the month of January, 2019. This begs the question, how many months have a prime number of days?

—Asamoah Nkwanta, Chairman & Professor
Morgan State University

Beauty, Numbers, and Literary Analysis Intersect at The Museum of Mathematics

Bonnie Ponce and Timothy Ponce

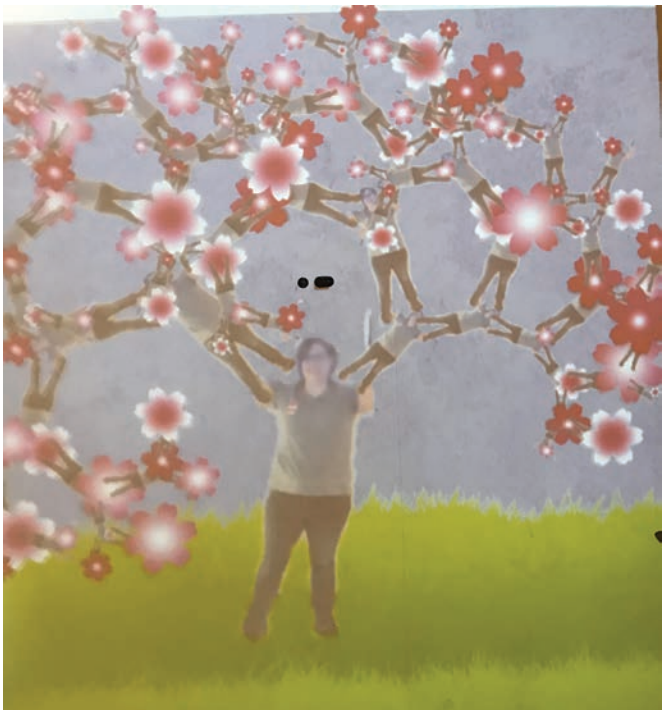
Nestled between Midtown and Lower Manhattan in New York, you will find a gateway to a world of intellectual inquiry, a door to a space where the most abstract of concepts become grounded in the tangible. We speak of The Museum of Mathematics, better known by the moniker MoMath. As Manhattan's only hands-on science center to date, MoMath seeks "to enhance public understanding and perception of mathematics (momath.org/about/)." After finding MoMath on several "what to do in New York" websites, Tim and I decided that we needed to go. We looked forward to visiting a space dedicated to teaching the general public about math.

But what exactly is a museum of mathematics? What could such an institution house? Those were the questions we had when the museum appeared on a travel website's list of best Manhattan museums. As the Managing Editor of Journals for the MAA (Bonnie Ponce) and a Lecturer in the Department of English at University of Texas Arlington (Dr. Timothy Ponce), we were not sure if the museum would be worth our time. After all, what could they possibly have on display that would interest either of us?

With open minds, we placed the museum on our travel itinerary. And we can both say that we were glad that we did. From capturing the beauty of the natural world to parallels with literary theory, MoMath shed light on our interests, non-mathematical though they be.

An Editor's Perspective

When I began assisting Dr. Scott Chapman in 2011 with *The American Mathematical Monthly,* I didn't know much about the world of mathematics. Dr. Chapman assured me that I would learn to appreciate the mathematics as I went along, and suggested that I begin reading some of the articles for enjoyment rather than for solely editing purposes. I took his suggestion and began to read *Monthly* articles that sounded interesting to me.



And Dr. Chapman was not wrong. As I read more and more, I realized that math exists all around us, making our world one of patterned beauty. One article in particular significantly impacted my idea of beauty in mathematics. Koukoulopoulos and Thiel's "Arrangement of Stars on the American Flag," published in the June/July 2012 issue of the *Monthly*, opened up an entire perspective of mathematics that I had never considered—that math sheds light on the beautiful complexity all around us if we only have eyes to see it.

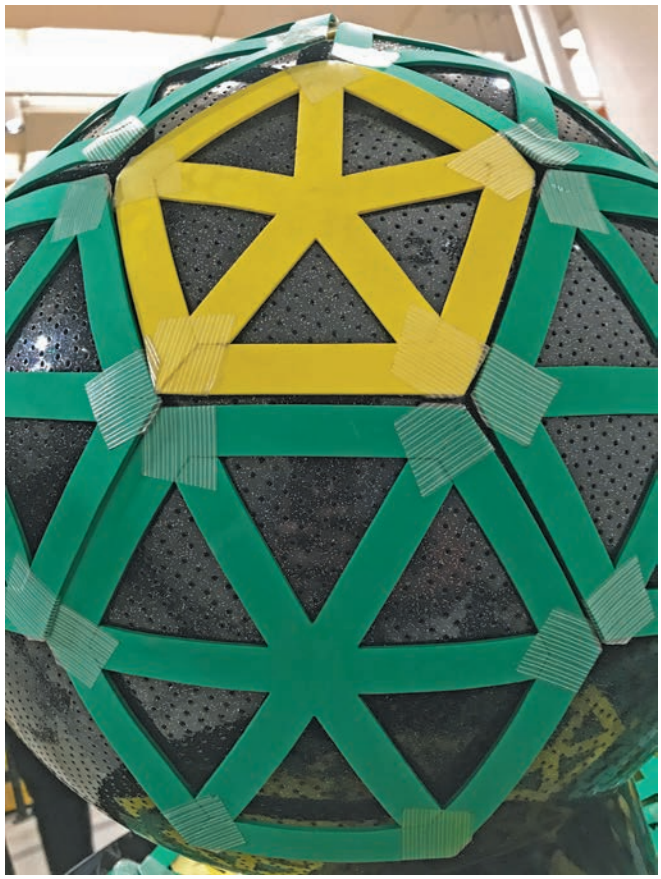
I hoped MoMath would intrigue me as that article had. I was not disappointed. It was exhilarating to see visual representations of mathematical topics, equations, and themes that I had read about. For instance, I had recently read several papers about fractal trees and I was excited to find that MoMath had an exhibit on fractals. In The Human Tree exhibit, a camera captures your image and movements, projecting you on a screen as a fractal tree. The more you stretch and move, the more complex the pattern. This exhibit literally brought the fractal trees to life, allowing me to create beautiful patterns with the stretch of my arm or the twist of my hand.

However, I think my favorite exhibit was the Finding Fifteen game, which took age old pattern-based games from my childhood to a new level. As a child, I never understood the complex patterns that lay beneath the surface of such simple games. But once I understood the mathematics behind the game, I gained a new appreciation of beauty birthed from its complexity.

All in all I found the MoMath Museum fascinating and engaging. It was a great opportunity to once again have my idea of mathematics turned on its head and see what math can look like, literally.

An English Lecturer's Perspective

Although it would be easy for one to assume that mathematicians and literary critics have little in common, both seek very similar knowledge. Part of the mission of MoMath—and I would argue the field of mathematics as a whole—is to help a "diverse audience to understand the evolving, creative, human, and aesthetic nature of mathematics. (momath.org/about/)" In other words, mathematics is not to be understood in a vacuum, but instead as part of a system of interactions between humans and the world around them.



In much the same way, literary critics interact with texts. John Milton, in his 1644 treatise *Areopagitica*, beautifully captures this human-text interaction by asserting that books "are not absolutely dead things, but do contain a potency of life in them to be as active as that soul was whose progeny they are." Thus, each person who reads a book will experience that book differently, much the same way that each student in a class has a different perception of their professor.

One exhibit at MoMath, the *Twist 'n' Roll*, captured this reality of reader/text interaction in a beautifully tangible way. The exhibit consists of a slightly slanted table top with various "tracks" printed on it, each curving and twisting in a unique direction. The participant is then asked to roll several objects provided down the table top to try and match the object to the correct track. Bonnie and I soon discovered that the only way to make any of the objects match their track was to take them apart, twist the pieces, and put them back together, yielding a new object. This is very similar to the interaction of text and reader. Just as each person who reads a text has a different interaction with it, so also does each object (despite being rolled down the same incline) produce a unique response.

Seeing more and more connections between literary studies and mathematics with each new exhibit I visited, I decided to take a break and spend some time at the museum's *Tessellation Station*, a gigantic magnetic wall with unique tiles available to make patterns. Standing side by side with two third grade students on a field trip, I began to make designs with the tiles. To my embarrassment, my fellow Tessellation enthusiasts were creating far more beautiful and complex patterns than I.

