Statistics and Quantitative Literacy

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Because much of the early work in quantitative literacy was led by statisticians—indeed, many K–12 programs in probability and statistics are named "quantitative literacy"—statistics bears a very special relation to quantitative literacy, with respect to both substance and education. This essay provides a perspective by leaders of statistics education on issues raised in the other background essays prepared for the Forum on quantitative literacy.

Setting the Stage

Who would disagree that college graduates, not to mention high school graduates, should be able to understand and correctly interpret disease or unemployment rates, the comparative costs of car or apartment rental agreements, and trends in the composition of the country's population? Yet many graduates are mystified by quantitative arguments, a mystification that ranges from minor confusion in some to functional innumeracy in others. Just as the information age is making the world more quantitative, however, the ability of people to deal with numerical issues of practical consequence is shrinking. It is past time to take seriously the challenge of improving the quantitative skills of graduates of U.S. high schools and colleges.

Before examining the role of statistics in the movement to improve quantitative literacy, it is wise to consider definitions of the key terms under discussion because there have been many different interpretations, even among enlightened readers. In fact, at the level of education under consideration here (high school and undergraduate) what some would call "statistics" might be termed "data analysis" or "statistical thinking" by others. We need to sort out the definitions of at least three different terms, all dealing with the same substance.

Statistics often is thought of as the keeper of the scientific method (although this may sound a little presumptuous to physical scientists) because it is the discipline that studies how to understand the world through the rubric of setting hypotheses, collecting data relevant to those hypotheses, analyzing the data, and drawing conclusions about the hypotheses from analysis of the data. Here "data" is to be understood broadly, because it well may include judgments of experts as in a Bayesian analysis. Although statistics has many elegant theories, its practice usually outstrips theory in the sense that many practical problems do not fit nicely into the assumptions of any theory.

This difficulty leads directly to *data analysis,* which can be thought of as following the rubric of the scientific method but with emphasis on answering real questions rather than trying to fit those questions into established theories. In data analysis, exploratory techniques stand alongside confirmatory techniques. Empirical evidence that a technique works often is taken as "proof" among data analysts who might choose to use such a technique in practice. "An approximate answer to the right question is better than an exact answer to the wrong question" is one of the mantras of the data

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analyst, the supreme example of whom is the late John Tukey (Tukey 1962). In today's complex world, data analysis is what most statisticians actually practice, and so it is quite appropriate that the subject be referred to as data analysis in standards and guidelines.

In reality, full-bore data analysis is more than most people need to deal with the statistical issues of everyday life and work. As a result, the third term, *statistical thinking,* comes into play. Statistical thinking is essential for anyone who wants to be an informed citizen, intelligent consumer, or skilled worker. It is the backbone of the contemporary emphasis on quality improvement because all levels of employees in a firm, from the CEO to the janitor, must have some notion of statistical thinking if a firm is to operate optimally. Using a quality-improvement definition, statistical thinking involves viewing life as made up of processes and viewing all processes as having variation. Once understood, variation can be broken down into that which can be reduced and that which must be managed.

This most basic of the three statistical terms might sound the most abstract, but we must keep in mind that processes can be simple and the sources of variation fairly obvious. Figuring out the gas mileage of a car is a process subject to variation, the most obvious sources of which are perhaps the grade of gasoline used and the style of driving. A person's health is likewise subject to variation, but here the sources of variation are many and sometimes difficult to detect. It is statistical thinking that keeps people from making rash decisions when accidents increase this month over last or one school has a slightly lower test score average than another school. The inherent variation in processes must be considered to determine whether change can be attributed to any cause other than pure chance.

Some might further differentiate between statistical thinking and *statistical literacy,* giving the latter a less formal definition than one involving processes and their variation. The ability to read a newspaper critically often is used as an attribute of a statistically literate person. The books by David Moore (2001) and Jessica Utts (1999) are good references for courses on statistical literacy, as is the Web-based Chance course (see www.dartmouth.edu/~chance). Because statistical thinking and statistical literacy are so close in the larger scheme of things, this essay uses the term statistical thinking when referring to this level of statistical education (which also may reduce the confusion over the many uses of the word *literacy*).

