A quadrilateral inside another one.

Connect each vertex of a convex quadrilateral $Q$ to the midpoint of the next (proceeding counterclockwise) side. The four connecting lines create an interior quadrilateral $I$. The ratio $\text{area}(I)/\text{area}(Q)$ can take any value in the interval $(1/6, 1/5]$. More generally, we determine what happens to $\text{area}(I)/\text{area}(Q)$ when the four midpoints are replaced by points which divide the sides in the ratio of $\rho$ to $(1 - \rho)$ proceeding clockwise. Here $\rho$ is any fixed number satisfying $0 < \rho < 1$. We also give a geometric characterization of the set of all quadrilaterals with maximal ratio. (Received September 19, 2007)