# Mathfest 2003 Boulder, CO



MAA and Pi Mu Epsilon Student Paper Sessions July 31 – August 1, 2003



#### PI MU EPSILON

Pi Mu Epsilon is a national mathematics honor society with over 300 chapters throughout the nation. Established in 1914, Pi Mu Epsilon is a non-secret organization whose purpose is the promotion of scholarly activity in mathematics among students in academic institutions and among staffs of qualified non-academic institutions. It seeks to do this by electing members on an honorary basis according to their proficiency in mathematics and by engaging in activities designed to provide for the mathematical and scholarly development of its members.

Pi Mu Epsilon regularly engages students in scholarly activity through its *Journal* which has published student and faculty articles since 1949. In addition, the society awards monetary prizes for mathematics contests and awards established by chapters.

Since 1952, Pi Mu Epsilon has been holding its annual National Meeting in conjunction with the summer meetings of the Mathematical Association of America (MAA).



#### MAA Student Chapters

The MAA Student Chapters program was launched in January 1989 to encourage students to continue study in the mathematical sciences, provide opportunities to meet with other students interested in mathematics at national meetings, and provide career information in the mathematical sciences. The primary criterion for membership in an MAA Student Chapter is "interest in the mathematical sciences." Thus, the Student Chapter program supplements, but does not compete with, the chapters of Pi Mu Epsilon. Currently there are approximately 225 active Student Chapters on college and university campuses nationwide. Students are also members of the MAA Sections in their geographic region. Many of the MAA Sections provide special activities for students at their regularly scheduled meetings.

#### J. Sutherland Frame Lecture

Friday, August 1, 2003 8:00 - 8:50 pm Grand Ballroom, Millennium Hotel

CHAOS GAMES AND FRACTAL IMAGES

Robert L. Devaney

Boston University

In this lecture we will describe some of the beautiful images that arise from the "Chaos Game". We will show how the simple steps of this game, when iterated millions of times, produce the intricate images known as fractals. We will describe some of the applications of this technique used in data compression as well as in Hollywood. We will also challenge the students present to "Beat the Professor" at the chaos game and maybe win his computer!

The J. Sutherland Frame Lecture is named in honor of the ninth President of Pi Mu Epsilon, who served from 1957 to 1966 and passed away on February 27, 1997. In 1952, Sud Frame initiated the student paper sessions at the annual Pi Mu Epsilon meeting, which is held at the Summer Mathfests. He continually offered insight and inspiration to student mathematicians at these summer meetings.

### Student Activities Schedule of Events

### All Student Activities Events will be held in the Millennium Hotel

| Wednesday | y, Ju | ly 30 |
|-----------|-------|-------|
|-----------|-------|-------|

| 5:30 pm - 6:30 pm  | MAA/PME Student Reception               | Outdoor Pavilion      |
|--------------------|---|-----------------------|
| Thursday, July 31  |   |                       |
| 9:00 am - 5:00 pm  | Student Hospitality Center              | Hospitality Suite 331 |
| 1:00 pm - 3:00 pm  | MAA Session #1                          | Flagstaff Room        |
| 1:00 pm - 3:00 pm  | PME Session #1                          | Sugarloaf Room        |
| 1:00 pm - 3:00 pm  | MAA Session #2                          | Canyon Room           |
| 1:00 pm - 3:00 pm  | PME Session #2                          | Trail Ridge Room      |
| 3:15 pm - 5:15 pm  | MAA Session #3                          | Flagstaff Room        |
| 3:15 pm - 5:15 pm  | PME Session #3                          | Sugarloaf Room        |
| 3:15 pm - 5:15 pm  | MAA Session #4                          | Canyon Room           |
| 3:15 pm - 5:15 pm  | PME Session #4                          | Trail Ridge Room      |
| 5:20 pm - 6:30 pm  | MAA Modeling Contest Winners            | Flagstaff Room        |
| Friday, August 1   |   |                       |
| 9:00 am - 5:00 pm  | Student Hospitality Center              | Hospitality Suite 331 |
| 1:15 pm - 3:15 pm  | MAA Session #5                          | Flagstaff Room        |
| 1:15 pm - 3:15 pm  | PME Session #5                          | Sugarloaf Room        |
| 1:15 pm - 3:15 pm  | MAA Session #6                          | Canyon Room           |
| 1:15 pm - 3:15 pm  | PME Session #6                          | Trail Ridge Room      |
| 3:30 pm - 5:30 pm  | MAA Session #7                          | Flagstaff Room        |
| 3:30 pm - 5:30 pm  | MAA Session #8                          | Canyon Room           |
| 3:30 pm - 5:30 pm  | MAA Session #9                          | Sugarloaf Room        |
| 6:00 pm - 7:45 pm  | PME Banquet                             | Outdoor Pavilion      |
| 8:00 pm - 8:50 pm  | J. Suthlerland Frame Lecture            | Grand Ballroom        |
|                    | Robert L. Devaney, Boston University    |                       |
|                    | Chaos Games and Fractal Images          |                       |
| Saturday, August 2 | 4)                                      |                       |
| 9:00 am - 3:00 pm  | Student Hospitality Center              | Hospitality Suite 331 |
| 1:15 pm - 2:00 pm  | MAA Special Session on Math Horizons    | Flagstaff Room        |
| I - The First      | Arthur T. Benjamin, Harvey Mudd College | 1 lagstan 100m        |
|                    | Jennifer J. Quinn, Occidental College   |                       |
| 2:10 pm - 3:50 pm  | MAA Student Workshop                    | Flagstaff Room        |
| The stop put       | Clayton Dodge, University of Maine      | r ragaran 100tti      |
|                    | Problems, Problems, Problems!           |                       |
| 4:00 pm - 4:50 pm  | MAA Student Lecture                     | Flagstaff Room        |
| *                  | Arthur T. Benjamin, Harvey Mudd College | T 1000001 1000111     |
|                    | The Art of Mental Calculation           |                       |
| 5:00 pm - 6:00 pm  | Student Problem Solving Competition     | Hospitality Suite 331 |
| - ^                | a of our position                       |                       |
|                    |   |                       |