As to the definition of *quantitative literacy* (QL), two of the many possibilities adequately cover the topic for purposes of this essay. The British report *Mathematics Counts* (Cockcroft 1982) popularized the term *numeracy* and defined it in part as "an 'at homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical demands of everyday life"(Cockcroft 1982, 11). More recently, the International Life Skills Survey, as quoted in *Mathematics and Democracy: The Case for Quantitative Literacy* (Steen 2001), offers a slightly broader definition of quantitative literacy as an "aggregate of skills, knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem-solving skills that people need in order to engage effectively in quantitative situations arising in life and work" (Steen 2001, 7).

There are strong ties between statistical thinking, data analysis, and quantitative literacy in terms of historical developments, current emphases, and prospects for the future. As pointed out in *Mathematics and Democracy* (Steen 2001), the American Statistical Association (ASA) conducted a National Science Foundation-funded project called Quantitative Literacy in the mid-1980s that produced materials and workshops to introduce mathematics teachers at the middle and high school levels to basic concepts of data analysis and probability. The project was built around a hands-on, active learning format that involved student projects and appropriate use of technology.

The ASA QL program was motivated by the Schools Project in England that had introduced statistics into the national curriculum, using the report *Mathematics Counts* (Cockcroft 1982) as one of the supporting documents. This report noted that statistics is "essentially a practical subject and its study should be based on the collection of data . . . by pupils themselves." To this end it urged "in-service training courses on the teaching of statistics not only for mathematics teachers but also for teachers of other subjects" as well as "teaching materials which will emphasize a practical approach" (Cockcroft 1982, 234). Even then, 20 years ago, the Cockcroft commission recognized that "micro-computers . . . offer opportunities to illuminate statistical ideas and techniques" (Cockcroft 1982, 235). All these points were taken to heart by the ASA QL team, and all are still valid concerns.

The emphasis on statistical thinking and data analysis that was introduced in Britain migrated to Canada and was picked up as a main theme for U.S. K–12 education by a Joint Committee of the ASA and the National Council of Teachers of Mathematics (NCTM). The ASA-NCTM QL project served as a model for the data analysis and probability strand in *Curriculum and Evaluation Standards for School Mathematics* published by NCTM (1989), a strand that is even stronger in the updated edition (NCTM 2000).

The movement to include data analysis and probability in the school mathematics curriculum thus has some of the same historical roots as the current QL movement, and has similar emphases. Properly taught, statistical thinking and data analysis emphasize mathematical knowledge and skills that enable an individual to cope with the practical demands of everyday life. They also develop knowledge, beliefs, dispositions, habits of mind, communication capabilities, and problem-solving skills that people need to engage effectively in quantitative situations arising in life and work. It is no accident that almost all of the examples given in the opening paragraphs of *Mathematics and Democracy* (Steen 2001) are statistical in nature.

Simultaneous with the K–12 effort, many statisticians began emphasizing statistical thinking at the college level. As mentioned above in the discussion of statistical literacy, excellent textbooks and other materials as well as numerous college courses have been developed around this theme. These deal with issues of quantitative literacy in much more authentic ways than almost any mathematics text seems to.

Because statistics and quantitative literacy share so much in common, we hope that statisticians and mathematics educators will work together to build a strong emphasis on QL in the school and college curriculum. Many statisticians would probably disagree with the statement in *Mathematics and Democracy* (Steen 2001) that QL is "not the same as statistics." Indeed, many think that a very large part of QL *is* statistics (statistical thinking or data analysis), just as the Cockroft commission thought that statistics was a large part of numeracy. In what follows, we take a more detailed look at the common ground between statistics and QL and suggest ways of building on that commonality for the good of all.

QL and Citizenship

Patricia Cline Cohen, quoting Josiah Quincy, notes in her essay that one of the duties of responsible government is to provide statistical knowledge about the general welfare of its citizens. Hard data "are to be sought and ought to be studied by all who aspire to regulate, or improve the state of the nation. . ." (Cohen, see p. 7). In fact, the very word "statistics" derives from its use to collect information on and about the state. A good example of the growth of statistics in government can be seen in the development and expansion of the U.S. Census Bureau over the years and the widespread uses to which its data are put. Developing an informed citizenry is one of the tasks of public education and, in light of the emphasis on data within the government, a large part of that task involves improving the quantitative literacy of all citizens. That statistics can be misused by politicians (and others) is one of the reasons citizens need some skill in statistical thinking and reasoning with data.

