Given a plane polygon $P$ of $m$ sides, erect squares on the sides of $P$ that all face inward or all face outward. These squares, together with the triangles interspersed between successive pairs of squares, form the first band. The second band consists of the squares erected on the remaining side of each triangle in band one, together with the quadrilaterals interspersed between these squares. Subsequent bands of $m$ squares and interspersed quadrilaterals are constructed in the same manner. Square banded $m$-gons have a number of surprising properties. For example, if $S_1$ and $S_2$ denote the sum of the areas of the squares in bands one and two, then $S_2 \geq 4 \sin^2(\pi/m)S_1$. Equality holds if and only if $P$ is affinely regular. Moreover, the non-negative value of $S_2 - 4 \sin^2(\pi/m)S_1$ can be identified geometrically, thereby extending a result of Euler from quadrilaterals to general $m$-gons. Other properties of square-banded polygons involve linear relations between the sums of areas of the squares in successive bands, the Fibonacci numbers, and some striking tiling patterns. (Received September 05, 2000)