Two teaching experiments were conducted in calculus classes in a large public university to validate and refine a framework characterizing understanding of the Riemann integral. Prior to the experiments, an initial framework was developed based on a mathematical decomposition of the Riemann sum definition of the definite integral in four layers: product, summation, limit, and function. Several of the layers also include sublayers that illustrate various ways of thinking about each layer. Analysis of the data from the teaching experiments guided modification of the framework to also reflect the cognitive development of students. Piaget’s structuralism was used as the theoretical perspective both for developing the study and for the analysis of the data. Structuralism is based on the idea that learners construct knowledge by abstracting actions and coordinations of actions within a self-regulating system. By examining these actions, researchers are able to infer students’ knowledge structures. In this talk, I will describe the layers and sublayers of the framework and explain how it can be used as a tool for data analysis as well as an aid in curriculum design. (Received September 26, 2006)