Everyone Can Teach Applications.

In linear algebra and equally in differential equations, second differences replace second derivatives. This is a terrifically valuable step in our three most basic courses:

Calculus: All students learn how \( \frac{dy}{dx} \) comes from \( \frac{\Delta y}{\Delta x} \). Until they see a little more, they miss a crucial point. We can choose forward or backward or centered differences (and we do). For second differences the good choice has coefficients 1, \(-2\), 1 and a division by \( \Delta x \) squared. Why?? This opens up a new understanding of \( y'' \).

Linear Algebra: My favorite matrices have \(-1, 2, -1\) on the three center diagonals. I write them on the day I meet the class, and ask for their properties. (Symmetry is always the first answer.) Are they invertible: yes. What are their pivots, and determinants, and eigenvalues and eigenvectors? All computations are beautiful, and these matrices are everywhere in applications—by their connection to \( -y'' \).

Differential Equations: Solving \( y'' + y = 0 \) is a pleasure. The solution is \( y = \cos(t) \) if \( y(0) = 1 \) and \( y'(0) = 0 \). What happens when \( y'' \) is replaced by a second difference in scientific computing? Again we have choices: Forward Euler, Backward Euler, Leapfrog, Trapezoidal Rule. Those four choices are controlled by their eigenvalues: Spiral out for \( |\lambda| > 1 \), Spiral in for \( |\lambda| < 1 \), Leap onto on ellipse or stay on a circle for \( |\lambda| = 1 \).

That choice is the reality of computational science. I will try to show why. (Received September 21, 2007)