A1. Find all ordered pairs \((a, b)\) of positive integers for which
\[
\frac{1}{a} + \frac{1}{b} = \frac{3}{2018}.
\]

A2. Let \(S_1, S_2, \ldots, S_{2^n-1}\) be the nonempty subsets of \(\{1, 2, \ldots, n\}\) in some order, and let \(M\) be the \((2^n - 1) \times (2^n - 1)\) matrix whose \((i, j)\) entry is
\[
m_{ij} = \begin{cases} 
0 & \text{if } S_i \cap S_j = \emptyset; \\
1 & \text{otherwise.}
\end{cases}
\]
Calculate the determinant of \(M\).

A3. Determine the greatest possible value of \(\sum_{i=1}^{10} \cos(3x_i)\) for real numbers \(x_1, x_2, \ldots, x_{10}\) satisfying \(\sum_{i=1}^{10} \cos(x_i) = 0\).

A4. Let \(m\) and \(n\) be positive integers with \(\gcd(m, n) = 1\), and let
\[
a_k = \left\lfloor \frac{mk}{n} \right\rfloor - \left\lfloor \frac{m(k-1)}{n} \right\rfloor
\]
for \(k = 1, 2, \ldots, n\). Suppose that \(g\) and \(h\) are elements in a group \(G\) and that
\[
gh^{a_1}gh^{a_2}\cdots gh^{a_n} = e,
\]
where \(e\) is the identity element. Show that \(gh = hg\). (As usual, \(\lfloor x \rfloor\) denotes the greatest integer less than or equal to \(x\).)

A5. Let \(f : \mathbb{R} \to \mathbb{R}\) be an infinitely differentiable function satisfying \(f(0) = 0, f(1) = 1,\) and \(f(x) \geq 0\) for all \(x \in \mathbb{R}\). Show that there exist a positive integer \(n\) and a real number \(x\) such that \(f^{(n)}(x) < 0\).

A6. Suppose that \(A, B, C,\) and \(D\) are distinct points, no three of which lie on a line, in the Euclidean plane. Show that if the squares of the lengths of the line segments \(AB, AC, AD, BC, BD,\) and \(CD\) are rational numbers, then the quotient
\[
\frac{\text{area} \triangle ABC}{\text{area} \triangle ABD}
\]
is a rational number.
B1. Let \( P \) be the set of vectors defined by
\[
P = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} \middle| 0 \leq a \leq 2, \ 0 \leq b \leq 100, \ \text{and} \ a, b \in \mathbb{Z} \right\}.
\]
Find all \( v \in P \) such that the set \( P \setminus \{v\} \) obtained by omitting vector \( v \) from \( P \) can be partitioned into two sets of equal size and equal sum.

B2. Let \( n \) be a positive integer, and let \( f_n(z) = n + (n - 1)z + (n - 2)z^2 + \cdots + z^{n-1} \).
Prove that \( f_n \) has no roots in the closed unit disk \( \{ z \in \mathbb{C} : |z| \leq 1 \} \).

B3. Find all positive integers \( n < 10^{100} \) for which simultaneously \( n \) divides \( 2^n \), \( n - 1 \) divides \( 2^n - 1 \), and \( n - 2 \) divides \( 2^n - 2 \).

B4. Given a real number \( a \), we define a sequence by \( x_0 = 1, \ x_1 = x_2 = a, \) and \( x_{n+1} = 2x_nx_{n-1} - x_{n-2} \) for \( n \geq 2 \). Prove that if \( x_n = 0 \) for some \( n \), then the sequence is periodic.

B5. Let \( f = (f_1, f_2) \) be a function from \( \mathbb{R}^2 \) to \( \mathbb{R}^2 \) with continuous partial derivatives \( \frac{\partial f_i}{\partial x_j} \) that are positive everywhere. Suppose that
\[
\frac{\partial f_1}{\partial x_1} \frac{\partial f_2}{\partial x_2} - \frac{1}{4} \left( \frac{\partial f_1}{\partial x_2} + \frac{\partial f_2}{\partial x_1} \right)^2 > 0
\]
everywhere. Prove that \( f \) is one-to-one.

B6. Let \( S \) be the set of sequences of length 2018 whose terms are in the set \( \{1, 2, 3, 4, 5, 6, 10\} \) and sum to 3860. Prove that the cardinality of \( S \) is at most
\[
2^{3860} \cdot \binom{2018}{2048}^{2018}.
\]