The Democratization of Mathematics

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Mathematics has been one of the primary engines in both the intellectual and material development of human society, especially in western civilization. Because of mathematics' broad influence, mathematics education impacts societies in both profound and practical ways. The development of mathematical and scientific knowledge that represents a reality beyond particular cultures and political systems is a central element in the world's shared social condition. Mathematical reasoning is transferable to any culture without loss of effectiveness; it does not depend on subjectivity, culture, or religion. The brute power of mathematical reasoning in human history comes from its practical utility. In combination with scientific experimentation, technology, and market economies, mathematical reasoning is a taproot of our material progress, which is still the driving force in restructuring the world's economic, cultural, and political systems.

Unfortunately, neither the need to understand mathematical reasoning as a distinctive approach to knowledge nor the practical need for applied mathematical skill is fully served by our current education system.[1] From school to college, mathematics follows an isolated trajectory of increasing difficulty and abstraction whose implicit purpose is to select and prepare the best mathematics students for graduate education in mathematically intensive fields. The isolation of mathematics is part of a larger pattern of academic specialization that creates virtually impregnable barriers between the discrete disciplinary silos of mathematics, science, and the humanities. Specialization obscures the animating ideas in those studies that are crucial to cultural literacy and democratic pluralism in modern societies. It discourages the development of an interdisciplinary "general curriculum" that fosters an appreciation for the healthy tensions between the rationalist perspective of mathematics and science and the subjective and spiritual perspectives of the humanities.[2] In addition, the isolation of the mathematics curriculum impedes broad dispersion of the practical uses of mathematics, thus erecting artificial barriers to learning and the development of applied disciplines.[3]

A more accessible mathematics curriculum is critical to closing the growing gap in the opportunity to learn and earn. Success in high school mathematics from algebra through calculus partially determines access to selective colleges, even among students who do not intend to pursue programs of study that require advanced mathematics. In similar fashion, higher levels of abstract mathematics are required for access to certain professions, even when high-level mathematical procedures are unnecessary in the day-to-day work of those professions.

Mathematics needs to become more accessible if it is to fulfill both its cultural and economic roles. Accessibility requires curricula to move beyond coverage of discrete operations to a deeper and more applied understanding. To fully understand mathematics as a key idea in our intellectual and cultural history, the walls that separate the disciplinary specialties in mathematics, science, and the

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humanities need to be lowered. Moreover, to fully exploit mathematics as a practical tool for daily work and living, mathematics needs to be taught in a more applied fashion and integrated into other disciplines, especially the applied curricula that now dominate postsecondary education.

Mathematics and Economic Opportunity

Quantitative reasoning is both a key element implicit in growing modern economies and a key asset for people who work in them. The economic value of mathematical reasoning has increased inexorably since around 3000 B.C., when the priests of Sumer, in present-day Iraq, began to use mathematical procedures to develop an agricultural calendar. Over time, the subsequent improvements in agriculture efficiency created food surpluses that freed up human labor for more productive pursuits (McNeil 1999). The resultant material progress stimulated increasing social complexity that, in turn, both generated and required ever-higher levels of mathematical reasoning abilities among the general population (Greenfield 1998; Neisser 1998; Schooler 1998).

The synergy between social complexity and reasoning ability continues. In Great Britain, scores on the Raven Progressive Matrices test showed that score levels that included the bottom 90 percent of the population born in 1877 included only the bottom 5 percent of the population born in 1967 (Flynn 1998). These increases in basic reasoning ability have occurred in spite of the fact that the highest fertility rates persist among the lowest scorers.

The value of mathematical reasoning has surged at each of the great economic divides: in the shift from agriculture to an industrial economy and most recently in the shift from an industrial to a knowledge economy. In the latest economic shift, the increasing value of reasoning abilities has ratcheted up the educational ante for good jobs from high school to postsecondary education. In 1959, only 19 percent of prime-age workers (ages 30 to 59) had any college education and, until the early 1980s, many good jobs were available for high school graduates and even high school dropouts, especially for men looking for blue-collar industrial jobs. Remarkably, however, since the 1980s when the new information economy took hold, the wage advantages of college-educated workers have continued to increase even as the supply of those workers has continued to grow. For example, even though the share of college-educated workers in the labor force increased from 37 percent in the 1980s to almost 60 percent in 2000, the wage premium for those with at least some college education over those with high school or less jumped from 43 percent to a whopping 73 percent over the same time period (Carnevale and Fry 2001).

Mathematical ability is the best predictor of the growing wage advantages of increased postsecondary educational attainment (Murnane, Willet, and Levy 1995). Improvements in mathematical skills account for at least half of the growing wage premium among college-educated women and is the most powerful source of the wage advantages of people with postsecondary education over people with high school or less. Moreover, although the wage premium for college-educated workers has increased across all disciplines, it has increased primarily among those who participated in curricula with stronger mathematical content, irrespective of their occupation after graduation (Grogger and Eide 1995).

