MEETING THE CHALLENGES OF IMPROVED POST-SECONDARY EDUCATION IN THE MATHEMATICAL SCIENCES

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The mathematical sciences play a foundational and crosscutting role in enabling substantial advances across a broad array of fields: medicine, engineering, technology, biology, chemistry, computer science, social sciences, and others. Due to this foundational role, the delivery of excellent post-secondary mathematics education is essential to the present and future well being of our nation and its citizens.

We greatly appreciate the engagement of PCAST in the challenges of post-secondary mathematics education. A key finding of the 2012 PCAST *Engage to Excel* report is that mathematics education is a critical component of all undergraduate STEM degrees. We share this perspective of mathematics education as an enabler of STEM careers, provider of broad mathematics literacy, and shaper of the next generation of leaders in our increasingly technological, data-driven, and scientific society.

The report also found that current deficiencies in mathematics learning are partly driving the loss of STEM majors in the early college years. We acknowledge many of the shortcomings highlighted by the report. The wake-up call delivered by PCAST has sharpened the awareness of the mathematical sciences community of the need for intensive, broad-scale efforts to address these problems. We emphasize that efforts by a great many in the mathematical sciences community predated PCAST's report, that progress is being made, and that plans are in place to broaden these to a community-wide effort.

Our task is to encourage and help lead constructive actions that will address the difficult and varied challenges facing post-secondary education in the mathematical sciences. How should mathematics educators improve developmental education in order to enable students to aspire to STEM careers? How should mathematical scientists in colleges and universities augment their cooperative efforts with "partner disciplines" to best serve the needs of students needing basic university mathematics? How should mathematical sciences departments reshape their curricula to suit the needs of a well-educated workforce in the 21st century? How can technology be best used to serve educational needs?

These questions must be answered in the context of a changing landscape. There are growing disparities in the preparation of incoming students. A third of all undergraduate mathematics students are enrolled in precollege level mathematics. At the other extreme, almost 700,000 high school students in the US completed a course of calculus this past year. The mathematical sciences themselves are changing as the needs of big data and the challenges of modeling complex systems reveal the limits of traditional curricula.

The NRC report *The Mathematical Sciences in 2025* eloquently describes the opportunities and challenges of this shifting landscape. This report should serve as a springboard for initiatives in mathematics education that more closely intertwine the learning of mathematics with the appreciation of its applications. However, the mathematical community alone cannot bring about the scale of changes called for in *Engage to Excel*. Building on all the activities in mathematics education underway or that have arisen as a result of the PCAST report, we ask for PCAST's help in promoting greater awareness, collaboration, and cooperation among all of the scientific disciplines who are working to prepare the STEM workforce of the future.

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Appendix A. Educational Activities

The mathematical community is engaged with educational activities on a national level. Its work extends from research into practices that promote student learning to programs that focus on bringing new innovations to larger numbers of students. Within the past five years, 41% of mathematics departments at research universities have added inter-disciplinary undergraduate courses with Biology.[4] National efforts include the *Mathways*[5] and *Statway*[6] programs that are assisting under-prepared students at two-year colleges, while *Modeling Across the Curriculum*[7] is working to embed computational learning and exposure to modeling and simulation in early STEM courses. We briefly mention the thriving community for Research in Undergraduate Mathematics Education[8] and two particularly noteworthy studies. The first, *Characteristics of Successful Programs in College Calculus*,[9] surveyed course coordinators, instructors, and students at 213 colleges and universities to learn the aspirations and experiences of those who study Calculus and the factors that encourage persistence and retention. The second is a study out of UC-Boulder into the effectiveness of Inquiry-Based Learning (IBL) methods as practiced at four prominent research universities.[10] More information on these studies is provided in Appendices B and C.

The increased engagement by our community with other STEM disciplines is evident from the STEM education activities of our professional societies and of the many academic departments to which its members belong. A representative snapshot of these activities is given below. Many of these activities involve significant fractions of the faculty members in academic departments at leading universities.

