This paper examines the value function with associated Lagrangian being measurably time-dependent. For such value functions, we cannot take the subgradient by the point evaluation, instead by the essential values of the Hamiltonians. Furthermore, we demonstrate that such value functions satisfy a subgradient" form of the Hamilton-Jacobi equations (Received September 14, 2010)