Those with stronger quantitative skills thus earn more than other workers. Data from the National Adult Literacy Survey (NALS) show that workers with "advanced/superior" mathematical literacy similar to that of the average college graduate earn more than twice as much as workers with "minimal" quantitative skills similar to average high school dropouts. Those with "advanced/superior" mathematical literacy earn almost twice as much as workers with the "basic" quantitative skills typical of below-average high school graduates. Moreover, the importance of quantitative skills in labor markets will grow in the future. Almost two-thirds of new jobs will require quantitative skills typical of those who currently have some college or a bachelor's degree (see Figure 1).

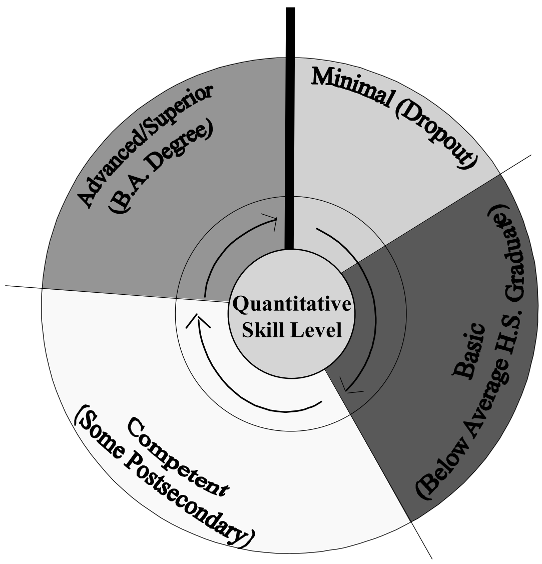
Success in the new information economy also appears to require a new set of problem-solving and behavioral skills. These skills, especially problem-solving skills, emphasize the flexible application of both mathematical and verbal reasoning abilities in multifaceted work contexts across the full array of occupations and industries. Such skills most often require the versatile use of relatively basic mathematical procedures more akin to "numeracy" and "quantitative literacy" than to higher knowledge of advanced mathematical procedures.

Who Pays for Innumeracy?

The growing importance of college-level cognitive skills, especially mathematical skills, in allocating economic opportunity is especially significant in the United States, where poorly educated individuals, not employers or governments, pay the price of educational inequality. Individuals who do not acquire college-level cognitive skills are forced into low-wage and low-benefits jobs. This is quite different from continental European labor markets, which have inherent incentives to educate and train all workers in the hope that their productivity will justify the earnings and benefits guaranteed by the European welfare states.

With no earnings or benefits guarantees, America is increasingly divided into math-haves and math-have-nots. Of course, teaching mathematics is not just about dollars and cents, but the inescapable reality is that ours is a society based on work and knowledge.

Figure 1: Quantitative Literacy and Job Opportunity, 2000-2010



Quantitative Skill Level	*Shares of the Labor Force (16-64) by Quantitative Skill Level, Percent Growth, Distribution of Jobs, and Average Annual Earnings of Year-round Workers (16-64)*
Minimal (Dropout)	**15% of the Labor Force** 13% Job Growth, 2000-2010 10% Share of New Jobs, 2000-2010 12% Share of All Jobs in 2010 2000 Earnings: $21,500
Basic (Below Average H. S. Graduate)	**24% of the Labor Force** 13% Job Growth, 2000-2010 22% Share of New Jobs, 2000-2010 25% Share of All Jobs in 2010 2000 Earnings: $26,900
Competent (Some Postsecondary)	**35% of the Labor Force** 15% Job Growth, 2000-2010 36% Share of New Jobs, 2000-2010 37% Share of All Jobs in 2010 2000 Earnings: $33,400
Advanced/Superior (Bachelorís Degree)	**26% of the Labor Force** 19% Job Growth, 2000-2010 31% Share of New Jobs, 2000-2010 26% Share of All Jobs in 2010 2000 Earnings: $48,000

Source: Authors Analysis of National Adult Literacy Survey, 1992; Current Population Survey, 2001; BLS Employment Projections, 2000-2010.

Unlike many of the continental European systems, there are minimal earnings and benefits guarantees for the unemployed or the underemployed in the United States. Even among those who are fully employed, wages and benefits depend on skill. We know that those who cannot get or keep good jobs are trapped in working poverty, underemployed, or unemployed. Eventually many of them drop out of the political system and withdraw from community life. In some cases, they may create alternative economies, cultures, or political structures that are a threat to the mainstream. If educators cannot fulfill their economic mission to help our youth and adults achieve quantitative literacy levels that will allow them to become successful workers, they also will fail in their cultural and political missions to create good neighbors and good citizens.