The mathematical sciences community has a long history of integrating evidence-based innovations into undergraduate teaching. Examples from the past three decades include the integration of calculator-based lessons into freshman and sophomore courses, the introduction of conceptual-based work in the calculus curriculum, the development of active-learning methods, the integration of computer-based lessons, major revisions in ordinary differential equations courses for engineers, the introduction of web-based class resources, the explosion in the development of new mathematics courses for life science majors, and the use of on-line videos.

The community is currently involved in experiments that flip classes, use computer-based testing, replace expensive static textbooks with interactive on-line resources, and more. The best practices that emerge from these experiments will also be adopted by the mainstream curriculum.

Many of the activities of the mathematical community directly address issues raised in *Engage to Excel*, including an anticipated "national conversation" on the report to begin in January 2014, intended as a major step in efforts to scale-up successful programs.

Identifying empirically validated teaching practices. In addition to the Calculus study (Appendix B) and IBL Centers (Appendix C), work on empirical validation of teaching practices includes:

- The *Calculus Concept Inventory*,[11] modeled on the idea of the Force Concept Inventory, was pilot-tested and validated in 2005. It has been used, for example, to show the effectiveness of the

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4 Source: CBMS *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the United States*

5 www.utdanacenter.org/higher-education/new-mathways-project

6 www.carnegiefoundation.org/statway

7 connect.siam.org/siam-nsf-workshop-on-modeling-across-the-curriculum

8 www.rume.org

9 www.maa.org/cspcc

10 www.colorado.edu/eer/research/steminquiry.html

11 math.unipa.it/~grim/21_project/21_charlotte_EpsteinPaperEdit.pdf

University of Michigan's approach to Calculus, which includes small classes taught by graduate students and post-docs who receive training and supervision in techniques of active learning.

- Arising from a strategic initiative of the American Statistical Association, the Consortium for the Advancement of Undergraduate Statistics Education (CAUSE)[12] is a national organization whose mission is to support, advance, and disseminate undergraduate statistics education in the four target areas of resources, professional development, outreach, and research.
- In the 1950s and '60s the MAA's Committee on the Undergraduate Program in Mathematics (CUPM)[13] undertook the creation of the modern undergraduate curriculum in mathematics to serve the needs of engineering and science majors. This committee continues to monitor and suggest refinements to this curriculum. In the early 21st century, CUPM launched the *Curricular Foundations Project* to build understanding within mathematics departments of the needs of partner disciplines.

Addressing the mathematics preparation gap:

- There are successful and widely implemented programs that supplement Calculus instruction with just in time delivery of precalculus topics by stretching the first term of calculus over two terms.
- The University of Illinois at Urbana-Champaign has been a leader in the use of placement exams that incorporate and direct students to online tutoring.
- Greater attention is being paid to issues of advising and its use as a tool for recruitment. The University of Arizona is nationally known for its efforts to recruit and advise Hispanic students with the potential to succeed in mathematically intensive majors.
- The American Statistical Association (ASA) and National Council of Teachers of Mathematics (NCTM) are working together to support high-quality training and professional-development opportunities that will help prepare teachers lead classes on statistics.
- The Conference Board of the Mathematical Sciences (CBMS) has produced *The Mathematical Education of Teachers II (MET2),*[14] which focuses on the mathematics and statistics preparation of K–12 teachers in response to the Common Core State Standards. The ASA board recently funded a project to create a companion report on the statistics education of teachers.