Frustrated, I stepped back to see my project anew. It was at this moment that a profound truth struck me. This attempt to "re-see" something from a new perspective—referred to as "revision" in the writing community—is truly the heart of all intellectual inquiry. There have been many moments in my research where I have found myself frustrated. Yet that frustration fueled my passion to re-see the subject from a new vantage point, opening up new possibilities that had been previously obscured. To paraphrase Queen Kathrine from Shakespeare's *Henry VIII,* we allow our "drops of tears" to "turn to sparks of fire."

From displays of beauty to parallels with literary studies, MoMath has something for everyone. If you ever find yourself in Manhattan with a free afternoon, know that it can be well enjoyed at the Museum of Mathematics (11 East 26th St, NY, NY 10010). ■

Bonnie Ponce is the managing editor for journals at the MAA. She can be reached at bponce@maa.org. *Dr. Timothy Ponce is a lecturer in the Department of English at University of Texas Arlington.*

Inclusion-Exclusion in Mathematics and Beyond

Who stays in, who falls out, why it happens and what we could do about it

Eugenia Cheng

Eugenia Cheng speaking about mathematics and Bach at the School of the Art institute of Chicago. Photo Credit: Paul Crisanti of PhotoGetGo.

I have had a somewhat unusual career path. I started out as a standard math professor at a standard research university. I did the usual things: an undergraduate math degree, a masters, a PhD, several post-docs around the world. Then I achieved the holy grail of academic careers: tenure. After living in three countries in four years I could finally settle down. And then I left.

I now teach art students at art school part time, and spend the rest of my time writing popular math books, touring the world speaking publicly, visiting schools, working with teachers, making art, performing piano concerts, and doing more math research than I ever had time to when I was a full-time professor.

Throughout all this there has been the added detail of being a woman in the male-dominated realm of mathematics. When I was starting out I didn't want to talk about this as I didn't want to draw attention to it. However, I now think it is important to talk about it, and moreover, that over the course of my unusual journey I have learnt some things that I should share as widely as possible in the hope of helping other people.

Background

I used to teach math majors at research universities: the Universities of Cambridge, Chicago and Sheffield. They were all students who had done very well in high school math, and knew they wanted to go

into careers in math and math-related fields: science, engineering, finance, accountancy. It was very male dominated. The last Honors Calculus class I taught at the University of Chicago was less than 30% female, and this was in 2014.

I gradually came to decide that while the tenured professorship was what I'd been working towards for about ten years, it wasn't making the best use of my abilities. I have been brought up to believe that it is important to figure out what you're good at and figure out how to make best use of your combination of abilities to do something worthwhile for the world. I decided that meant I should try to reach more people than those who already like math, so I started doing more outreach and popularisation work, and this eventually lead to an unusual teaching position at the School of the Art Institute of Chicago.

This is an elite art school, but it is a liberal arts school so there are people there teaching some of everything. Students are required to take two science classes (and math counts as science), but they could instead choose to take molecular biology, quantum physics, waves, or various other science classes. My classes are not "remedial math" as many people assume, but are about highly abstract mathematics as a way of thinking. I teach category theory to art students.

These are mostly, but not exclusively, students who did not do well in high school math, and mostly ended up hating or fearing it, or both. They will have careers in art, design, photography, fashion, film-making, and so on. And as it happens, my classes are very female dominated, with typically less than 10% of my students being male. (The figure is around 40% in the school as a whole.)

Learning about math trauma from my art students

One of the wonderful things about teaching art students is what I learn from them. Teaching at its best is a two-way process, with students learning from teachers and teachers learning from students. My art students are very serious students, but they just happen not to be math students. They are very interested in thinking, and they are very self-aware, so I have the opportunity to talk to them in depth about their past math experience, to discover what put them off in the past and what can reverse that.

One of the first assignments I give them is to write about their past math experiences. There are some clear themes that emerge from this about what put them off math in their earlier education: memorization, especially times tables and formulae; getting things wrong; a false dichotomy between art and math; and a perceived lack of creativity in math and science.

This makes me sad because they're almost the same reasons that I love math, but turned on their head. I loved math the most at school because I *didn't* have to memorize anything —I could just understand it and then work everything out. I never memorized my times tables (but I did understand them enough to be able to do them quite fast). Math at school does tend to be about right and wrong answers, but not when you get to a research level, where much of it is more about constructing different worlds in which different things are true and building rigorous justifications. And I know that math is creative, it just is often not taught that way.

Re-thinking my story

My own experience with math goes something like this: when I started as a grad student I decided I should hide all aspects of femininity as much as possible, so as not to give anyone a chance to stereotype me and say I was no good because I was a woman. Once I was tenured I felt more secure and started allowing some femininity in, but only outside work. This created a strange double life, so I tried to merge the two and be feminine at work too. Then I realised I didn't like my job, and left. (Note that in reality it was less abrupt than that, but that might be what it looked like.)

A few years on the other side I realised that none of this makes sense: in fact everything I do is "feminine" because I am a female person. We as a society ascribe genders to character traits in a way that is outdated and obstructive. Men are put under pressure to "man up," not show their emotions, to endure pain and not do anything too "girly" like sing, dance, or drink pink drinks. Women are advised to be "more like men" in order to be successful: be more ambitious, risk-taking, have more self-confidence. But when they do they get accused of being aggressive, angry, or "masculine" as a criticism. I believe that very few people benefit from this.

New Language

I came to realise that we need new language in order to separate character traits from gender, and have less divisive conversations in which people don't have to get defensive about "not all men" or "not all women" being a certain way. Because indeed men are not all the same and nor are women. We need a way to talk about character traits without reference to gender, and new axes along which to measure them.

I have come up with the following words: "ingressive" and "congressive." The etymological idea is that "ingressive" is about "going into" things, and "congressive" is about "bringing things together."

The first important point is that I am not claiming that this is a clean dichotomy. People aren't just one or the other. They can be somewhat one and somewhat the other, under different circumstances. Importantly these are also not fixed characteristics, but just types of behaviour and like all behaviour they can be learnt. I realised that I have spent most of my life learning to be more ingressive to be successful in an ingressive world, but that I didn't like myself that way. In my new portfolio career, I have been unlearning that ingressive

Eugenia Cheng exploring math with a convergent sequence of scones at the University of Sheffield, UK.

behaviour and finding that I can be successful while remaining true to my congressive self. I think it's more like a 2-dimensional plane and we can be anywhere on it.

It's also important to note that these can be both character traits of individuals and traits built into systems, just like sexism and racism.

Characterisations

Here are some ways that I characterise ingressive and congressive behaviour.

Ingressive: competitive; enjoys winning and being right; calls out answers; likes solving problems; overestimates own abilities; acts in own best interest.

Congressive: collaborative; dislikes losing and being wrong; prefers situations in which there are no winners and losers; reluctant to speak in class; likes discovering, creating and building rather than solving problems; underestimates own abilities; acts with the community in mind.

I suspect that more ingressive people are more likely to enjoy high school math, whereas more congressive people are more likely to be put off by it, with all its tests, exams and emphasis on answers and problem solving. I myself didn't enjoy many aspects of pre-university math classes except when we had open-ended projects for exploration.

Congressive math

The new terminology has helped me focus on how I want to change the way I teach math to draw my art students in: I have focused on ways to present math in a congressive light. I consider ingressive math to focus on written symbols, outcomes, facts, rules, formulae, solving problems, and calculating answers. I always look for more ways to make it congressive by focusing on visual and physical aids, processes, discovery, investigation, choices, opinions, reasons, and instead of problems and answers, building structures and uncovering relationships. My research field, category theory, arguably is much more about finding relationships between different concepts and building theories about that, rather than picking a problem and solving it. One of my favourite demonstrations investigates the relationship between circles and a torus, and involves closing a slinky into a circle and inserting a bagel inside it. [See image on p. 35.] Essentially this is a demonstration that the product space of a circle and a circle is a torus, something you wouldn't typically see unless you were a math major taking topology. But putting a bagel inside a slinky is something anyone of any age and any level of expertise can appreciate, and has the added bonus of being enormously satisfying to do, just as math should be.