According to Cohen, statistics are a powerful tool of political and civic functioning, and at our peril we neglect to teach the skills required to understand them. In large measure, Cohen equates quantitative literacy with statistics and makes a strong case for including statistics in everyone's education. With this, statisticians certainly can agree. They would not agree, however, with Cohen's statement that "statistics has become a branch of mathematics." Statistics has many roots, including business, engineering, agriculture, and the physical, social, and biological sciences; it deals with many issues that would not be considered mathematical. Emphasis on context is one such issue; emphasis on the design of studies is another. Although statistics uses mathematics, the key to statistical thinking is the context of a real problem and how data might be collected and analyzed to help solve that problem. Some would say that the greatest contributions of statistics to modern science lie in the area of design of surveys and experiments, such as the demographic and economic surveys of the Census Bureau and the Bureau of Labor Statistics and the experiments used in many health-related studies.

In fact, statistics has much broader uses than its mathematical roots might suggest, and many, including the federal government itself, are attempting to enlighten citizens about the proper collection, analysis, and interpretation of data. One example of this is the effort of the FedStats Interagency Task Force to develop a statistical literacy program for users of the Federal Statistical System. A related effort is embodied in a recent report from the National Research Council entitled *Information Technology Research for Federal Statistics,* which talks about the importance of literacy, visualization, and perception of data:

> Given the relatively low level of numerical and statistical literacy in the population at large, it becomes especially important to provide users with interfaces that give them useful, meaningful information. Providing data with a bad interface that does not allow users to interpret data sensibly may be worse than not providing the data at all, ... The goal is to provide not merely a data set but also tools that allow making sense of the data. (NRC 2000, 20)

These and other efforts by the federal government to improve statistical literacy are supported by Katherine Wallman, chief statistician of the US government, who said in a 1999 speech (Wallman 1999):

> Electronic dissemination is truly a boon to national statistical offices anxious to make their data more accessible and useful—and to user communities equipped to handle the wealth of available information. But this technology remains to a degree a bane, for while we have taken monumental strides in making our nation's statistics electronically available, attention to documentation in electronic media has lagged. And I continue to argue, as I have for almost a decade, that the gap between our citizens' computer literacy and their "statistical literacy" remains significant.

Citizens encounter statistics at every turn in their daily lives. Often, however, they are ill-equipped to evaluate the information presented to them. Fortunately, quantitative literacy initiatives show prospects of enhancing the statistical literacy of the next generation. Our ideal would be students who can use statistics to keep their fingers on the pulse of humanity, as envisioned by the great Belgian statistician and social scientist Adolph Quetelet:

> I like to think of the constant presence in any sound Republic of two guardian angels: the Statistician and the Historian of Science. The former keeps his finger on the pulse of Humanity, and gives the necessary warning when things are not as they should be. The Historian . . . will not allow humanity to forget its noblest traditions or to be ungrateful to its greatest benefactors. (Walker 1945, 10)

QL and the Workplace

Everyone agrees that business needs workers with QL skills, but according to Linda Rosen and her colleagues in their essay, it is not at all clear what those skills are or how urgently they are needed. In fact, the types of skills needed vary from business to business, and it may require some serious research to sort out the best set of skills for the workforce of tomorrow. Rosen offers sound advice, emphasizing notions of communication and cooperation that are similar to skills that often are seen as part of QL itself. In particular, she urges advocates of quantitative literacy to better document the existing level and anticipated need of QL in the workplace, to raise general awareness about the importance of QL in today's workplace, and to engage educators to help upgrade the QL skills of the workforce based on identified quantitative needs (Rosen et al., see pp. 43–52).

These recommendations fit well with current efforts in the statistics community to build bridges between the academic community and business, industry, and government to ensure an effective statistics education for the workforce of the future. Somewhat surprisingly, however, the level of skills attached to quantitative literacy varies greatly among those quoted by Rosen, ranging from merely knowing basic arithmetic to making "judgments grounded in data." If such judgments are thought of in the sense of statistical thinking and data analysis, they are much deeper than basic mathematical skills and require an educational component that is not found in traditional mathematics courses. Statistical thinking has a stochastic component (could this variation be caused by chance alone?) that is essential to intelligent study of business, industry, and government processes.