Diversifying pathways to STEM careers:

- Many colleges and universities now use courses other than Calculus—for example, discrete mathematics, linear algebra, or statistics—as entries to the serious study of mathematics.
- Both the US Military Academy at West Point and Macalester College have refocused introductory Calculus from a study of differentiation and integration to the study of dynamical systems.
- SIAM's *Modeling across the Curriculum* has, as its overall objective, to "engage and keep young people in STEM disciplines, from K-12 through undergraduate (and graduate) studies, and into the workforce." Its major themes include:
    - Expansion of modeling in K-12.
    - Development of a high school one semester or one year modeling course with stratified content.
    - Development of modeling-based undergraduate curriculum.
    - Assessment of the effects of such efforts on college STEM readiness and retention in STEM majors.
    - Development of a repository of materials for all aspects and levels of math modeling instruction and understanding. This includes but is not limited to course lesson plans, articles, books, web sites, videos, contests, problems and solutions.

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12 www.causeweb.org

13 www.maa.org/cupm

14 cbmsweb.org/MET2

- ASA is working on new pathways through the Mathematical Sciences that emphasize Statistics, Statistical Modeling, and Computational Mathematics. This both attracts students who might not have considered career in the Mathematical Sciences and meets a very real need.
- The Summer Institute for Training in Biostatistics (SIBS)[15] hosts undergraduates majoring in quantitative areas for six weeks, exposing them to statistical principles and career opportunities in biostatistics, and encouraging them to pursue graduate training.
- The Mathematical and Theoretical Biology Institute[16] at Arizona State University is a widely recognized model for engaging students, especially those from underrepresented minorities, in the mathematical sciences through a program of educational, research, and mentoring programs. In 2011, it was recognized with the Presidential Award for Mentoring in Science, Engineering, and Mathematics.[17]
- MAA's *Math & Bio 2010*[18] report, published in 2005, reflects the recognition that the Biological Sciences have joined Engineering and the Physical Sciences as an area with rich mathematical connections that must be reflected in the undergraduate curriculum.

Catalyzing widespread adoption of best practices:

- MAA began Project NExT (New Experiences in Teaching)[19] in 1994. It provides an extensive introduction to the profession for new PhDs in the Mathematical Sciences and provides extensive information about effective ways of approaching teaching and learning as well as continuing networking and mentoring. It has now mentored close to 1500 faculty.
- The CUPM Curriculum Guides[20] provide guidance on undergraduate curricula and instructional practices that support all students studying mathematics. They are widely disseminated among department chairs and discussed at sectional and national meetings.
- The Research in Undergraduate Mathematics Education (RUME) community has been pro-active in publicizing research results in formats that are accessible to mathematicians. An example is the MAA publication *Making the Connection*.[21]
- SIAM has been involved in various STEM activities through its Education Committee and other groups. One example is the recent Mathematics in Industry[22] report, which surveyed about 25 companies as well as recent graduates of masters and doctoral programs who took non-academic positions to analyze training for non-academic careers.
- AMS is pushing education issues through the activities of its Committee on Education and has endorsed a statement of policy[23] concerning the role of mathematicians in college-level teaching of STEM students.
- AMS publicizes Innovations in College-Level Mathematics Teaching,[24] showcasing examples of ongoing educational collaborations.
- AMS also undertook and has published an influential study on the use of web-based homework.[25]

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15 www.nhlbi.nih.gov/funding/training/redbook/sibsweb.htm
16 mtbi.asu.edu
17 paesmem.net/awards/institutions/228
18 www.maa.org/mtc/projectreport.html
19 archives.math.utk.edu/projnext
20 www.maa.org/cupm
21 www.maa.org/ebooks/notes/NTE73.html
22 www.siam.org/reports/mii/2012
23 www.ams.org/policy/govnews/pcast-statement
24 www.ams.org/programs/edu-support/Innovations-College-Level
25 www.ams.org/profession/leaders/webassess

- ASA published and has heavily promoted the GAISE Reports (Guidelines for Assessment and Instruction in Statistics Education).[26]
- *The Journal of Statistics Education*[27] is a free, online, international journal focusing on the teaching and learning of statistics.
- The Academy of Inquiry Based Learning[28] provides support and resources for the adoption of active learning strategies. Several hundred enthusiasts, as well as many who are simply curious, attend its annual meeting. Its primary outreach is to new faculty.
- There has been considerable experimentation with how to best incorporate internet-based resources into instruction. These run the gambit from videos, animations, interactive hyper-linked text, expert systems, on-line forums, and many other new formats. National meetings bring mathematicians together to compare notes and gain a greater sense what works best.