Congressive teaching/outreach

Along with presenting the mathematical material in its most congressive possible light, I also aim to make the classroom and my teaching style as congressive as possible. I think traditional math teaching is very ingressive: old-style lecturing, asking the students questions and then telling them if they are right or wrong, setting them exercises and tests. Instead I use low floor/high-ceiling projects in every session, so that there is a low barrier to entry and essentially no limit to what can be learned from the activity. For example, we explore platonic solids without me saying what the platonic solids are in advance—I just invite them to explore what highly symmetrical 3-dimensional objects they can build by cutting out regular polygons and sticking them together. Some of them will build the platonic solids, some of them will build other polyhedra that are highly symmetrical but aren't quite platonic solids, and some of them will build a dinosaur. And if they do build a dinosaur what they discover is that pentagons are not a useful shape for building a dinosaur, but equilateral triangles are very versatile. This is a profound mathematical point that is used at the highest levels of research where triangulations are the key to deep proofs, for example in the classification of surfaces up to homotopy.

This has taught me to aim to be as congressive as I can in the outreach activities I devise as well. I think a lot of traditional math outreach has been somewhat ingressive: math competitions, math games and puzzle books. Those can make math fun for some people, but only to those who enjoy competitions, games and puzzles. I typically enjoy none of those things. However, I love crafts and would always prefer a math craft to a math game. I strongly support Julia Robinson Mathematics Festivals, where there are various tables of mathematical activities and children can spend as much or as little time as they like at each, with no particular aim. I am lucky that I was not put in for any math competitions when I was a teenager as I would have hated it and possibly concluded that I was not cut out to be a mathematician. I worry that other

congressive people have the same experience and that this leads to us shutting out congressive people from our field.

The mistake of ingressive filters

The upshot is that I think we are presenting math too ingressively to children, which may be putting off more congressive children. Moreover, we are using ingressive filters to select people for higher levels of math, which doesn't just put off congressive children but actually keeps them out. This starts with timed tests and exams in school which are likely to test speed more than depth. Many leading math researchers, including Fields Medallist Sir Andrew Wiles, talk publicly about how exams are not a good predictor for future research success. We should think hard about whether they're a good predictor for anything at all, other than at an extremely broad level of correlation. We should think about whether we use them just because they're an easy way of ranking people, and whether we are comfortable with the false negatives that result. I sweated blood to do well enough in my finals to get the PhD place I wanted. I scraped in and many people who did much better than me in exams fell by the wayside when they tried to do research. If I hadn't done well enough to get my first choice PhD place I was ready to conclude that I wasn't good enough to be a mathematician. More ingressive students simply move on to their second choice, or their third choice, or fourth choice, sure that they are good enough and that the system is unfair.

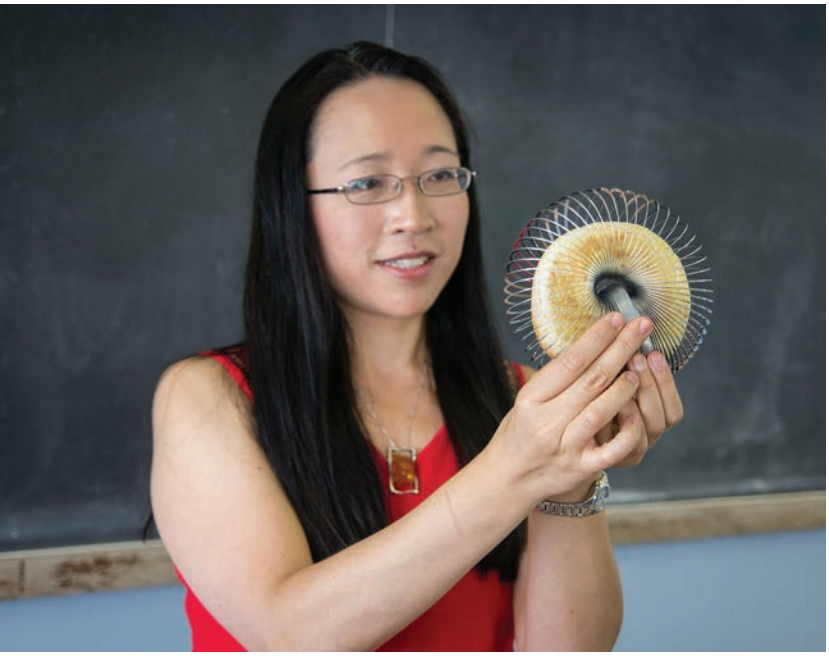
Being congressive in an ingressive world

It is hard to be congressive in a world that is structured in such an ingressive way, but in the last few years I have developed my own portfolio career and have gradually found ways to be successful while remaining congressive. While it might look like I'm doing ingressive things like putting myself forward, promoting my activities, and asking to be paid for my work, my motivations are deeply congressive: I am always thinking about the good I can do for the world and the best ways I can reach as many people as possible, especially those who might otherwise be neglected by the existing system. It looks like I took a huge risk by resigning from tenure, but actually I constructed a wide safety net around me before doing so. It looks like I stepped way out of my comfort zone to become largely freelance, but in fact I expanded my comfort zone gradually and always stayed inside it. It looks like I have a lot of self-confidence, but in fact I have a large network of supporters and enough external validation that gives me that confidence. Before I changed my life around, I had managed to make myself ingressive enough to get by in the ingressive system up to a point, but it made me unhappy. I realised I was becoming someone I didn't want to be. Now I am becoming who I really want to be, I am happier, and feel that I am having a better impact on the world.

Conclusion

My dream is that many (or even most) people would prefer a more congressive world, and that together we can work towards that future. It is hard while the ingressive people hold power and will try to keep it, but we can find ways to reward and value congressive behaviour despite that, at individual and classroom-type levels, and gradually we will find ways to change our structures as well. If we believe it is impossible then it certainly will be. There are individuals, organisations and cultures that are more congressive than others, and so I believe that my dream congressive world is not just an improbable dream but a possible reality. ■

Eugenia Cheng is Scientist in Residence at the School of the Art Institute of Chicago and Honorary Felllow of the University of Sheffield, UK. She is author of popular math books How to Bake Pi, Beyond Infinity *and* The Art of Logic in an Illogical World.



Eugenia Cheng inserting a bagel into a slinky. Photo Credit: University of Sheffield.

GET MORE

Check out the interview of Eugenia Cheng, "Searching for Categories," in the February 2018 issue of *Math Horizons*.

Border Walls, Assault Rifles, and Data Science

William Yslas Vélez

Border walls are a means of separating "ourselves" from "them." Sometimes border walls are built of metal, cement, or rock. At times border walls are a natural barrier—a desert or river—but sometimes they are more subtle, a mode of dress or speech. Border walls are supposed to function as barriers, but often they are symbolic. (See bit.ly/2DsQsdg for Francis Su's discussion of symbolic borders.) Insiders think these walls protect them from outsiders. But given any physical structure, creativity and persistence will find a way over, under, through or around.

At my retirement party in September 2017, mi primo hermano (in the Mexican community first cousins signify a closer relationship as the literal translation, cousin-brother indicates), Carlos Vélez-Ibañez, Regents Professor of Anthropology at Arizona State University, gave me a copy of his latest book (*The US–Mexico Transborder Region, Cultural Dynamics and Historical Interactions*, edited by: Carlos Vélez-Ibañez and Josiah Heyman, The University of Arizona Press). On page 65, I came to the section: *Border Walls: States of Exception and Necropower*. It was an eye-opener:

> "For example, Spanish border agents at Melilla do not officially register migrants apprehended at its border fence, as is required by Spanish law. Instead, they often turn migrants over to Moroccan authorities, who then leave them in the desert to perish." [page 67]
>
> "…the United States constructed the border fence along sections of its border with Mexico that is a combination of walls and barriers, sometimes including concertina wire and CBP (Customs and

Photo above: William Yslas Vélez and his cousin, Carlos Vélez-Ibañez

The views in this article do not necessarily represent the views of the MAA.

Border Protection) agents armed with M-16 assault rifles." [page 67]

On March 22, 2018, the following appeared on the front page of the Arizona Daily Star: "Agent in border killing acted as 'executioner,' US argues."

A sudden realization came to me; the border wall is not the issue; it is how that border wall is guarded. The guards posted along border walls can be given necropower—power over life and death.

Mathematical Border Guards

As a mathematician reading these passages, an image immediately came to mind. College algebra and precalculus are walls that students need to surmount if they are to proceed onto STEM fields in college. In my mind, I pictured pre-college teachers shooting at these students as they tried to surmount this wall. On the other side of that wall were the college teachers also shooting down those students.

A wall forces an individual to expend limited resources or getting into debt on attempts to get across that wall.

