Pi on Trial and Other Prime Suspects

Tommy Ratliff

Mathematical constants are rarely the basis for legal thrillers, but sometimes reality can be stranger than fiction. Such was the case when Judge Michael H. Simon of the U.S. District Court in Oregon released a ruling on March 14, 2012 (yes, the date is significant) with the memorable conclusion: "Pi is a non-copyrightable fact, and the transcription of pi to music is a non-copyrightable idea."

At dispute in the case was a musical composition by Michael Blake called "What π sounds like." Blake assigned consecutive digits to the C-major scale and used the circle of fifths to assign digits to chords. He used this correspondence and the first 32 digits of π to create his composition and posted a video online in 2011, which received quite a bit of attention. Lars Erickson, whose "Pi Symphony" is also based on the digits of π, filed a lawsuit asserting copyright infringement since he had registered his composition with the U.S. Copyright Office in 1992. Judge Simon ruled in favor of Blake, and now both compositions can be found online.

Of course, there is nothing special about using π for the basis of a musical composition. Any other constant could conceivably be used, although not all would be sonically pleasing. *Math Horizons* readers, if swayed by the April 2012 Aftermath column, may prefer Blake's composition based on τ.

There are many other well-documented instances of mathematics providing artistic inspiration. For example, the annual Bridges Conference is dedicated to exploring the connection between art and mathematics, and a perusal through back issues of *Math Horizons* will turn up many other examples. Additionally, mathematicians are adept at finding mathematical representations in unexpected places, some of which can be seen at the Found Math gallery at the MAA website. One of my personal favorites is the O! statue in Omaha, which certainly wasn't intended as a factorial.

Perhaps surprisingly, there is also precedent in the legal system for the reverse process of content from another field inspiring interesting mathematical constants. One of the earliest cases was the generation by Phil Carmody in 2001 of an illegal prime number. That is not a typo. Publishing the 1905 digit prime could have been considered illegal at the time in the United States under federal law!

The story begins with the Content Scrambling System (CSS), which is used to encrypt DVDs to prevent piracy and restrict the geographic region where each DVD can be played. A large pushback came from many users who felt that purchasing a DVD should allow the owner to access the content without restriction. As was perhaps inevitable, the encryption scheme was reverse engineered, and the resulting program, called DeCSS, was widely distributed on the Internet in 1999.

The program was incredibly short and, when expressed as a C program, could even be printed on a T-shirt. However, distributing the DeCSS program was illegal in many countries, including the United States under the Digital Millennium Copyright Act (DMCA), so wearing this shirt was technically breaking the law in the United States.

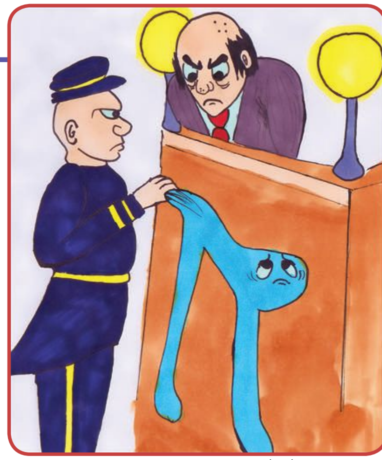
The DVD-Copy Control Association immediately obtained an injunction against websites that distributed DeCSS, including some that sold the T-shirt. Carmody devised a scheme to hide the algorithm in plain sight: He would embed it in a prime number large enough to qualify as a special prime to be included at the Prime Pages website.

He took the C-source code for the DeCSS program and used the compression utility gzip to transform it to a binary file, which is simply a very long sequence of zeros and ones. However, this can also be viewed as the base two representation of a single, very large integer, which we will call b. Not surprisingly, the value b is not prime. However, Carmody took advantage of the way that the decompression utility gunzip works to reverse the compression performed by gzip. Whenever gunzip reads a null byte consisting of all zeros in a binary file, it terminates and ignores whatever follows.

Carmody looked at integers of the form $b \cdot 256^Y + X$. Since $256 = 2^8$, the term $b \cdot 256^Y$ will have a long sequence of zeros in its binary representation to act as a terminator for gunzip, and X can then be chosen to ensure that the integer is prime. He guessed that Y needed to be at least 156 for the prime to be large enough to be mathematically significant, and he restricted X to be between 1 and 255 to maintain the sequence of zeros created by the $b \cdot 256^Y$ term.

Carmody used software to generate probable primes of this form very quickly, but verifying that a specific value is prime is much more computationally intensive, requiring many days of computing using the Elliptical Curve Primality Proving (ECPP) algorithm. Eventually, he found that the integer $b \cdot 256^{211} + 99$ is prime.

What made this prime sufficiently special to be archived is that it was, at the time, the 10th-largest integer proved to be prime using ECPP. This is quite remarkable. If you treat this prime number as a binary file and process it with gunzip, then you immediately obtain the code that allows you to bypass the encryption safeguards on DVDs. Since distribution of the algorithm was illegal, Carmody had created an illegal prime and, technically, publishing this prime could be viewed as a violation of the DMCA.


Charlotte Torrence

This does seem somewhat absurd. The prime number was interesting and worthy of archiving in its own right, but because it could be interpreted as divulging proprietary information, its distribution could have legal ramifications. No charges related to the illegal prime were ever filed, and the injunction against distributing DeCSS was overturned in 2004 when a California court ruled that CSS had been widely available on many websites worldwide in many forms and was no longer a trade secret. Thus, thankfully, *Math Horizons* broke no laws in the publication of this article.

This same procedure could be followed to embed any clearly articulated concept into a prime number that is sufficiently interesting on its own to be worthy of archiving. There may be technical hurdles, depending on the size of the initial concept, but theoretically this would be possible. How should we interpret the resulting value? Is this just an interesting prime number, or does it represent someone's proprietary intellectual property? Is the intent of the individual who generated the number relevant?

Without trying to untangle the legal ramifications, there is a clear mathematical takeaway message: The same number can have many different meanings, depending on the context. π is the ratio of the circumference and diameter of a circle, and its decimal representation is the basis for a pleasing melody. $b \cdot 256^{211} + 99$ is both an interesting prime number and the encoding of an algorithm for breaking DVD encryption. 1729 is both the rather dull number of a taxicab in London and the smallest number that can be written as the sum of two cubes in two different ways. If we know how to listen, any number may have many interesting stories to tell. ■

Further Reading, Viewing, and Listening

Michael Blake, "What π sounds like," *http://www.youtube.com/watch?v=wK7tq7L0N8E*.

Lars Erickson, "Pi Symphony," *http://www.pisymphony.com/gpage.html*.

Vi Hart, "Singing Pi-Gram," *http://www.youtube.com/watch?v=4C9PALaDh2U*.

Phil Carmody, "Illegal Prime Number?," *http://fatphil.org/maths/illegal1.html*.

MAA Found Math Gallery, *http://maa.org/FoundMath/*.

The Bridges Organization: Art and Mathematics, *http://bridgesmathart.org/*.

Prime Pages: *http://primes.utm.edu/*.

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http://dx.doi.org/10.4169/mathhorizons.21.1.18