Encouraging partnerships among stakeholders:

- Encouraging such partnerships is the main purpose of the NRC report, *The Mathematical Sciences in 2025*.[29]
- MAA launched the Curriculum Foundations Project[30] in 2000 as a series of workshops to build understanding within the mathematical community of what partner disciplines need for their undergraduates from departments of mathematics.
- There are eight mathematical institutes funded by the Division of Mathematical Sciences (DMS) of the National Science Foundation, many of which were founded specifically to encourage partnerships among stakeholders. All of them have been active in fostering dialog around educational issues. Much of their outreach activity is directed toward encouraging the careers of under-represented groups in the mathematical sciences. Collectively, they have been a significant player, on both the level of research and of personal contacts, in creating bridges between the mathematical sciences and other fields, and have had an impact on the culture of the mathematical community. These institutes have strengthened connections between the mathematical community and application areas, and between different areas of mathematics and statistics. This vitality has had a major impact upon the culture of our community that is evident in *The Mathematical Sciences in 2025* report.
- Two NRC units, the Board on Mathematical Sciences and their Applications (BMSA)[31] and its Committee on Applied and Theoretical Statistics (CATS)[32] play critical roles in developing in building and strengthening partnerships with other disciplines. *The Mathematical Sciences in 2025* is a BMSA report. There have been more mathematical NRC reports produced since 2003 than in the entire earlier NRC history.[33]

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26 www.amstat.org/education/gaise

27 www.amstat.org/publications/jse

28 www.inquirybasedlearning.org/

29 www.nap.edu/catalog.php?record_id=15269

30 www.maa.org/features/currfound.html

31 sites.nationalacademies.org/DEPS/BMSA

32 sites.nationalacademies.org/DEPS/BMSA/DEPS_047575

33 Recent reports can be found at www.nap.edu/moretitles.php?org=BMSA

Appendix B: ***Characteristics of Successful Programs in College Calculus***

David Bressoud, Macalester College, Project Director

This appendix presents a few of the results of the NSF-supported[34] national study of *Characteristics of Successful Programs in College Calculus* (CSPCC), a five-year project run by MAA. In fall 2010 we surveyed Calculus I coordinators, instructors, and students at 213 colleges and universities across the United States, ranging from research universities to two-year colleges. This past fall, we conducted follow-up case study visits at 16 of the institutions that were shown to have calculus programs that have higher than expected rates of persistence and retention. This section begins with selected summative data and concludes with observations from the case studies.

I. Selected Summative Data. Roughly 300,000 students study Calculus I in college or university each fall, of which 111,000 (37%) are at research universities. While the most common intended major among Calculus I students at research universities is Engineering (34%), the Biological and Life Sciences come close with 31%. The Physical, Computer, and Mathematical Sciences combine to constitute only 10% of Calculus I students. Over 70% of Calculus I students at research universities have already completed a calculus course while in high school.

From the CSPCC survey data, 67% of research mathematics departments provide professional development activities or speakers who address methods for improving undergraduate instruction, and 62% of instructors stated that the scholarship of teaching and learning is either moderately or very valued by their colleagues in the department

While most Calculus I instructors at research universities (64%) believe that lecture is the most effective means of teaching, only 27% agree or strongly agree that this is the best approach to teaching, with 37% mildly agreeing. At the same time, 74% have a strong interest is raising their own awareness of how students learn, and 92% have a strong interest in improving their own teaching.

Clickers are available for the use of instructors at 64% of research mathematics departments, and 33% of departments provide training and support for their use. The uptake of this tool has been slow. So far, only one of the 73 surveyed research universities requires the use of clickers in the calculus course, one additional university recommends their use, and 7% of instructors use them.