Higher levels of quantitative literacy increase both individual and national income. Sweden is one of the most quantitatively literate countries in the world. If the levels and distribution of quantitative skills in the United States mirrored those of Sweden, a back-of-the-envelope calculation suggests that we could increase GDP by $463 billion and reap as much as $162 billion in additional federal, state, and local tax dollars.[4]

Mathematics and International Economic Competitiveness

Our ability to produce mathematically literate citizens is also critical to the performance of the American economy in global competition. Although data from the National Assessment of Educational Progress (NAEP) show that American performance on mathematics is improving, our scores on international tests are consistently sub-par. The recent Third International Mathematics and Science Study (TIMSS) is the latest in a steady drumbeat of reports showing that U.S. students do not measure up globally. Among 38 nations tested in TIMSS, we are significantly behind 14, even with 6, and doing significantly better than 17. You do not have to have the test scores of a rocket scientist to know that in the new high-tech economic world mathematics and science education is a key asset in global economic competition.

But, if the United States is so bad at mathematics and science, how can we be so successful in the new high-tech global economy? If we are so dumb, why are we so rich?

Just look at the numbers: Japanese students are among the front-runners in the TIMSS study, but the average purchasing power of American families is 40 percent greater than the average purchasing power of Japanese families. In general, members of the European Union outperform us on TIMSS. In 1998, however, the U.S. per capita income towered over that of the European Union nations—$32,413 versus roughly $25,000 in West Germany, Belgium, Denmark, and the Netherlands and roughly $22,000 in Italy, France, Sweden, and the United Kingdom (Mishel, Bernstein, and Schmitt 2001). During the same period, U.S. unemployment has been consistently less than half the European level.

How can we reconcile our educational failures in mathematics and science and our economic success in the high-tech global economy? The first answer is that although the United States may not have, on average, the world's best overall stock of mathematically skilled graduates, because of our size we have more top students—and our economic agility allows us to use their abilities more effectively.

Our sheer size, therefore, allows us to be both mediocre in mathematics and science and number one in the world economy. The U.S. population, for instance, is roughly four times the size of France, Italy, or the United Kingdom and three times the size of Germany. Our student population is only twice as large as the Japanese school-age population but our size advantage still prevails. In the TIMSS data on eighth-grade students, the Japanese ranked fifth in mathematics and we ranked eighteenth. Sixty-four percent of Japanese eighth-graders scored in the top quartile of international benchmarks in mathematics compared with 28 percent of U.S. students. But because our eighth-grade population is twice as large as the eighth-grade population in Japan, there are 970,000 U.S. students in the top international quartile compared with 928,000 Japanese eighth-graders.

Although more is not always better, in this case it often is. For instance, we have four times as many workers as France, Italy, or the United Kingdom. Four pretty good engineers tackling a business problem often outperform one very good engineer working alone. Similarly, four companies in the software business competing directly against each other in the highly competitive U.S. product market are likely to produce better software than a single company elsewhere.

A second advantage that allows the United States to get away with relatively low levels of mathematical and scientific literacy is the flexibility that allows us to make better use of what talent there is. In the United States, minimally regulated labor markets allow employers enormous agility in hiring, paying, and allocating workers, and also allow workers more job flexibility. Pay varies with performance, and there are virtually no wage, benefits, or job guarantees. Our flexibility optimizes returns on capital investments, human and machine. With no substantial safety net, individuals, not employers or governments, pay the price of underinvestment or obsolescence of human capital.

America's characteristic flexibility also means that employers do not need to rely on the nation's homegrown mathematics and science talent. Immigration is a major source of talent among technical professionals. For instance, more than 40 percent of all engineers and almost half of all civil engineers are foreign born (National Science Foundation 2002). In addition, U.S. companies are free to produce offshore if they cannot find the talent at home at the right prices.

In Europe and Japan, by comparison, access to jobs and pay is highly regulated by skill certification and seniority. Jobs are protected shelters from economic and technological change. There is a place for everyone in the European and Japanese economies—and everyone stays in his or her place. The results? Job security and structural rigidity in a world of economic and technological changes.

The problem with the current American strategy in global competition is that our advantages will not last. We cannot remain a first-rate economic power with second-rate mathematical and scientific literacy. In global economies, all forms of advantage are temporary. The European and Japanese versions of highly planned economies surged in the 1970s but lost out to American flexibility in the 1980s. Eventually, our competitors will narrow our economic lead as they learn how to create their own versions of agility and scale. At that point, the competition will really come down to who has the best human capital—especially in a world in which people are no longer nation-bound and in which technology and financial capital ignore national boundaries as they hop across borders from one entrepreneurial opportunity to the next.

