David J Schmitz* (djschmitz@noctrl.edu), 30 N. Brainard St., Naperville, IL 60613, and Charles W Gatz. Inverse Preserving Functions.

If \( f \) is a function and \( n \) a positive integer, then the exponent \( n \) in \( f^n \) is applied to the outputs of the function (as in \( \sin^2(x) = [\sin(x)]^2 \)). However, the “exponent” \(-1\) does not follow this pattern. Instead of indicating the use of the reciprocal (or inverse) of each output from \( f \), the notation \( f^{-1} \) denotes the inverse function of \( f \) in the case when \( f \) (or a suitable restriction of \( f \)) is bijective (as in \( \sin^{-1}(x) = \arcsin(x) \)). This exceptional exponential syntax can certainly be confusing for students in pre-calculus and calculus courses. But do there exist functions where the two possible interpretations of the exponent \(-1\) lead to the same result? In this paper we uncover conditions for when a bijective function \( f : G \to G \) from a group \( G \) to itself satisfies the identity \( f^{-1}(x) = [f(x)]^{-1} \) for every \( x \in G \). We call such functions inverse preserving. In addition to finding examples of such functions defined on cyclic and dihedral groups, we discovered two (relatively elementary) inverse-preserving functions defined on the multiplicative group of non-zero real numbers that pre-calculus and calculus students may have possibly encountered. (Received September 04, 2007)