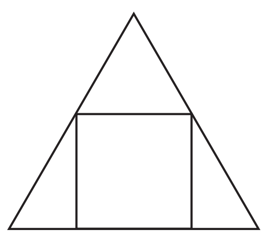
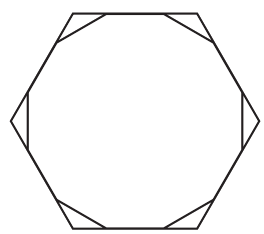
Problem 220, “Inscribe-A-Gon,” ended with this open question: for which $m, n \geq 3$ can a regular m-gon be inscribed in a regular n-gon? Specific cases discussed in the February 2009 Playground were

- all $m = 3$ and all n
- all $m = 4$ and all n
- $m = 2n$
- m dividing n

Do these four cases exhaust all of the possibilities?

In an attempt to solve this problem, we might first try to prove that the only possibilities with $m > n$ occur when $m = n + 1 = 4$ and when $m = 2n$, as shown in the second figure below for $n = 6$:





Here are three initial observations. Notice that m can't be larger than $2n$: since no three vertices of an m-gon are colinear, each side of the n-gon can contain at most two of the vertices of the m-gon. Notice also that if $m > n$, by the Pigeonhole Principle at least one side of the n-gon contains two of the vertices of the m-gon. Finally, if two vertices of the m-gon appear on a side s of the n-gon, they must be the same distance from the midpoint of s. To convince yourself of this last fact, think about what would happen to the m-gon and n-gon if you reflected them about the perpendicular bisecting line of s.

Readers are encouraged to try and complete the analysis for $m > n$, or offer cases not listed above, or provide any other useful information in an effort to solve this problem. Useful contributions will be acknowledged as they are received.