The Demographic Twist

If we are to retain the lead in the global economic race and the good jobs that go with it, we will at some point have to rely on homegrown human capital for our competitive edge. Eventually, we will have to close the education gap; however, because of demographic shifts we face at home, that may be surprisingly difficult.

A simple thought experiment demonstrates the likelihood of a shortage of workers with college-level quantitative literacy. We know that retirements begin aggressively after age 55, especially for men, and that retirement ages have been declining steadily. By 2020, about 46 million baby boomers with at least some college education will be over 55 years of age. Over the same period, if we maintain current attainment rates in postsecondary education, we will produce about 49 million new adults with at least some college education—a net gain of about three million (Carnevale and Fry 2001).

Historical and projected increases in the share of jobs that will require at least some college-level mathematical literacy far exceed this small increase in the college-educated population, however. Official projections on the share of jobs that will require at least some college education through 2020 are unavailable, but the U.S. Bureau of Labor Statistics projects a 22 percent increase by 2010 in such jobs. If the trend continues, we will experience a net deficit in workers with mathematical skills at or above the "some college" level of more than 10 million workers by 2020 (Carnevale and Fry 2001).

The Curriculum Mismatch

One way to close the emerging gap that will arise with the retiring baby boom is to align the secondary school curriculum more closely with college requirements and labor market needs. The current structure of the secondary school mathematics curriculum creates a mismatch between college admission requirements and what students choose to study once they enter college. College entry requirements tend to demand at least three years of mathematics—preferably geometry, algebra I, and algebra II; selective colleges usually expect trigonometry, calculus, and statistics as well. Advanced Placement (AP) courses in mathematics are a plus in getting accepted at selective colleges. In 2001, there were 181,000 enrollments in AP Calculus and 41,000 in AP Statistics. By way of comparison, there are roughly 150,000 seats for first-year students in the nation's top 146 colleges.

Once they enter postsecondary education, the vast majority of students increasingly avoid the highly quantitative academic silos of mathematics, science, and engineering in favor of less-quantitative curricula with a more applied focus. Of the 1,184,000 bachelor's degrees conferred in 1998, roughly 175,000 were in mathematics (12,000), science, and engineering. More than 200,000 were in the liberal arts, literature, social science, history, and humanities. The remaining two-thirds of bachelor's degrees were awarded in applied majors outside traditional academic disciplines. For example, there were 233,000 bachelor's degrees in business; 17,000 in parks, recreation, leisure, and fitness studies; 50,000 in communications; 52,000 in the visual and performing arts; 17,000 in home economics; and 25,000 in protective services (U.S. Department of Education 2000). The same pattern is seen in the expansion of applied associate degrees, certificates, certifications, and customized training in two-year colleges (Carnevale and Desrochers 2001). Of the 555,000 associate degrees conferred in 1996, 115,000 were awarded in the liberal arts and sciences, general studies, and humanities and only 758 were conferred in mathematics (U.S. Department of Education 2000).

The apparent mismatch between high school mathematics and college degrees raises two natural questions about mathematics education: Are mathematics courses creating artificial barriers to college entry? And are the majority of college students who do not continue their mathematics education getting enough mathematics? Perhaps these are the wrong questions. Advocates for a shift in focus toward quantitative literacy and numeracy over the traditional abstract curriculum and teaching methods would argue that Americans are not taking too much mathematics but are taking the wrong kind of mathematics in high school and not enough applied mathematics in the majority of college majors.

There also appears to be a mismatch between the mathematics students take in high school and the mathematics used on the job. Mathematical skills are the best general proxy for demonstrating the increasing economic returns to reasoning ability in the new economy. It is much less clear, however, that the content and methods of the current mathematics curriculum are aligned with the uses of mathematics in the world of work. Most Americans seem to have taken too little, too much, or the wrong kind of mathematics. Too many people do not have enough basic mathematical literacy to make a decent living even while many more people take courses in high school such as geometry, algebra, and calculus than ever will actually use the mathematical procedures taught in these courses.

The pattern of too little, too much, or the wrong kind of mathematics seems to persist in college. Most people abandon mathematics after high school even though a vast majority of jobs require increasing levels of quantitative literacy. The same holds even in mathematics and science disciplines: postsecondary institutions produce more Ph.D.s in quantitative disciplines than are required to fill college teaching positions, but not enough to fill K–12 mathematics and science teaching positions or enough to meet private sector needs for technically qualified managers and other professionals (Romer 2000).