It is important to realize that data, information, and knowledge are a part of a hierarchy: an event yields observations called *data,* which are collected and processed into *information,* which is analyzed and combined with human intelligence to produce *knowledge*. *Wisdom* is the product of knowledge, judgment, and experience. Such taxonomies are important in new fields such as data mining—the process of discovering knowledge through data. As these fields become increasingly important to society, the statistical aspects of thinking intelligently about data and its uses (and misuses) become critical. Most often, the teaching of statistics only reaches the information stage because moving to the higher stages of knowledge and wisdom requires setting the information inside a framework in which to make intelligent judgments. If statistical thinking is a part of the framework, issues such as context (including the surrounding science) and variation are taken into account. How to go beyond the information level in understanding the world around us is one way to phrase the key intellectual challenge of QL. Statisticians surely agree with those cited by Rosen who argue that the core mathematics curriculum must be "something more than arithmetic proficiency."

Although business leaders may be confused about the details of what QL is and how much of it they want, as Rosen suggests, most enlightened leaders of business and industry see the advantages of quantitative thinking quite clearly in at least one area, that of quality control and productivity improvement. The total quality management (TQM) effort is giving way to the Six Sigma improvement initiative, which has become extremely popular in the past several years. In addition to generating a great deal of discussion within statistics and quality-control circles, it has been one of the few technically oriented initiatives to generate significant interest from business leaders, the financial community, and the popular media. Hitching the QL wagon to the Six Sigma star would be one way to move QL higher on the agenda of business leaders.

QL and Curriculum

A central theme of QL is that the meaning of "literate" must be expanded to include quantitative literacy and that the latter, like the former, must be addressed across the curriculum. This theme is clearly stated in the essay by Randall Richardson and William McCallum, who enunciate two main criteria for a QL curriculum: it must go beyond the basic ability to read and write mathematics to the development of conceptual understanding, and it must be engaged with a context, be it humanities, business, science, engineering, technology, or everyday life (Richardson and McCallum, see pp. 99–106).

Richardson and McCallum argue, along with many others, that QL cannot be regarded as the sole responsibility of teachers of mathematics, whether in high school or college. It is the responsibility of those in other disciplines to help provide basic tools and conceptual understanding and to model the use of mathematics as a way of looking at the world. In short, QL should be the focus of mathematics across the curriculum. The nurturing of QL across the curriculum, however, requires strong administrative support and significant institutional change.

Those experienced with teaching statistics suggest that one way to garner administrative support and foster institutional change is to tie much of QL to the statistics curriculum, everywhere it is housed. The very lifeblood of statistics is context, and the current teaching of statistical thinking and modern data analysis is built around conceptual understanding (calculations are done by machine). Because it is *used* across the curriculum, in most colleges and universities statistics already is *taught* across the curriculum. It would make practical as well as pedagogical sense to anchor the expansion of QL to the statistics teaching efforts of colleges and universities. Indeed, some postsecondary institutions ranging from liberal arts colleges (Mt. Holyoke) to large research universities (Ohio State) have centered much of the quantitative reasoning component of their general education requirements on statistics courses.

QL and Mathematics

Closely related to the issue of curriculum is the relationship between QL and mathematics. Deborah Hughes Hallett asserts in her essay that QL is the ability to identify and use quantitative arguments in everyday contexts, that it is more a habit of mind than a set of topics or a list of skills. QL is more about how mathematics is used than about how much mathematics a person knows. For this and other reasons, a call to increase QL is a call for a substantial increase in most students' understanding of mathematics. It is, therefore, not a dumbing down of rigor but an increase in standards. According to Hughes Hallett, this increase is essential because "the general level of quantitative literacy is currently sufficiently limited that it threatens the ability of citizens to make wise decisions at work and in public and private life" (Hughes Hallett, see p. 91).

Statisticians will find it interesting (and gratifying) that probability and statistics are the only subject areas that Hughes Hallett mentions specifically. Indeed, she finds the absence of these subjects in the education of many students remarkable given that they are so "extensively used in public and private life." Simply requiring more students to study advanced mathematics is not the answer: they actually must be taught QL by solving problems in context. Courses must demand "deeper understanding," which will require a coordinated effort to change both pedagogy and assessment.

