Problem 220, “Inscribe-A-Gon,” ended with this open question: for which $m, n \geq 3$ can a regular $m$-gon be inscribed in a regular $n$-gon? Specific cases discussed in the February 2009 Playground were

- all $m = 3$ and all $n$
- all $m = 4$ and all $n$
- $m = 2n$
- $m$ dividing $n$

Do these four cases exhaust all of the possibilities?

In an attempt to solve this problem, we might first try to prove that the only possibilities with $m > n$ occur when $m = n + 1 = 4$ and when $m = 2n$, as shown in the second figure below for $n = 6$:

Here are three initial observations. Notice that $m$ can’t be larger than $2n$: since no three vertices of an $m$-gon are colinear, each side of the $n$-gon can contain at most two of the vertices of the $m$-gon. Notice also that if $m > n$, by the Pigeonhole Principle at least one side of the $n$-gon contains two of the vertices of the $m$-gon. Finally, if two vertices of the $m$-gon appear on a side $s$ of the $n$-gon, they must be the same distance from the midpoint of $s$. To convince yourself of this last fact, think about what would happen to the $m$-gon and $n$-gon if you reflected them about the perpendicular bisecting line of $s$.

Readers are encouraged to try and complete the analysis for $m > n$, or offer cases not listed above, or provide any other useful information in an effort to solve this problem. Useful contributions will be acknowledged as they are received.