A substantial share of Americans have too little mathematics. Almost 40 percent of the workforce does not have sufficient quantitative literacy for jobs that pay more than $26,900, on average (see Figure 1). These people tend to be in job categories that are growing more slowly than average and in which inflation-adjusted wages are declining. Their quantitative literacy is similar to that of a high school dropout or below-average high school graduate. At best they can perform a single arithmetic operation such as addition or subtraction when the numbers are given and the operation

Table 1.
Mathematical Literacy Paradigm from the National Adult Literacy Survey

Skill Level	Approximate Educational Equivalence	NALS Level	NALS Competencies (Quantitative)	NALS Examples (Quantitative)
Minimal	Dropout	1	Can perform a single, simple arithmetic operation such as addition. The numbers used are provided and the operation to be performed is specified.	—Total a bank deposit entry
Basic	Average or below-average high school graduate	2	Can perform a single arithmetic operation using numbers that are given in the task or easily located in the material. The arithmetic operation is either described or easily determined from the format of the materials.	—Calculate postage and fees for certified mail —Determine the difference in price between tickets for two shows —Calculate the total costs of purchase from an order form
Competent	Some postsecondary education	3	Can perform tasks in which two or more numbers are needed to solve the problem and they must be found in the material. The operation(s) needed can be determined from the arithmetic relation terms used in the question or directive.	—Use a calculator to calculate the difference between the regular and sale price —Calculate miles per gallon from information on a mileage record chart —Use a calculator to determine the discount from an oil bill if paid within 10 days
Advanced	Bachelor's or advanced degree	4	Can perform two or more operations in sequence or a single operation in which the quantities are found in different types of displays, or in which the operations must be inferred from the information given or from prior knowledge.	—Determine the correct change using information in a menu —Calculate how much a couple would receive from Supplemental Security Income, using an eligibility pamphlet —Use information stated in a news article to calculate the amount of money that should go to raising a child
Superior	High-achieving, college-educated populations	5	Can perform multiple operations sequentially, and also can find the features of problems embedded in text or rely on background knowledge to determine the quantities or operations needed.	—Use a calculator to determine the total cost of carpet to cover a room —Use information in a news article to calculate the difference in time for completing a race —Determine shipping and total costs on an order form for items in a catalog

Source: Carnevale, Anthony P., and Donna M. Desrochers. 1999. *Getting Down to Business: Matching Welfare Recipients to Jobs that Train.* Princeton, NJ: Educational Testing Service; Barton, Paul E., and Archie LaPointe. 1995. *Learning by Degrees: Indicators of Performance in Higher Education.* Princeton, NJ: Educational Testing Service.

described— determining the difference in price for theater tickets to two different shows, for instance (see Table 1).

Those who get the best jobs have taken the most mathematics. We estimate that three-fourths of those in the top-paying 25 percent of jobs have at least one year-long high school credit in algebra II. More than 80 percent have taken geometry. Twenty-seven percent of those in the top-paying jobs have at least a semester of pre-calculus and roughly 20 percent have taken calculus. Among the rest of those in the top half of the pay distribution, more than half have taken algebra II and more than two-thirds have taken geometry in high school. In the bottom half of the distribution of earnings in American jobs, roughly three-quarters have at least a year-long credit in algebra I, 63 percent have geometry, and slightly fewer than half have algebra II.

Clearly, algebra II is the threshold mathematics course taken by people who eventually get good jobs in the top half of the earnings distribution. And the more mathematics beyond algebra II, the better the odds of eventually landing a job in the top 25 percent of the earnings distribution. Yet even a casual analysis of the distribution of occupations demonstrates that relatively few of us—fewer than 5 percent—make extensive use of geometry, algebra II, trigonometry, or calculus on the job. In the year 2000, there were 146 million people in the workforce. Roughly three million were in "computer and mathematical occupations," including actuaries and statisticians. There were roughly 1.5 million engineers and architects and 1.2 million life, physical, and social scientists. In addition, there were 132,000 secondary school science teachers and 180,000 secondary school mathematics teachers (Hecker 2001). In spite of these realities, in 1998, 75 percent of high school students took geometry, 63 percent took algebra I, 62 percent took algebra II, and 18 percent took calculus (U.S. Department of Education 2000).

The mismatch between high school mathematics courses and the quantitative literacy required on the job suggests that a large share of Americans have either too much mathematics or the wrong kind. What mathematics skills are required for good jobs in the new economy? The threshold appears to be the skills associated with people who have some postsecondary education. These workers tend to have "competent" mathematical literacy—level 3 on the NALS scale. These also are the jobs that are expected to add the most new positions over the next decade and that tend to pay $33,400, on average (see Figure 1). Workers whose mathematical skills are similar to those of people with some postsecondary education can, typically, perform quantitative tasks to solve problems when the appropriate numbers and operations are not given directly but can be determined from the words used in the problem. An example would be the calculation of miles per gallon using a mileage record chart (see Table 1).

People in the most highly paid jobs tend to have the overall mathematical literacy skills of those who are advanced or superior (at levels 4 and 5 in the NALS hierarchy), skills typical of college graduates. The quantitative literacy characteristic of people with bachelor's degrees or better does not, on average, rise much above independent application of basic mathematical operations in complex situations (see Table 1). Yet people with this level of quantitative literacy are in jobs that pay $48,000, on average, per annum. This is the second fastest-growing set of jobs (see Figure 1).