Although there is much to agree with in Hughes Hallett's essay, statistics educators would probably disagree with the claim that ". . . the teaching of probability and statistics suffers from the fact that no one can agree on when or by whom these topics should be introduced." The statistics community played an important role in developing the NCTM standards (1989, 2000) and offers strong support for the data analysis and probability strand contained in these recommendations. Similarly, ASA has been involved in the expansion of the data analysis and probability section of the National Assessment of Educational Progress (NAEP) framework for the 2004 examination. The NCTM recommendations for all grade levels, which are reflected in the NAEP framework, call for instructional programs from prekindergarten through grade 12 that enable all students to:

> . . . formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them; select and use appropriate statistical methods to analyze data; develop and evaluate inferences and predictions that are based on data; understand and apply basic concepts of probability (NCTM 2000, 48).

With NSF support, ASA has developed a series of supplemental materials for teaching modern data analysis in the elementary, middle, and high school grades called, respectively, Exploring Statistics in the Elementary Grades, Quantitative Literacy, and Data Driven Mathematics. (See the education section at www.amstat.org or the Dale Seymour section of Pearson Learning at www.pearsonlearning.com.) These materials support and enhance the NCTM recommendations, and thus also the kinds of quantitative literacy that Hughes Hallett seeks.

The Advanced Placement (AP) Statistics course has become quite popular among high school teachers and students; its course description (see http://apcentral.collegeboard.com/repository/ap01.cd_sta_4328.pdf) reflects modern trends in data analysis that now are being emulated in some college courses. Statistics educators discovered long ago that classroom activities, laboratory activities, and group projects really work. The Mathematical Association of America (MAA) publication *Teaching Statistics: Resources for Undergraduate Instructors* showcases many examples of materials and programs that support this approach (Moore 2000). That the statistics community has rallied around these ideas is evidenced by the promulgation of good resources for hands-on, active teaching of statistics at both the school and college levels.

At the college level, both ASA and MAA have prepared guidelines concerning the undergraduate teaching of statistics. The ASA "Curriculum Guidelines for Undergraduate Programs in Statistical Science" encourages a broad range of programs that offer all students useful options beyond the traditional introductory course:

> Undergraduate statistics programs should emphasize concepts and tools for working with data and provide experience in designing data collection and in analyzing real data that go beyond the content of a first course in statistical methods.

> The detailed statistical content may vary, and may be accompanied by varying levels of study in computing, mathematics, and a field of application. (ASA 2001, 1)

Reports from the MAA (CUPM 1993) recommend that all undergraduate mathematical sciences majors should have a data-centered statistics course. Taken together, the standards, guidelines, and curriculum materials fashioned by the statistics community (with support from the mathematics community) give solid evidence that many pieces of the "coordinated effort" needed to improve quantitative literacy are in place. The QL reform that may be coming should make good use of the projects and related ideas already afloat within the statistics education community.

To be honest, however, many statistics courses still are taught in a manner that misses the QL point. This is partly because tension always exists between breadth of coverage and deep understanding—the latter of most importance to QL. Although the statistics education community may have reached consensus on how to deal with the tension, this consensus does not always play out easily in the classroom. Courses serve many clients, some of whom demand coverage of many specific topics in statistical inference.

Jan de Lange's paper, also about QL and mathematics, introduces two new and important ideas (de Lange, see pp. 75–89). First, it extends the definition of quantitative literacy to the term "mathematical literacy" because of the indisputable fact that much more in mathematics is useful besides numbers. Indeed, many aspects of statistical thinking (which de Lange includes under the name "uncertainty" as one of his core phenomenological categories) are not about numbers as much as about concepts and habits of mind. For example, the idea of a lurking variable upsetting an apparent bivariate relationship with observational data is a conceptual idea, part of statistical thinking but not particularly about numbers. The notion that designed experiments are more reliable than observational studies is another very important nonquantitative idea.