Is that image relevant? From my early years in college I recall the following directive aimed at a large class of students in a science or engineering class-"Look to your left, look to your right. Only one of you will be here at the end of the semester." That is someone displaying necropower, the power to destroy a student's hope of a STEM career! Academia has built walls that must be surmounted, and we, as mathematics teachers, are the border guards.

Let's review the impact of border walls.

A wall expresses superiority.

We made it over that wall. In fact, it wasn't even a wall for many of us mathematicians, so many of us cannot begin to understand how it is possible that others cannot get over that wall. We learned this material, passed the exams; why can't others? And if they cannot, then they do not belong on our side of the wall. We have standards; moreover we have tenure, the academic equivalent to having an assault rifle. Some see it as their duty to maintain the culture of mathematics as one of precision and rigor. Tenure allows us to guard those walls with fierce determination.

Gated communities

Look at the ten best research mathematics departments in the USA. You will notice that underrepresented minorities are not part of that faculty, and there are few women among that group. These departments have been very effective in maintaining this wall to keep these two populations out. When one looks further, one also sees that underrepresented minorities are almost invisible among the graduate and undergraduate students in these departments. The reality is that graduate students at these top mathematics departments are the potential faculty at the top ten schools. This particular border wall has proven to be effective in keeping out under-represented minorities. For more thoughts on this, see bit.ly/2QsApi8.

A wall forces an individual to expend limited resources or getting into debt on attempts to get across that wall.

Students who complete high school without sound mathematical preparation must take remedial courses. This takes away from the ability to complete a STEM degree in four years and it often forces students into debt. Since resources are limited, a non-STEM degree is more financially viable.

A wall forces an individual to find a way around, a way that can lead to danger, and to death.

Many students, in particular underrepresented minority students, have to begin their college studies at community colleges. Without a plan to pursue a STEM degree, students avoid mathematics courses until later, and then, after three years, realize that they have used up much of their financial aid; therefore transferring to a four-year university is not possible. We have lost those students.

We Are Not Border Guards

We should not view ourselves as border guards—rather the gift that we have for doing mathematics is one that should be shared with our students. Our mathematics courses are vital for tomorrow's jobs and we should view our instruction in mathematics as one that motivates students to the continued study of mathematics. Mathematics should be the pump into STEM careers, and not the stopper (Mathematics Instruction, An Enthusiastic Activity, William Yslas Vélez, On Teaching and Learning Mathematics, AMS Blogs, August 1, 2014).

A wall forces an individual to find a way around, a way that can lead to danger, and to death.

Border walls can be changed, or we can devise a way around them that can lead to STEM fields. In fact, a different view of these walls would make them more palatable. College algebra and precalculus should be viewed as the bridge to the future and we, in the mathematical community, should stand on both sides of that bridge, encouraging students to succeed.

Currently there is an opportunity that presents itself to mathematics departments: data science. This new endeavor is coming to our four-year institutions. How will mathematics

departments deal with this subject? Already we hear comments from mathematicians that express disdain for the lack of mathematics needed for this topic. We have heard these same comments before aimed at applied mathematics, statistics, and computer science—all areas which greatly benefitted from the expertise of mathematicians and are now vital to scientific endeavors.

I hope that mathematics departments will use this opportunity to aggressively join in the development of data science. Courses in this subject can show students how useful and necessary mathematical ideas can be (see, for instance, bit.ly/2HljbtK). Entry-level courses in data science can be viewed as a bridge to STEM careers, and perhaps even to the further study of mathematics. I would like to see mathematics departments enthusiastically join in the efforts to develop data science programs and welcome students into our realm of quantitative thinking. ■

William Yslas Vélez is a retired professor of mathematics at the University of Arizona.

Mathematical Sciences Semesters in Guanajuato, Mexico

William Yslas Vélez

For some years I have thought that a semester-long program in mathematics in Latin America, like the Budapest Semesters in Math or the Math in Moscow program, would be a valuable asset to students around the world. In particular, a program like this in Mexico would be attractive to Hispanic students in the U.S, an opportunity for intense study of mathematics combined with an immersion in the Mexican culture.

A number of years ago, CIMAT (Centro de Investigación en Matemáticas) in Guanajuato, Mexico, sent a delegation of mathematicians to the mathematics department at The University of Arizona (UA) to discuss possible collaborations. The UA responded by sending a team of ten mathematics faculty to CIMAT to continue these conversations and I took the opportunity to propose that CIMAT establish semester-long mathematical programs, whose courses would be taught in English. I am pleased to announce that these programs will begin in August, 2019. Here is the website for details, list of courses, and application procedures: mathsciencesgto.cimat.mx/es. The fall semester will explore mathematical tools for modeling, the spring semester will study mathematical tools for data science and the summer program will be partial differential equations.

An attractive feature of this program is that the prerequisites are third semester calculus and linear algebra to allow college students to add an international component to their undergraduate education early in their mathematical career. Moreover, tuition, travel and living expenses are within reach of students who could not have previously studied abroad.

CIMAT is an accredited institution of higher education, so credit earned there will transfer to the student's home university. The University of Arizona is currently working to establish an agreement with CIMAT where credit would be granted by the University of Arizona.

An added attraction is that CIMAT is located in the beautiful city of Guanajuato, a city full of history and beauty. Students will appreciate living in such a historic city, with its many cultural activities.

MAA Project NExT

NEW EXPERIENCES IN TEACHING

Launch the NExT stage of your career

MAA Project NExT (New Experiences in Teaching) is a year-long professional development program for new or recent Ph.D.s in the mathematical sciences. The program is designed to connect new faculty with expert teachers and leaders in the mathematics community and address the three main aspects of an academic career: teaching, research, and service.

Recent program sessions have included:

- getting your research and grant-writing off to a good start,
- innovative teaching and assessment methods and why they work,
- finding your niche in the profession,
- attracting and retaining underrepresented students,
- balancing teaching, research, and service demands,
- starting an undergraduate research program, and
- preparing for tenure.

Application deadline: April 15, 2019

projectnext.maa.org

projectnext@maa.org

MAA Project NExT Fellows join an active community of faculty who have become award-winning teachers, innovators on their campuses, active members of the MAA, and leaders in the profession.

MAA Project NExT welcomes and encourages applications from new and recent PhDs in postdoctoral, tenure-track, and visiting positions. We particularly encourage applicants from underrepresented groups, including women and minorities. Applications for the 2019 cohort of MAA Project NExT Fellows are due on **April 15, 2019** and can be found at projectnext.maa.org.

Project NExTers at MAA MathFest in Denver.

LGBT Math—Out of the Closet

Frank A. Farris

The world was very different in 1984. In my interviews at the JMM Employment Registry in Louisville, I no more would have told potential employers that I was gay than I would have confessed to embezzlement. And no one asked. I was presentable, with a good degree, and imagined that my unacceptable nature was detectable only to myself.

After a long day in our interview clothes, my college friend Steve Kass and I propagated a little prank. We made a neat sign to set by the message board announcing

Association of Gay Mathematicians

8 pm

and added the address of a local gay bar. This is the story of how an idea we found laughable—that gay, lesbian, bisexual, and transgendered mathematicians would one day be celebrated at the JMM—is now a fact that young people take for granted.

The first meeting

Steve Kass and I had planned to spend a little time at the bar that evening, wondering whether our sign would bring any other mathematicians to the site. After about half an hour, we saw two men with conference badges peeking out from their clothes. It was amusing to pretend that we had identified them as mathematicians only from their demeanor, though that was probably not out of the question. We passed a happy social evening, laughing together at the small joke that there could ever be such a thing as an Association of Gay Mathematicians. Looking back I see that our conversation offered us exactly what people now gain from the LGBT Math receptions that happen every year: the feeling of support, the

Photo above: Frank Farris and Bill Beeman at their wedding. Photo by Sam Kessler.

exchange of facts about how those like us are treated, and the comfort in knowing that we were not alone.

Since I cannot contact S and B to ask their willingness to be named in this article, I will stick to initials. S taught at a small liberal arts college in the Midwest, while B was a teaching specialist at a large southern university. Our common experience was of "passing," meaning that we pretended to belong to the dominant culture of straight people.

We all were considered excellent teachers and all used language particularly well. I do not recall whose idea this was, but we agreed that part of our skill in teaching came from childhoods spent having to do that extra calculation of what others might think of our behavior and speech, modifying to make ourselves more acceptable. Just the little tricks involved in finding words to avoid identifying the gender of the person one is "seeing" are money in the bank when it comes to presenting complicated mathematical ideas to our students.