It appears that the requirement for mathematical literacy in labor markets (and by implication in society) is one of an ascending ability to use basic mathematical operations with increasing independence and in situations of increasing complexity. This suggests that the way we teach mathematics may not be aligned with the uses we make of mathematics in most jobs.

Does the fact that only 5 percent of us use advanced mathematics on the job mean that we should stop teaching algebra, geometry, trigonometry, or calculus in high schools? Not necessarily. In the current educational curriculum, these higher-level courses are the means by which people learn higher-level reasoning skills even if they are not directly applicable on the job. For instance, the core competencies of computing, measuring, and manipulating shapes as well as the ability to solve problems by understanding factors and their relationships and the ability to assess the likelihood of events are consistent with the core competencies implicit in algebra, geometry, trigonometry, calculus, and statistics. Too many students, however, get bogged down in the abstract procedures that remain the focus of much of the current mathematics curriculum. Others know the formulas and procedures but do not understand what they know well enough to use mathematics outside mathematics class. We certainly should not throw out the current curriculum without a superior alternative in place, but ultimately we will need a curriculum that teaches these higher-level quantitative reasoning skills in a more applied and accessible context in which the goal is both knowledge and understanding.

How Did the Mismatch Arise?

The current mismatch between the core mathematics curriculum and our growing need for quantitative literacy is primarily an accidental product of recent history. Prior to World War II, elementary and secondary education included both academic and vocational tracks culminating in the "comprehensive" high school that prepared most students for work and a few for college. The college preparatory curriculum emphasized the traditional core subjects of mathematics, science, and humanities. For the most part, colleges extended studies in these disciplines beyond the introductory core curriculum as preparation for the professions and college teaching.

In the first few decades following World War II, this core academic curriculum experienced explosive growth. First, the Cold War and then Sputnik made sorting the best mathematicians and scientists an urgent priority in the K–16 system. Because the liberal arts curriculum was viewed as a cultural and political bulwark against communism, the humanities were supported along with mathematics and science, but to a lesser degree. The federal government fully funded university research and development in mathematics and science. Student aid increased massively, beginning with the GI Bill of Rights and culminating in the National Defense Education Act. The baby boom expanded the 18- to 24-year-old student population to outsized proportions. Public funding and demography created a tidal wave of new demands for college. The rising tide raised all the boats in the traditional college curriculum. The government provided the financial means for college and, in the 1960s, offered added motivation because the Vietnam conflict gave every male a good reason to stay in school and go to college. Unprecedented economic growth provided funding and robust job markets for college graduates from 1946 through 1973.

As a consequence, between 1946 and the mid-1970s, a massive education system was built around the core set of discrete disciplines in mathematics, science, and the humanities whose implicit purpose was to reproduce the college professoriate at the top of each disciplinary hierarchy. The rapid expansion in college faculty jobs and the growth of federally funded, university-based research and development justified the disciplinary pipelines that ran from middle school to graduate school.

In the 1970s, all the economic, demographic, and geopolitical forces that created the American Golden Age, and the Golden Age of higher education, lost momentum. The Cold War receded as communism began to collapse under the weight of its own inherent contradictions. As the Vietnam conflict wound down, young men no longer needed college deferments. The baby boom became the baby bust ending the ready supply of 18- to 24-year-old students. The postwar economic boom gave way to stagflation. As college-educated baby boomers flooded the job market, the wage premium attached to traditional college degrees was cut in half. Education became a mature industry, no longer subject to exponential growth (Menand 2002). To fill the empty seats left by the graduating baby boomers and to be more responsive to market realities in the stagflationary 1970s, postsecondary education moved toward vocationalism in its curriculum and toward nontraditional adults in its student population.

Throughout the 1970s, higher education managed to maintain enrollment levels. The college wage premium never fell below 30 percent and the demand for college-level talent, especially in vocational majors, continued to grow. After the Volker recession wrung the inflation out of the economy in the early 1980s, growth resumed and "skill-biased technology change" accelerated economic restructuring. With this restructuring, the college wage premium grew rapidly both in traditional college-level jobs and also in a growing share of jobs that previously did not require college. The share of prime-age adult workers with at least some college jumped from 2 in 10 in 1959 to 6 in 10 in the late 1990s. Even more stunning is the fact that the wage premium for college-educated workers, compared with high school educated workers, has increased by 70 percent since the 1980s, even though the supply of college-educated workers increased by 60 percent. As a result, even though the number of high school graduates declined by 700,000 between 1979 and 1991, college enrollments only dropped by 14,000 students. In the late 1990s, the college-age population began to surge again. Over the period between 1979 and 2000, this population declined by 250,000 students but admission standards and enrollments increased substantially and acceptance rates declined.