Almost all Calculus I instructors at research universities (92%) want to convey the relevance of calculus to their students. Two-thirds (67%) frequently or always look for applications when planning their lessons. Almost all (98%) look for applications at least occasionally.

Additional summative data from the study have been published as “The Calculus Student: Insights from the MAA National Study”.[35]

II. Effective Practices. From the case study visits conducted at research universities with significantly better than expected rates of persistence and retention,[36] seven departmental or institutional practices emerged that appear to contribute to this success:

1. **Attention to local data.** The department regularly collects, analyzes, and acts on data on the effectiveness of its programs, adjusting placement procedures and organization of courses as problems are identified.

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[34] NSF #0910240. Opinions expressed here do not necessarily represent those of NSF.

[35] Bressoud, D., M. Carlson, V. Mesa, C. Rasmussen. 2013. The Calculus Student: Insights from the MAA National Study. *International Journal of Mathematical Education in Science and Technology.* Available at www.tandfonline.com/doi/full/10.1080/0020739X.2013.798874

[36] This means that student intention to continue the study of calculus at the end of the course was significantly higher than our model predicted based on student-level variables that included academic background, gender, race/ethnicity, intended major, and intention to continue beyond Calculus I when the course began.

2. **Placement and Remediation.** In addition to monitoring placement procedures, especially for critical subpopulations such as women and other underrepresented students, the best programs have designed effective means of bringing students with weaker backgrounds up to speed so that they do succeed in calculus while losing as little time as possible toward completion of the degree.
3. **Coordination and Communication.** Those who teach calculus meet regularly to discuss difficulties and successful strategies. Particular attention is paid to mentoring those who are teaching calculus at this university for the first time.
4. **TA Training.** Whether Teaching Assistants are used to teach their own small calculus classes or run recitation sessions, there is a substantial training program to prepare them for this role combined with continual monitoring of their effectiveness in the classroom.
5. **Active Learning.** Calculus instructors are encouraged to use and experiment with active-learning strategies. One institution has biweekly teaching seminars led by the math faculty or invited experts. No particular instructional approaches are prescribed for faculty at any of the institutions.
6. **Rigorous Courses.** While procedural fluency is still highly valued, the most successful calculus programs challenge students mathematically. They select problems that required students to delve into concepts, work on modeling, or even explore proofs. Course assessments go well beyond template problems.
7. **Learning Centers.** There is a well-run and well-utilized tutoring center. In some cases this center is devoted entirely to Calculus. Tutors receive training, and there is an established procedure for identifying students most of need of this support and encouraging them to take advantage of it.

Links to publications and further details of this study are available at maa.org/cspcc.

Appendix C: ***From innovation to implementation: Multi-institution pedagogical reform in undergraduate mathematics***

Sandra Laursen, Ethnography & Evaluation Research, University of Colorado Boulder, Project Director

Inquiry-Based Learning (IBL) is a recent name for an approach with a long history in mathematics that includes the Moore Method, employed by R.L. Moore at UT-Austin, and the active-learning and project driven approaches that appeared in the late 1980s. Five universities have established privately funded "IBL Centers" within their mathematics departments. The study described in this appendix was conducted at four of these departments that share general aims for students' intellectual development or mathematical maturation, and a general pedagogical approach emphasizing student creation, communication and critique of ideas.

Their activity was quite varied, with ~40 IBL courses but few in common across sites. The courses were varied in content, student audience, and forms of IBL. Departments had distinct contexts and cultures, though all had some record of prior innovation around math education.

This quasi-experimental study thus examines a realistically messy, multi-site implementation of educational reform. This is what reform looks like when implemented on a scale that matters. The measures used were broad, not content-specific, to accommodate the variety of courses and sites. Data include 300 hours of classroom observation, 1100 surveys, 110 interviews, 220 tests, and 3200 academic transcripts gathered from >100 sections at 4 campuses over 2 years.