Perhaps the most important thing that came of our conversation was the question, "If there are four of us, could there be more?" What if we could enjoy the same kind of networking and information exchange at every national meeting?

The San Francisco Earthquake

That year, I was hired by Santa Clara University, a Jesuit university where I continue to teach today. Over the years, things changed slowly on my own campus: Students at SCU formed a support group for gay and lesbian students; when I came out to a student, she dismissed the news as an "open secret."

With more openness came a backlash. In 1992, Colorado had passed its nasty Amendment 2 by popular vote. This law prevented any jurisdiction within Colorado from recognizing sexual identity as a class protected from discrimination. Looking ahead to the JMM scheduled for Denver in January, 1995, the leadership of both the MAA and AMS became uncomfortable with the idea of meeting in a state with such a law. Ken Ross, outgoing Associate Secretary of the MAA, tells an anecdote about the decision:

> At the January 1993 meeting in San Antonio, the AMS and MAA agreed to move the meeting from Denver. This was a rare and electronically-interesting joint decision. My local computer guru insisted that I take his $4000 black-and-white heavy laptop (I was afraid it would disappear) to the meeting. So the joint AMS-MAA resolution was crafted by the MAA Board on my computer, then I went down the hall to the AMS Council and read the resolution; they changed a few words, then I went back to the Board, etc., with fast convergence. Everyone was impressed with this use of modern technology.

Ken Ross tells me it cost the two associations $10,000 to move the meetings to San Francisco, but the gain in good will, from my perspective, was immeasurable.

There in San Francisco, Jim Humphreys, Robert Bryant, and others organized a reception for lesbian, gay, bisexual, and transgendered mathematicians (LGBT Math, now called Spectra) at a bar safely distant from the conference site in the San Francisco Hilton. It changed my world to see the crowd: perhaps 60 people of all ages, including a past president of the MAA and an author I had long admired. It startled me to see these successful, happy people enjoying both a freedom to express their sexual identity and a sense of secret fellowship, fostered by the fact that many were not "out," even to their professional colleagues.

The LGBT Math reception became an annual event, with a listserv organized by Ron Buckmire of Occidental College. The most popular topics of conversation involved the gay-friendly quality of one's home institution. Pre-tenure faculty expressed fears that they had to pretend not to be different until tenure might make them safe. Even with tenure, some wondered whether a "moral turpitude" clause might terminate their employment if they were too open. It is important to remember that most states had no anti-discrimination laws to protect people with variant sexual identities. California's Unruh Act did not explicitly identify sexual orientation as a protected class until 2005, although courts had held that it actually extended protection before that time.

Another popular comparison was the degree to which one's school permitted or fostered associations of LGBT students. The more forward-looking places had these in place by the early '80s; at some schools there remain none.

The concept that one's employer might offer any benefits to a same-sex partner would have been laughable in 1984. Perhaps without being explicitly told, we knew that our schools wanted us to say we were single, even if we were not. And yet,

Frank Farris and Bill Beeman at Huntington Lake in 1993.

school after school started to offer equal benefits to same-sex partners.

And now I'm married

It's hard to capture how much the world has changed since Steve and I put up that sign as a joke that there could ever be an organization for gay mathematicians. I can find some sympathy with people who have found it difficult to embrace the cultural shift, but it is truly marvelous to see so many LGBT mathematicians living happy and open lives. That said, struggles are not over. Despite clear scientific evidence that gender is not a matter of a single X or Y chromosome, we hear politicians blustering to put people into boxes based on testing a single gene and to remove trans soldiers from the military.

On my own campus, it's only been a few years since my university has started to signal that my sexual orientation is a positive contribution to the diversity of our community. One small token of the shift: At a reception to celebrate our Rainbow Resource Center, supporting LGBT students on campus, they had not just any old refreshments, but really good, catered refreshments. We had arrived. Though the picture is rosy now, I should not minimize the damage it did to my own career to have lived all those years in discomfort, hearing the official church doctrine that referred to homosexuality as "inherently disordered."

My partner, William Beeman, attended that first reception with me in 1991 and has now been back to many receptions since then. Having become a couple in 1984, during my last year teaching at Brown University, we lived apart as he retained his job in the Anthropology department at Brown until moving to the University of Minnesota in 2007. With many various sabbaticals and other leaves taken in each other's cities, we registered as domestic partners in California after a 1999 law made it possible. We married in 2014, but have never requested benefits from one another's university, though we may some day.

Our lives have been improved wonderfully by efforts to end discrimination. Even so, I feel sorrow for those in our profession who still suffer as I did from feelings of being fundamentally unacceptable, for whatever reason. And it's a double sorrow to think of those who left mathematics over feelings of exclusion. We have work to do. ■

Frank Farris, Professor of Mathematics and Computer Science at Santa Clara University, has served as editor of Mathematics Magazine *and as the MAA Chair of the Council on Publications and Communications. His book,* Creating Symmetry: The Artful Mathematics of Wallpaper Patterns, *was published by Princeton University Press in 2015. His undergraduate degree is from Pomona College (1977) and his PhD from MIT (1981). Awards include the Distinguished Teaching Award from the Golden Section of the MAA and Best Photograph, Painting, or Print at the 2018 Mathematical Art Exhibition at the Joint Mathematics Meetings.*

Seeking Nominations for the Deborah and Franklin Tepper Haimo Award for Distinguished College or University Teaching

This award honors college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions.

Each year at most three college or university teachers are honored with this award. Recipients of the Haimo Award receive $1,000 and a certificate of recognition; recipients must be members of the Association (teaching in the U.S. or Canada). At least one of the Award recipients must be a current Section nominee. The Section nominee may be the current recipient of the Section Award for Distinguished Teaching or a previous recipient of a Section Award for Distinguished Teaching from any Section. An individual may win the Haimo award at most once. Nominations for the Haimo award are due to the secretary of the MAA (secretary@maa.org) by **March 1**.

For more information and nomination forms go to maa.org/awards/haimo.

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JULY 31 - AUG. 3, 2019

MAA MathFest heads to

Cincinnati, Ohio

in 2019

Visit **maa.org/mathfest** to learn about the speakers, panels, sessions, and more for mathematicians, students, and enthusiasts.

A Department's Commitment to Gender Inclusivity

Mako E. Haruta

I joined the University of Hartford mathematics department in 1992, and for well over a decade was one of two female faculty in a department of 13. Through new hires, the mathematics faculty made a concerted effort to change that ratio. Roughly 18 years after I was hired we reached 30%, and today just over 50% of our full-time tenured and tenure-track faculty are women.

Over the course of a year, I brainstormed ways to celebrate this departmental milestone. A group photo of the women faculty seemed ideal. I wanted to depict a more modern and up-to-date image of who we are as women researchers and teachers. Images from popular TV series and movies featuring strong female leads, such as *Hidden Figures, Madam Secretary, Wonder Woman,* and a variety of crime dramas, provided inspiration for poses that conveyed strength and confidence. Lesly Juarez, a local professional photographer, completely understood what we were going for and was excited to participate. In April 2018, we met for a photo shoot in the newly renovated Mortensen Library. As we relaxed beforehand, we found ourselves sharing our experiences as women mathematicians. There was also a lot of laughter and fun that helped dispel the natural self-consciousness of being in front of a lens during a photo shoot. The behind-the-scenes candids by Devanney Haruta (Brown '16.5, Music, Mathematics) perfectly capture the positive energy of the event and reflect what we enjoy on a daily basis in our department. Both in and outside of the classroom, the people in our department strive to provide support, mentoring, and instruction in a

Back row, left to right: Jean McGivney-Burelle, Michelle Rabideau, Virginia Anne Noonburg (Emerita), Alicia Marino.
Front row, left to right: Aslihan Demirkaya, Larissa Schroeder, Mako Haruta (Chair), Hwayeon Ryu.
Photo Credit: Lesly Juarez unsplash.com/@jblesly *and* jlesly.myportfolio.com/

All of the candid photos on this page are courtesty of Devanney Haruta, devanney_haruta@alumni.brown.edu

way that builds our students' ability and confidence. We want our photo to be an inspiration for other faculty and students alike in the larger mathematics community.