Since the late 1990s, the number of college-age youth has surged again and will not peak until 2015 (Carnevale and Fry 2000). This demographic surge, in combination with the high college wage premium, will create enormous pressure to align mathematics curricula with job requirements.

Matching Curriculum to Needs

The recognition of the need for a broader and more applied mathematics curriculum has grown appreciably since the 1970s as a result of new occupational skill requirements and new forms of work organization. The vast majority of new jobs requiring postsecondary education created since the 1970s emerged in service occupations (e.g., management, business services, education, health care, computer services) that did not require advanced mathematical operations but whose incumbents did need quantitative literacy at the level of people who had at least some postsecondary education (see Table 1). The shift toward a high-skilled service economy required more and better integration of quantitative and verbal reasoning abilities. Problem solving in high-skilled service jobs is embedded in complex social interactions that mix both quantitative and verbal reasoning (Carnevale and Desrochers 2001; Carnevale and Rose 1998). Consequently, employers and educators began focusing on analytic, problem-solving, and critical thinking skills in the 1970s, and national assessments of quantitative literacy were developed in the 1970s and 1980s.

The landmark report *A Nation at Risk,* issued in 1983, called for high standards for all students in mathematics as well as curricula that would teach students to "apply mathematics in everyday situations" (U.S. Department of Education 1983). The call for more applied and accessible curricula has been a persistent theme in education reform, but it has proven far easier to outline more rigorous mathematics standards for all students than to develop and implement effective new curricula.

Ironically, with notable exceptions, the traditional mathematics curriculum increasingly dominates secondary education and admission to postsecondary institutions. Indeed, the "back-to-basics" tone of education reform tends to strengthen the traditional academic silos in high school. Studies show that 56 percent of students completed at least three years of mathematics in 1998, compared with only 14 percent in 1982 (Roey et al. 2001).

The growing share of students who complete the traditional mathematics curriculum at least through algebra II represents both a remarkable achievement and a new opportunity. The next step in making the mathematics curriculum more accessible will be to shift toward a more applied context. A stronger emphasis on applications should improve teaching and learning for all students and will align high school mathematics more closely with college studies, work, daily life, and citizenship.

The sequence of abstract high school mathematics courses that prepares students for advanced degrees in mathematics and science is still crucial to our advanced economy, but moving the entire school-age population through the academic hierarchy from arithmetic to calculus as a sorting strategy for producing elite mathematical talent required of a small share of college majors and fewer than 5 percent of the workforce does not match well with our more general needs for applied reasoning abilities and practical numeracy. Even now students take more mathematics courses in elementary and secondary school than any other subject except English, but the narrowly focused sequence of courses from arithmetic to calculus surely is not the only way to produce reasoning abilities and does not necessarily lead to a more applied quantitative literacy (Steen 1997; Steen 2001).

The Democratization of Mathematics

The remedy for the widening cultural, political, and economic gulf between those who are literate in mathematics and those who are not is the democratization of mathematics. Democratization does not mean dumbing down. It means making mathematics more accessible and responsive to the needs of all students, citizens, and workers. The essential challenge in democratizing mathematics applies to the sciences and humanities as well. The challenge is to match curricula to cultural, political, and economic goals rather than continuing the dominance of discrete disciplinary silos.

Jacques Barzun ends his history of the last 500 years of western civilization with a disturbing vision of the future. Barzun foresees the globalization of western culture, with the exception of fierce pockets of resistance both within and outside the advanced economies. He invents a fictional historian who, looking back from the year 2300, writes:

> The population was divided roughly into two groups; they did not like the word classes. The first, less numerous, was made up of men and women who possessed the virtually inborn ability to handle the products of techne and master the methods of physical science, especially mathematics—it was to them what Latin had been to the medieval clergy. . . . It validated their position over the masses who by then could neither read nor count. . . . He, and more and more often she, might be an inventor or a theorist, for the interest in hypothesis about the creation of the cosmos and the origin of life persisted, intensified, from the previous era. The sense of being close to a final formulation lasted for over 200 hundred years. . . . It was from this class—no group—that the governors and heads of institutions were recruited. . . . On the workaday plane, the dictates of numerical studies guided the consumer, the parent, the old, and the sick. (Barzun 2000, 799)

Barzun's scenario is disturbing because it comes a little too close to home. As we begin the twenty-first century, the juggernaut of western science, mathematics, and technology seems to be increasingly compartmentalized and closed off from the mass of citizens at home and abroad. Many people live and work in a world that is driven by mathematical and scientific forces beyond their understanding or control. When these forces are embodied in new technologies and disseminated by market economies, they often are experienced as a threat to job security as well as to established traditions and institutions.

