Setting aside their dog-eared manuscripts, the experts have spoken: Perelman’s preprints prove the Poincaré conjecture. A triumph in geometry has resolved an age-old conjecture in topology. But how does his proof change the relationship of geometry to topology? In fact, Perelman proved the far deeper geometrization conjecture of Thurston, that any three-dimensional manifold can be cut into pieces each with a simple, natural shape. Now, topology predicts whether a three-manifold really wants to be round, flat, hyperbolic, or another of eight geometries.

But does this natural geometry of a three-manifold tell us what it looks like? Dusting off the classical notion of a Heegaard splitting, a way to cut a three-manifold into two multi-handled coffee cups, one sees a further decomposition that may unlock a complete geometric blueprint, or DNA sequence. Like the genome, this blueprint may hold the key to deep questions elusive until now.

In this talk, I will work from the ground up, describing how to slice a three-manifold along spheres and donuts into its geometric pieces, describing some of these geometries and their implications (Poincaré), and finally asking “what do three-manifolds really look like on the inside?” (Received September 26, 2006)