We found that in sharing the photo with students and other mathematicians, we began to receive more invitations for outreach, including a podcast featuring women in math and presentations to women in STEM high school groups. At the University of Hartford, Drs. Alicia Marino and Hwayeon Ryu will be running the first meeting of Beyond the Binary in Spring 2019, a gender inclusive mathematics conference for high school and college students (details below). We are fortunate to have a highly collaborative and mutually supportive department community that celebrates all of our diversity, and this photo is just one of the many examples!

How to do your own photo!

Images are powerful and have the potential to influence how we think about ourselves and how others view us. Updating the images of women in mathematics can be a creative and collaborative process. Here are some ideas on how to make it happen. Be bold and have fun!

- **Choose your angle.** Decide what you'd like the photo to convey and who will be in it, whether it's a group or an individual in your department, or yourself.

- **Kickstart the process**. Talking to one or two others you'd like to have in your photo is a good way to raise interest.
- **Update your image.** Do a little research online by browsing professional photographers' sites to find inspiring images. Read an article about poses to get some insights.
- **Find inspiration**. Popular media is a great place to get inspiration. In particular, TV series and movie characters can provide visual representations of ideals such as strength, courage, and wisdom. ■

Mako E. Haruta is an Associate Professor of Mathematics and Chair of the Departments of Mathematics and Physics at the University of Hartford. More photos from behind the scenes can be found at hartford.edu/math.

Beyond the Binary Conference

On April 27, 2019 the University of Hartford will host the one-day undergraduate mathematics conference "Beyond the Binary." It is an inclusive conference designed to promote gender diversity in mathematics and explore career opportunities for all. There will be industry representatives leading a panel discussion, an academic panel discussion, one-on-one networking opportunities, and students can present their work at a poster session. There will be two plenary talks bookending the conference, one by Emily Willson and one by Dr. Karamatou A. Yacoubou Djima. The organizers of the conference are Drs. Alicia Marino and Hwayeon Ryu, along with the undergraduate ambassador team: Zaria Santos '20, I'ana Johnson '21, and Kelly Insalaco '22. The conference is generously funded by the Institute for Advanced Study's Women and Mathematics Program, Lisa Simonyi, and the University of Hartford. More information is at beyondthebinary2019.com.

Departments

CELEBRATIONS OF PROJECT NEXT

Developing Professional Friendship

Matt Boelkins and Matt DeLong

Twenty-five years ago, T. Christine Stevens and Jim Leitzel proposed that the MAA develop an innovative professional development program to help jumpstart the careers of new mathematics faculty. Shortly afterward, the Exxon Corporation came to the MAA looking for a project to fund. The Executive Director at the time, Marcia Sward, connected the two and MAA Project NExT ("New Experiences in Teaching") was born. Project NExT provides professional development opportunities for faculty in their first two years after completing their PhD and helps them transition to academic positions. Workshops are held at three major national meetings—MAA MathFest during their first summer as Fellows, the Joint Mathematics Meetings, and the subsequent MAA MathFest, where one year's Fellows welcome the new cohort.

In the 25 years since its initiation, about 2000 faculty in the mathematical sciences have joined the MAA Project NExT community. All this year we'll be sampling the stories of those mathematicians. What was the program like in its early years? How have the resulting friendships, collaborations, and mentoring opportunities transformed professional lives? What does MAA Project NExT mean to individuals? To the mathematical community?

Stay tuned for more stories like this one...

***In summer 1999, we were both members of the 99–00 Brown** Dot cohort of MAA Project NExT, which (aptly) was held at Brown University in Providence, RI. At a gathering for Fellows from institutions in Indiana, Illinois, and Michigan, we met for the first time; through the first three national conferences we attended together in concert with NExT, we became close friends. At many national meetings since, we've even occasionally been confused for one another. Now 20 years later, we're honored to have this opportunity to reflect on what Project NExT has meant to us, both individually and as friends.*

Matt Boelkins (MB): What's your favorite memory from that first NExT gathering in Providence?

Matt DeLong (MD): So many things from that workshop have stuck with me over the years. Bob Devaney's approach to teaching Differential Equations shaped how I taught that course for the next 19 years. Ed Burger's course on teaching Liberal Arts Mathematics fundamentally reframed how I thought about teaching mathematics and its role within a liberal arts education. Chris Stevens and Aparna Higgins enthusiastically communicated their personal excitement for every last Fellow. And Joe Gallian's "Just Say Yes!" inspired me to do just that. But my favorite memory is of getting to know you and a few other Fellows from the Midwest after a full day of sessions as we took an evening stroll along WaterFire, the famous public art display on the river in Providence.

What are some things that you learned from participating in those early Project NExT workshops that have shaped the educator you are today?

MB: My eyes opened to a whole world of new possibilities. Sessions on calculus reform, how to develop effective computer labs with a computer algebra system, and how to engage undergraduate students in research all had a major impact on my work in both the near and long term. I got immediately better at developing labs for students, thought long-range about what my goals should be for calculus (which in some sense ultimately led to writing a calculus text based in active learning), and got started on a research project with a student that eventually led to several joint papers and future work. For all the positive effects NExT has had on my professional life, the professional friendships I've developed have been even better.

When I first started attending post-NExT JMM and MAA MathFest conferences, I always found a roommate to make travel more affordable. While I'm fond of my colleagues from my home institution, because I spend so much time with them already, I prefer not to room with them at conferences. I think you and I first started sharing a hotel in New Orleans in 2011, and that now-regular practice has led to us getting to share time in many different cities away from formal conference activities and to develop a long-lasting friendship. I've so appreciated the chance to talk about things both professional and personal that benefited from the perspective of someone outside of my home environment.

MD: Indeed. We've roomed together in New Orleans, Seattle, San Diego, and several other interesting cities. Especially as you moved into MAA leadership, the accommodations were

Matt DeLong and Matt Boelkins at the MAA Pavilion at JMM 2019.

a significant step up from the early MAA Project NExT days in the dorms at Brown! Regardless of the setting, the company was always a pleasure. When we didn't room together, we always made it a practice of finding time for dinner or a nightcap, whether it was San Antonio, Washington, or Denver. I'm grateful to the MAA for many things, one of which is that the meetings have allowed me to appreciate a significant number of US cities with good friends.

True friendships are often forged through deep waters. Thank you for being an incredible support to me at that JMM in 2011 in New Orleans. You were the first person with whom I talked after hearing the news that my good friend and collaborator, Dr. Dale Winter, had passed away suddenly the day before he was supposed to fly to the meetings. You were an empathetic listener who gave me space to grieve, both during those days at the meetings and at times since. Our professional lives are only a part of who we are, and so our professional associations serve us best when they recognize our full humanity. I have always appreciated you for taking that perspective, and the MAA for helping to create such spaces within our professional lives.

Speaking of sharing our full selves, one of my favorite MAA Project NExT memories from my time on the leadership team was when we had a plenary speaker suddenly take ill and you stepped in on a moment's notice. I texted you while you were in the Board of Directors' meeting to tell you of our dilemma just ahead of the scheduled talk, and I was amazed when you responded with, "I think that I could do something." We were even more amazed at how your transparent and moving presentation impacted that cohort of Fellows.

MB: I think this has been one of the biggest lessons that MAA Project NExT taught me: to be unafraid to share my full self. The 2017 talk to NExT Fellows on time management, prepared on an hour's notice over lunch, was an opportunity to not only provide some brief advice, but to share some real failures in my professional and personal life and hope that others could learn from those experiences. Hard lessons are often the lessons that teach us the most, and learning that not only could I work too much, but I could work so much that I no longer liked my work, was an important part of my growth. Having only an hour to prepare for the conversation meant that I had no choice but to be honest and direct, and that was absolutely best.

I can recall many times in NExT interactions where brave colleagues spoke of things that weren't going well for them. A particular one on the NExT listserv came from a colleague who was really struggling post-tenure and was asking for help. The message was honest and heartfelt and led to one of the best threads I'd ever read: a warm combination of people saying "I hear you", "I feel that way sometimes too", and "this is what I did when I felt that way." It was one of the best examples of how NExT provides a community for us all that reminds us we are not alone and instead are in this work together, even if most of us work at different institutions from one another.

Having a friend you can turn to outside your home department is a key part of academic survival and thriving, I think.

