We investigate Hopf algebra structures that can be introduced on incidence and path coalgebras. This general problem is an open question of interest; we look at the particular case when the Hopf algebra is required to have a nonzero integral (the non-(co)comutative generalization of the algebra of functions on a compact group). Such a Hopf algebra is known to have the "co-Frobenius" property. We give the classification of co-Frobenius coalgebras which are subcoalgebras of full path (quiver) coalgebras but which have a basis of paths, show how some of these coalgebras are related to category theory and homological algebra, and determine Hopf algebra structures on these. (Received September 21, 2010)