I. What is IBL? The implementation of inquiry-based learning

Classroom observation was used to verify that the IBL classes (designated by the Centers) were indeed different from non-IBL sections of the same course. On average over 60% of IBL class time was spent on student-centered activities including problem presentations, discussion, small group and computer work, while students in non-IBL courses spent 87% of class time listening to their instructors talk. IBL courses also showed more variety in classroom activities, and more student leadership and question-asking. They were rated more highly for a supportive classroom environment, students' intellectual contributions, and in-class feedback to students on their work.

Differences in instructor-centered behaviors were small, suggesting that the observed differences between IBL and non-IBL classrooms lie more in teachers' choice of instructional activities than in their intent or interest in students. Individual engagement in meaningful mathematical tasks and collaborative processing of mathematical ideas are central in students' accounts of processes that contributed to their learning.

II. Student outcomes: Selected results

1. After an IBL or comparative course, IBL students reported higher learning gains than their non-IBL peers, across cognitive, affective, and collaborative domains of learning.
2. IBL students' attitudes and beliefs changed pre-to-post-course in ways that are known to be more supportive of learning, compared to students who took the non-IBL sections.
3. In later courses, students who had taken an IBL course earned grades as good or better than those of students who took non-IBL sections, despite having "covered" less material.
4. On a research-based test of students' ability to evaluate proofs, IBL students showed evidence of greater skill in recognizing valid and invalid arguments and of the use of more expert-like reasoning in making such evaluations. The volunteer sample consisted of only high-ability students; no instructors gave the test to all students during class time.
5. On a validated test of mathematical knowledge for teaching, pre-service teachers who had taken a math course targeted to their needs (K-6, K-8 or 6-12, site-dependent) scored above the mean for a large national sample of in-service teachers. All groups' scores improved pre to post (effect size 0.8), but rose most for students who scored lowest on the pre-test. No non-IBL sections of these pre-service courses were available for comparison.

Results by gender

6. Non-IBL courses show a marked gender gap: across the board, women reported lower learning gains and less supportive attitudes than did men (effect size 0.4-0.5). Women's confidence and sense of mastery of mathematics, and their interest in continuing in math, was lower. This difference appears to be primarily affective, not a real difference in women's preparation or achievement. (post-survey, pre/post survey; academic records)
7. This gender gap was erased in IBL classes: women's learning gains were equal to men's, and their confidence and intent to persist similar. IBL approaches level the playing field for women, fixing a course that is problematic for women yet with no harm to men.

Results by achievement group

8. When sorted by prior achievement, the grades of most students (IBL and non-IBL alike) dropped in subsequent courses as course work became more difficult. But grades of initially low-achieving students who had taken the IBL course rose 0.3-0.4 grade points, unlike their low-achieving, non-IBL peers, and unlike their higher-achieving classmates.

III. Current and forthcoming publications

Laursen, S., Hassi, M.-L., Kogan, M., Hunter, A.-B., & Weston, T. (2011). *Evaluation of the IBL Mathematics Project: Student and Instructor Outcomes of Inquiry-Based Learning in College Mathematics*. (Report to the Educational Advancement Foundation and the IBL Mathematics Centers) Boulder, CO: University of Colorado, Ethnography & Evaluation Research. http://www.colorado.edu/eer/research/steminquiry.html

Hassi, M.-L., & Laursen, S. L. (2013, in review). Transformative learning: Personal empowerment in learning mathematics.

Kogan, M., & Laursen, S. L. (2013, in review). Assessing long-term effects of inquiry-based learning: A case study from college mathematics.

Laursen, S. L., Hassi, M.-L., & Hough, S. (2013, in review). Inquiry-based learning in mathematics content courses for pre-service teachers.

Laursen, S. L., Hassi, M.-L., Kogan, M., & Weston, T. J. (2013, in review). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study.

Laursen, S. L. (manuscript in preparation). Critical teaching decisions in undergraduate IBL mathematics classrooms.