MD: I think that word "survival" is not hyperbole. I know that I've turned to a handful of MAA Project NExT friends (you, Francis Su, Talithia Williams, Alissa Crans, among others) for counsel and support on major life decisions or events, and I have often returned the favor as well. I am grateful that the MAA's first core value is Community. Many of us have found it to be a place that nurtures those close professional friendships.

Beginning with MAA Project NExT, we have also found the MAA to be a place that supports academic thriving within broader networks by cultivating "participation in mathematics through outreach and partnership" and by strengthening "community through collaborative activities," the two components of its core value of Community. MAA Project NExT asked us to envision our role in the profession, gave us models of people doing interesting and exciting work, and then supplied a support network as we transitioned into our own active participation in the Association and the broader mathematical community.

The contrast between our first experiences at the national meetings and at present is stark. I recall in the early years wondering who all of those people were and what they were doing there (and how Chris and Aparna seemed to know everyone)! Since then, as one "Just Say Yes!" moment led to

another, we have participated in Section NExTs, Section executive boards, MAA committees, Project NExT leadership (MD), MAA leadership (MB), PRIMUS editor-in-chief (MB), MathPath academic director (MD), and other professional subcommunities. Now being at the national meetings feels like being with a gigantic group of colleagues, as over the years we've worked with many of them on one project or another.

MB: One of the things that overwhelmed me in my first NExT experiences was the sheer number of opportunities that were presented. For many of those, I felt inadequate, both in available talent and time. Early on, I tried to do too much. As I shared in the 2017 talk to NExT Fellows on time management, just after tenure, I nearly had a nervous breakdown as a result of overworking myself, undercaring for myself, and generally losing perspective.

One of the lessons I learned in that experience—which felt like total failure at the time—was that it's a long career, and we have to play a long game: there's lots of time in a full career to get projects done, and it doesn't all need to be tackled right away. Many times I'd told my kids "People are more important than things", but I think that I myself lost sight of the fact that "people are more important than accomplishments." A renewed focus on building relationships with my students and my colleagues (both at Grand Valley and throughout MAA) rejuvenated me, and now a decade after a dark time, I find myself living a better life. Your friendship has made a huge positive difference in that improvement, as twice a year for the last decade we've connected in person at national meetings, talked honestly about how we're both doing, and held one another accountable to the most important things to which we aspire. Thank you for that.

Your last sentence above regarding the national meetings has me even more excited for both Baltimore and Cincinnati, where I'll not only get to see you, but also spend time with my professional colleagues and friends who don't work at my home institution. I also see more clearly now that being a regular conference participant and MAA member is itself a long game. Over 20+ years, my professional network has expanded, fantastic new opportunities have come my way, and I've benefited so much from the greatest resource of the MAA: its members. I'm grateful for all of the lessons those members have taught me, for how MAA Project NExT introduced me to this community, and especially for how it led to the privilege of being friends with you. ■

Matt Boelkins is Professor of Mathematics at Grand Valley State University in Allendale, MI. Matt DeLong is Professor of Mathematics at Marian University in Indianapolis, IN.

MEET A MEMBER

Brian P Katz (BK)

What is your current position and how long have you been there?

I'm am an associate professor in the Department of Mathematics and Computer Science at Augustana College, in Illinois, where I have been for the last decade. This year, I am on leave and am teaching high school in Manhattan, in part to learn how to support the future teachers I mentor more effectively.

Why are you a member of the MAA?

I joined the MAA near the end of graduate school as part of attending conferences, but I found more than meetings—I found a professional home where teaching was seen as a vital element of our discipline. Unless you work at an institution that builds a culture around open classrooms, teaching can be isolating, in part because of how we are evaluated. The MAA was a place where I found new, challenging ideas and where people engaged in the shared work of thinking about our teaching together in public. And on an institutional level, I have been drawn toward the work of our discipline that needs to be housed in the MAA, including the *Instructional Practices Guide*, editorial work for MAA publications, and the SIGMAAs.

Over the last decade, I feel that the MAA has stepped up in a new way to lead the discipline in rethinking what equity, diversity, inclusion, and justice mean for mathematics. I'm proud to be part of an organization that invited such a diverse group of speakers for MAA MathFest 2018, that empowers Dave Kung to make Project NExT a fulcrum of equity work for both a more diverse and a more culturally competent professoriate, and that hosts subgroups including the IBL and RUME SIGMAAs as they draft and approve statements on equity and justice. In particular, I channel my work through the MAA because, as an organization, it understands that de-humanized and colorblind definitions of what it means to do mathematics would allow our discipline to be used as a continued tool of oppression and therefore it accepts responsibility for re-humanizing these definitions, at the levels of both the individual and our community as a whole. A decade ago, I would have been surprised to know that now so many mathematicians see fluency on issues of power and identity as central to their work, and this change came in no small part through the work of the MAA and its membership.

The MAA's values include community, communication, inclusivity, and mathematics teaching and learning. What would you like to see MAA do to better address these values?

The MAA is doing good work in support of these values, but I believe that we should be more proactive in leading the disciplinary community with these goals. The MAA supports the work of developing a culturally competent and diverse faculty, but I would like to see the MAA set a goal of making sure that all mathematicians are culturally competent and that this goal is seen as integral to our work, rather than optional or for a subset of our community. The MAA supports our development as teachers, but it does not have a journal focused on teaching written for a readership of mathematicians. PRIMUS is filling this role for the community, but I would like to see these two organizations collaborate to guarantee that mathematicians have access to peer-reviewed ideas about teaching and learning. The MAA has started taking a firm and public stand in support of student-centered pedagogies, and I would like us to make similarly public statements about equity and justice. I was impressed by Ami Radunskaya's quick action after learning that travel to an AWM conference could be inequitable, and I hope the MAA can become this nimble and decisive in response to similar issues.

I'm worried about the MAA and AMS splitting with the end of the JMM because it could reintroduce a false choice between teaching and research and am even more worried that caring about equity could be seen as a subset of caring about teaching. I'd like to see the MAA take responsibility for this across the whole mathematics community, even among mathematicians who may not be MAA members.

Describe the MAA in four words.

Colleagues, mentors, friends, home. ■

Departments

PUZZLE PAGE

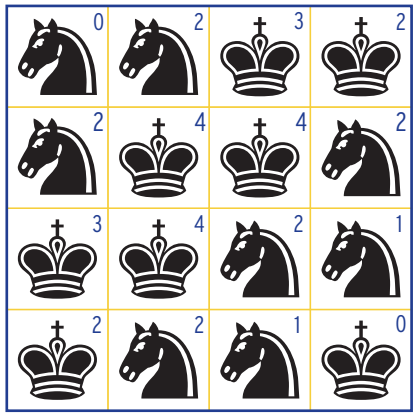
Knights and Kings

—David Nacin

Fill in the cells with knights and kings so that every row and column contains the same number of each. The numeric clues indicate the number of pieces of the same type that the piece in that cell can attack. For example, a clue of 4 in a cell indicates that there is either a knight in that cell that can attack four other knights, or a king in that cell that can attack four other kings. ■

David Nacin is a professor at William Paterson University. He enjoys designing and studying puzzles that involve groups and Lie algebras, partition identities, the motion of chess pieces, and other mathematical structures. He maintains a free puzzle blog at quadratablog.blogspot.com.

Sample solved 4 by 4 puzzle



2	2	1	1
2	2	2	2
1	2	4	3
1	2	3	2

1	4	4	2	1	1
2	5	5	3	1	1
1	5	2	5	3	3
1	2	5	3	5	4
3	2	2	3	3	3
1	1	4	2	3	0

1	2	1	2	3	3	2	1
1	1	4	3	3	3	4	2
2	3	4	3	2	2	4	4
2	3	5	4	5	3	4	3
2	3	3	4	3	5	4	4
3	4	3	5	4	6	2	2
4	4	3	3	3	3	2	1
2	3	1	3	3	1	2	0

Never a Dull Moment

—Stephen Kennedy

Never a Dull Moment: Hassler Whitney, Mathematics Pioneer
Keith Kendig
385 pp; Softcover, ISBN: 978-1-4704-4828-8

Hassler Whitney came from distinguished American and mathematical stock: California's Mt. Whitney is named for a grand uncle; his maternal grandfather was Simon Newcomb, distinguished mathematical astronomer and fourth president of the AMS; Whitney himself is named after his great-great-grandfather Ferdinand Hassler, the first head of the US Coast Survey in the early nineteenth century. Whitney, a tinkerer with mechanical devices from childhood and devoted reader of *Popular Science* magazine, intended to pursue physics. He was admitted to Harvard for graduate study in physics in 1930. Whereupon he realized with dismay that physicists had to remember stuff, which he was bad at.

