Learning mathematical concepts through object manipulation.

We mean by object manipulation the activity of manipulating a number of objects in an ordered way, which includes ordinary juggling. We think it promising for learning mathematical concepts because of the following reasons. (i) It can bring liveliness to a class. (ii) Novel substantiation of mathematical concepts and the skillful manipulation help students to learn the concepts in a tangible way. (iii) It gives students a chance to see a close relationship between mathematics and other fields. First, we make a natural correspondence between a kind of manipulation called palm rolling, in which we set several balls in a palm and roll them, and the groups such as the dihedral and tetrahedral groups and the permutation group. Next, we consider a kind of manipulation called cigar box, in which a set of boxes aligned in a sequence are shuffled to form another sequence. Such a change of sequences naturally corresponds to an element of the permutation group. We further demonstrate some ideas of substantiating mathematical concepts such as the induction. Finally, we comment on the impact obtained through some class experiences at our university. (Received October 05, 2004)