The wall of ignorance between those who are mathematically and scientifically literate and those who are not can threaten democratic cultures. The scientifically and mathematically illiterate are outsiders in a society in which effective participation in public dialogue presumes a grasp of basic science and mathematics. Their refuge is a deep mistrust of technocratic elites that often leads to passive withdrawal from public life or an aggressive and active opposition to change. In extreme cases, withdrawal leads to alienation and a retreat to various forms of secular nihilism or religious fundamentalism that explicitly reject the mathematical and scientific rationalism at the heart of western culture (Castells 1997; Gellner 1992). Citizens who are resigned to being cogs in some incomprehensible machine are not what the founders of the American republic had in mind, nor does such a society put its best foot forward in the global cultural dialogue.

From a purely economic point of view, the prospects for reforming the current mathematics curriculum to encourage broader numeracy are promising—perhaps inevitable. Expanding the pragmatic reach of mathematics and science as a tool for work has powerful backing. It serves our material interests in economic growth and individual economic opportunity. It is powered by the relentless invisible hand of market forces that increasingly disciplines educational investments according to their economic returns. In the new economy, quantitative literacy has increasing value. Increasing efficiency in the production and dissemination of mathematics education is now being driven by powerful market forces and by the authority of governments that pay the bills.

In the short haul, pragmatic economic needs and career requirements are probably a healthy spur to reform in mathematics education, but the alignment of mathematics education with economic requirements can miss the mark and go too far. The ultimate goal in making mathematics more accessible is democratization not commodification. Advances in mathematical reasoning abilities need to serve our individualistic culture and our participatory politics as well as our economy. Over the long haul, we will need to be vigilant to ensure that the economy does not hijack mathematics education.

The advocates for quantitative literacy already are leading the way toward a more accessible mathematics curriculum that serves both economic and cultural purposes. We need to continue support for their work. They offer a more engaged approach that would teach the ability to use mathematics seamlessly in varied social contexts and in different communities of practice (Ewell 2001). According to the mathematical historian Joan Richards, "When teaching mathematics is seen as a way of teaching people how to think, it can no longer be isolated" (Richards 2001).

Notes

1. The debate over whether mathematics should be taught as an abstract deductive system or in a more applied fashion sets up a false choice between purists and advocates of quantitative literacy and numeracy. The validity of mathematics is founded on deduction but it develops, and is most easily understood, in applied contexts. Similarly, the distinction between mathematical and verbal reasoning is also artificial. In the real world, reasoning is a cognitive soup of words and numbers that assumes the shape of social contexts. (Cole 1996; National Research Council 2000; Scribner 1997; Scribner and Cole 1997).
2. The most powerful call for a "general curriculum" comes from a study released in 1945, sponsored by James Conant at Harvard, officially entitled *General Education in a Free Society* and unofficially known as the "Red Book." The report argued for interdisciplinary learning to foster an appreciation of the pluralism of ideas—the rational, subjective, and spiritual—at the heart of western culture. The general curriculum was viewed as an antidote to single-minded ideologies and fanaticism. In 1945 that meant communism and fascism. In the twenty-first century, it applies to the global clash of cultures. The development of a general curriculum remains difficult in the context of specialization in the academic disciplines and the rise of vocationalism.
3. Teaching and learning that takes advantage of the synergy between applied and abstract knowledge can be deeper and more accessible, if done properly (see Barton 1990; Berryman and Bailey 1992; National Research Council 1998; Resnick and Wirt 1995; Schoen and Zubarth 1998; Steen 1997; Steen 2001; Wood and Sellers 1996).
4. Data from the International Study of Adult Literacy shows that workers in Sweden have the following distribution of quantitative literacy: Level 1 (lowest): 5 percent; Level 2: 17 percent; Level 3: 40 percent; and Level 4/5 (highest): 38 percent. In contrast, the distribution of workers' quantitative literacy in the United States is much lower: Level 1 (lowest): 16 percent; Level 2: 24 percent; Level 3: 33 percent; and Level 4/5 (highest): 27 percent (OECD 1995). To estimate the increases in GDP and taxes that would occur if we had a quantitative literacy distribution similar to Sweden's, we first calculated the number of workers in the United States at each literacy level and, second, applied the distribution of literacy in Sweden to the total number of workers in the United States to estimate how many workers would fall at each skill level if the United States' quantitative literacy levels resembled Sweden's. Taking both of the distributions, we multiplied the average earnings of U.S. workers at each skill level by the number of workers at each level and summed to get aggregate earnings. The difference in aggregate earnings using the U.S. and Swedish distributions provided an estimate of the potential increase in GDP. We then multiplied the estimated increase by 35 percent to capture the additional federal, state, and local taxes that would be paid by these more-skilled workers.

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