De Lange's second important idea is that if mathematics were properly taught, the distinction between mathematical content and mathematical literacy "would be smaller than some people suggest it is now." The issue is part of the aforementioned tension between breadth of coverage and depth of understanding, but it also suggests a resolution of the dilemma of QL courses. Separate courses in QL create serious problems. First, students are pigeonholed into those capable of taking "real mathematics" and those who will only need QL, thereby entrenching two classes of students in a structure that serves the nation poorly. Second, although all students need to be quantitatively literate, there is growing evidence that those who take regular mathematics courses (and who in a segregated system may not encounter much QL) are not learning many of the critical thinking skills they need.

QL and Articulation

Articulation of the K–16 mathematics curriculum is difficult to attain because it involves inextricably linked political and policy issues. Michael Kirst's essay (Kirst, see pp. 107–120) outlines the main areas of political tension: between professional leadership and political consensus, between flexible and specific standards, between dynamic standards and reasonable expectations for change, between professional leadership and public understanding of standards, between expectations and requirements. Progress toward improving articulation requires a clear signal up and down the line as to what is required. Part of that signal should be a clear message about QL.

As subject-matter standards and examinations have evolved in recent years, one of the widespread changes has been increased emphasis on data analysis and statistics; however, one of the main limiting factors is the quality of materials for teachers. "Any attempt to change mathematics curriculum," Kirst observes, "must involve rethinking textbook creation and adoption policies." Another limiting factor is the ever-present standardized examination. Multiple-choice basic skills tests do not adequately emphasize complex thinking skills such as statistical inference and multistep mathematical problem solving.

The statistics community would argue that an emphasis on statistics and QL in the mathematics curriculum could help alleviate some of these tensions. The movements to infuse school mathematics with data analysis and to enhance undergraduate statistics offerings owe much of their success to the fact that leaders from business and industry supported the efforts. It helped, of course, that these efforts began when quality improvement was a high national priority; that theme is still important for garnering support for statistics among business and political leaders. Another theme that allows statisticians to enter doors that might be more difficult for mathematicians to open is data: everyone is collecting tons of it and few know what to do with it. The public understands something of these issues. Indeed, many see the need for statistics education much more clearly than they see the need for mathematics education (although they might view statistics as a part of mathematics).

Will college faculty buy into an articulated program in mathematics education that includes a strong component of QL? Statistics faculty are likely to do so, if the success of the AP Statistics course and the support for the changes promoted by the NCTM standards and the NAEP framework are any indication. A QL emphasis would not look as radically new to a statistician as it might to a mathematician.

QL and Assessment

Many of the exhortations in the background essays about the importance of assessment to a successful QL program are subsumed in the comprehensive and detailed paper by Grant Wiggins (Wiggins, see pp. 121–143). In Wiggins' view, echoed by others, "we have often sacrificed the primary client (the learner) in the name of accountability." Wiggins seeks to put the interests of the learner back in the center of assessment.

Assessment plays a central role in QL reform. Wiggins argues for a realignment of assessment with QL that puts more emphasis on open-ended, messy, and "authentic" assessment tasks. Much of this realignment will require challenging changes in the focus of traditional instruction, including much more formative (diagnostic) assessment. To develop reliable examples of high-quality assessment strategies that are focused on a few big ideas will require significant collaboration. In addition, instructors will need training to design, administer, and grade these new types of assessment.

Wiggins makes much of "context" but seems to use the term in at least two different ways. One relates to determining the source of a problem (who is asking the question, how was the information gathered, who is the answer for, what are relevant issues in the discipline that may affect the solution). Another suggests a more philosophical, historical point of view (where do laws or theorems come from, are they debatable, can you understand the history and how it affects our present state of knowledge). Although historical perspective is important, Wiggins seems to overemphasize the role of this type of context for beginning students. To statisticians, the first definition of context is absolutely essential for any problem; the second, although helpful for some problems, is not nearly as essential.

Data analysis problems usually have a built-in context that may make them easier for teachers to attack (although not many such examples are found in Wiggins' essay). They have less of the baggage of the years of formalism that has accompanied mathematics instruction and that can be difficult for new teachers to break free from.