Whitney had an unusual background in mathematics, upon arrival at Yale for his undergraduate education he asked James Pierpont's permission to skip Calculus 1 and enroll in Calculus 2. (Whitney had taught himself calculus.) Pierpont's angry rejection of this request kept Whitney from enrolling in any mathematics class for several years. As a senior he finally enrolled in Pierpont's graduate-level complex analysis class, hoping that Pierpont would not remember their earlier encounter.

In spite of this background, upon arrival at Harvard he switched from physics to mathematics immediately and thereupon began a lifelong pattern. He looked around for an easily stated problem upon which he could practice his tinkering instinct. He wanted a problem, easily stated and understood which did not require lots of technical machinery. He wanted a problem that would yield, at least somewhat, to devoted study of geometric examples. He found such a problem in the four-color theorem of graph theory. Of course, he didn't solve it, but he discovered: a new characterization of planarity; a simple condition that guaranteed the existence of a Hamiltonian cycle; and some tricks for computing chromatic polynomials. It was enough for a PhD under G.D. Birkhoff.

All of this work is examined in Kendig's fascinating biography. To give just one example, consider a planar graph G and its dual G'. If, for any cycle in G we remove the corresponding edges from G', the result becomes disconnected. Whitney proved that planar graphs are characterized by this property. That is, if given a graph H there exists a graph H' and a bijection g on the edges such that $g(C)$ (where C is any cycle in H) disconnects H', then H must be planar. It is easy to see in Kendig's description of this work its roots in Whitney's geometric messing around. You can see a similar genesis in Whitney's definition of matroids. By fooling around with pictures of graphs and minimal spanning trees you see clear analogies with vector spaces and bases. Kendig is simply phenomenal at tracing Whitney's inspiration in examples and pictures, it feels as if you are looking over Whitney's shoulder as he creates. (Kendig is especially to be commended as Whitney hid all this geometric motivation in his formal papers.)

Kendig is equally good at explaining the origins of almost everything Whitney did. His extension theorems, his embedding theorems, his work on singularities, and his (co-)invention of the cup product all followed a similar pattern. He would mess around with geometric examples until he found the ones that perfectly typified the behavior he was trying to understand. Then he would express the result formally for publication, hiding all traces of the geometric inspiration. Kendig digs it all out and lays it bare.

Keith Kendig first met Whitney at the Institute for Advanced Study in the 1950s. With shared passions for music and geometry, the two became lifelong friends. As a result, Kendig became very familiar with Whitney's mathematical thinking, that familiarity is on full display here. He also learned the Whitney family lore and Whitney's habits and hobbies: the mountain climbing, the tinkering, the chamber music. This closeness makes for a very intimate, personally and mathematically, biography of a great mathematician.

All this talk of history gives me the chance to remind you of the latest MAA Notes volume—*The Courses of History: Ideas for Developing a History of Mathematics Course*. Amy Shell-Gellasch and Dick Jardine have collected descriptions of 33 different courses in mathematics history. There are courses for future secondary teachers, courses for mathematics majors, courses for general education students. There are courses on special topics: calculus, probability, complex analysis. I'm sure many readers of *MAA FOCUS* have found themselves in the position of teaching history of mathematics despite a lack of professional preparation and training. (Full disclosure: I have, and the course I designed is described in this volume.) *The Courses of History* is a great resource for anyone in that position. Recall that all new MAA Notes volumes are available as a benefit of membership. Log on to your MAA account (click on "My Profile" in the upper-right corner of the MAA webpage). Look in your Member Library. ■

Stephen Kennedy (Carleton College) manages acquisitions for MAA Press. Contact him if you are interested in writing a book for the MAA: kennedy@maa.org.

Departments

BOOK REVIEW

Mathematics for the People

—Adriana Salerno

A review of "*The Equidistribution of Lattice Shapes of Rings of Integers of Cubic, Quartic, and Quintic Number Fields: An Artist's Rendering*" by Piper Harron
Hardcover, Birkhäuser, 1st ed. 2019 edition (available 9/11/2019, available for pre-order on Amazon), ISBN: 978-3319765310

If you are at all on social media, you may remember that a few years ago the prologue of a number theory thesis went viral—this is possibly the first time you heard of Piper (aka The Liberated Mathematician, aka @pwr2dppl). The fact that this was a Princeton PhD thesis advised by recent Fields Medalist Manjul Bhargava might have already gotten it noticed by the mathematical community, but it was its unique, honest, and vulnerable take on what doing mathematics is (and isn't) that really took it beyond just "good thesis" status, shared and read on social media by the mathematical and non-mathematical alike. When the first line of a PhD thesis reads "Respected research math is dominated by men of a certain attitude," you know you are in for a special and challenging ride, in more ways than one. The wise people of Birkhäuser also took notice, and soon they will be publishing Piper's thesis, in its entirety, as one of their volumes.

The main goal of the thesis is to prove the theorem:

> **Theorem:** *For $n = 3$, 4, and 5, when isomorphism classes of S_n-number fields of degree n are ordered by their absolute discriminants, the lattice shapes of the rings of integers in these fields become equidistributed in the space of shapes as the discriminant goes to infinity.*

The level is great for anyone from students to professional mathematicians, although the intended audience seems to be somewhere in the middle. As carefully detailed in the preface and introduction, each chapter has a "laysplanation" section for the more inexperienced reader, a section for mathematicians (with the more formal definition-theorem-proof-rinse-repeat structure) and a section for details (which she calls the "weeds"). This is the rare book where you will find comics, poems, pop-culture references, and funny diagrams and pictures in the midst of some serious and deep mathematics.

Laysplanation

And now I offer you my own attempt at a laysplanation of the results in the book: S_n number fields are extensions of the rationals whose Galois group is S_n. The absolute discriminant is a sort of key feature (an invariant) of a number field that measures the "size" of the ring of integers of the number field. The lattice shapes have to do with how you embed these rings of integers into real space, and you can think of a set of all possible shapes as the space of shapes. What Piper does in the thesis is then to prove that if you let the discriminant go to infinity, the shapes associated to these special number fields will be equidistributed (i.e., with no discernible pattern) in the space of all shapes.

Each chapter that follows ("how it's gonna go down") breaks down a gigantic formula (at the top of page 10) into accessible pieces, giving the relevant definitions and intermediate steps in a structured way while also reminding the reader of what the ultimate goal is. It almost unfolds like a novel (and many of the side remarks and framing make it clear this is not accidental), and the reader follows the heroine through her quest to prove her theorem.

Personally, I found the "laysplanations" to be the most illuminating, because they contained wonderful analogies about some abstract mathematical objects I already knew and understood, but with a whole new insight. As a professional mathematician who is also an educator, I really appreciate the care and effort that it takes to make complex topics so accessible. This is a true testament to the ability of this author to connect with people at many different stages of their careers. For example, in Chapter 2, the author expertly develops an analogy for parametrization, equivalence, and group action (that is better than any "formal" explanation I have ever seen) through a game of counting bicycles. She carefully develops the analogy and it leads right where it should: careful and mathematically correct definitions.

Honesty

The tone and overall honesty of the writing is, as mentioned above, really special. Often, we see books that just give us a polished, final version of something that probably took years to compile (and maybe about mathematics that took hundreds of years to develop). This gives the erroneous idea that mathematics just flows out of someone's head and onto the paper. The book (and the thesis that preceded it) make no such assumptions, and in fact lay bare the process of mathematics: one of confusion, re-evaluation, and progress through failure. This will be EXTREMELY valuable for advanced undergraduates and graduate students to read, even if, as I suspect, some professional mathematicians might not appreciate this approach as much. I do think professional

mathematicians could learn a bit of humility from the way this book is written, and that the profession as a whole could benefit from such transparency and honesty about our processes. Like Piper says: "This may be challenging for happy mathematicians to read through; my only hope is that the challenge is accepted."

I invite you all to join this very worthwhile challenge. And who knows, maybe you will love learning a little more algebraic number theory in the process. ■

Adriana Salerno is an Associate Professor of Mathematics at Bates College. She is originally from Caracas, Venezuela, and obtained her PhD in Mathematics, with a focus on Number Theory, at the University of Texas at Austin.

The Mathematical Association of America
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