Wiggins differentiates between "meaning making" and "statistical reasoning," whereas statisticians would not see these as so different. His interpretation of "meaning making" as "what is mathematics and why does it matter" seems a bit narrow. Many levels of reasoning and conceptual understanding are important in mathematics even when historical perspective is incomplete. The focus should be on students' abilities to reason with their own knowledge and "understand how it works," even if their ability to question and debate is limited. Mathematics that is relevant to students' direct experiences is more meaningful to many beginning students than philosophical debates. The important message is that different experiences are meaningful to different students, and teachers need to be ready to provide students with a variety of contexts.

One of the main goals of mathematics education reform surely should be, as Wiggins claims, to make assessment design "more public, collaborative, and subject to ongoing peer review." This cannot be overemphasized, but teachers need more examples of how to do this, particularly for lower-level students. Although many of Wiggins' examples are quite grand, what teachers need are simpler tasks that could be assigned on a daily basis to help students learn to interpret and test their understanding. Fortunately, statistics educators have been thinking about authentic assessment for some time; B. L. Chance (1997) and J. B. Garfield (1994) give good overviews of current thinking on authentic assessment.

Conclusion

Statistics and quantitative literacy have much in common. Although few would disagree with this, statisticians would probably argue that QL is mainly statistics while mathematicians and mathematics educators tend to argue that QL is only partly statistics. Statistics emphasizes context, design of studies, and a stochastic view of the world. Although statistics is clearly not the same as mathematics, nor even a part of mathematics, it uses mathematics as one of its main tools for practical problem solving. Being one of the most widely used of the mathematical sciences, statistics is well entrenched in many places across the curriculum. At the K–14 level, statistics already has embarked on a program that emphasizes active learning, much in the spirit recommended by modern cognitive science. All this suggests that students will reap dividends if the two disciplines work together.

Although statistics education has gained acceptance (even respect) over the past 15 years as a key component of the K–12 mathematics curriculum, this acceptance does not always translate into classroom practice. The taught curriculum is far from reconciled with the recommended curriculum. In addressing this challenge, statistics and QL should be mutually reinforcing. Simply put, statistics has opened the door for quantitative literacy. In his background essay on curriculum in grades 6–12, Lynn Arthur Steen argues that in a balanced curriculum, "[D]ata analysis, geometry, and algebra would constitute three equal content components in grades 6 to 8 and in grades 9 to 11" (Steen, see p. 66). "Real work yielding real results," he emphasizes, "must begin and end in real data" (see p. 59).

On the pedagogical side, statistics educators have learned to emphasize both engagement and relevance. There is ample evidence that both teachers and students like a hands-on, activity-based approach to data analysis (the type recommended earlier in this essay), and that students learn better through this approach. Two teachers using data analysis materials in an algebra course and a teacher of AP Statistics have noticed how data analysis not only adds valuable content to the curriculum but also improves attitudes:

> The [data analysis] materials allow the students to construct knowledge based on their experiences, and these materials provide activities and experiences to guide the students to good concept-based skills. The students understand what and why they are doing things.
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> * * *
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> Almost all of the students were amazed by the fact that some of the mathematical concepts that they study (logs and exponentials) are actually used in such situations. I must also say that I find it very exciting to engage in these topics as well!
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> * * *
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> I would like to echo the comments about the value of an early statistics education. Yesterday, our AP Psychology teacher told me how much difference she sees between students with a stats background and students without. She said the difference was like "night and day," especially with project work. Our science teachers are saying the same thing. I guess what I am saying is what a lot of us already believe: a knowledge of statistics enriches every other discipline and life in general. Three cheers for statistics!

At the college level, statistics is one of the most widely required or recommended courses in the mathematical sciences, and the same emphasis on data analysis with hands-on activities and laboratory experiences is permeating these courses. AP Statistics is widely accepted, even emulated, by many college programs and can form one of the paths for articulating a QL message between schools and colleges. Strong ties between ASA and MAA can help cement the path.

As noted above, Adolph Quetelet emphasized the importance for science of both statisticians and historians of science. It seems appropriate, then, to end this review with a relevant observation from a historian of science, Theodore Porter:

> Statistical methods are about logic as well as numbers. For this reason, as well as on account of their pervasiveness in modern life, *statistics cannot be the business of statisticians alone,* but should enter into the schooling of every educated person. To achieve this would be a worthy goal for statistics in the coming decades. (Porter 2001, 61) (Italics added.)

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