#### Flagstaff Room (Millennium Hotel)

1:00 - 3:00 PM

1:00 - 1:15

### A Link Between Fermat and a Conjecture of Landau W. Andrew Pruett Millsaps College

For any  $a \in \mathbb{N}$ , is there a b < a such that  $a^2 + b^2$  is prime? This reduces to the question of how many arithmetic progressions with prime generators less than  $a\sqrt{2}$  are required to cover the interval [1, a]. This question arises when considering a conjecture of Landau which postulates the existence of a prime between  $n^2$  and  $(n+1)^2 \ \forall n \in \mathbb{N}$ .

1:20 - 1:35

### A TRAFFIC SIMULATION PROGRAM Eric Bengtson Augustana College

Computers can be used to simulate traffic. I will discuss the process of writing and revising a traffic simulation program, and will include my own examples. I will also describe the information that may become available from such a simulation.

1:40 - 1:55

# ECONOMIC MODELS WITH RANDOM ELEMENTS Andrew Boettcher Augustana College

A widely used model for the value of an asset includes an underlying growth rate and a random change in value. I will discuss this model and show how a positive growth rate is needed to offset the random changes in the model.

2:00 - 2:15

### KERMACK, MCKENDRICK, AND EPIDEMIOLOGY Robin Douglas Augustana College

I will present some of the history of the foundations of mathematical epidemiology, including information about Ross, Hudson, Kermack, and McKendrick. I will also compare the models of Kermack and McKendrick with models with slightly different underlying assumptions.

2:20 - 2:35

### SOME SUBGROUPS OF A SYMPLECTIC GROUP Michael B. Henry Augustana College

I will define the symplectic groups. I will show how to find many discrete subgroups, and will provide explicit formulas for the elements of some of these subgroups. Some properties of the subgroups will also be discussed.

2:40 - 2:55

### LINEAR TRAJECTORIES ON A POOL TABLE Rebecca Huberts Augustana College

An idealized shot on a pool table may bank off a long side or a short side before going in a pocket. We show that for a shot which banks many times before going in, certain sequences of long-short banking are impossible. For example, long-short-long-short-short is an impossible sequence.

Sugarloaf Room (Millennium Hotel)

1:00 P.M. - 2:35 P.M.

1:00-1:15

### HOW DOES IT DO THAT?! Rebecca Jungman South Dakota State University – South Dakota Gamma

Disappearing faces and pennies appearing from nowhere may be magic tricks, but they are also Mathematics! These are just two of the amazing properties of hexaflexagons. This presentation will look at the theory behind the hexaflexagon's unique properties. It will also feature instructions for making these delightful "flexing" puzzles.

1:20-1:35

GERBERT: THE MATHEMATICIAN WHO SOLD HIS SOUL TO THE DEVIL

Jennifer Webb

Hood College — Maryland Delta

This past May marked the 1000<sup>th</sup> anniversary of the death of Gerbert, the remarkable French mathematician and churchman who became Pope. I will report on the results of a summer research project, in which I tried to determine when and where Gerbert learned about Arabic mathematics.

1:40-1:55

# THINKING THROUGH THE FOURTH DIMENSION Delilah Whittington Millsaps College – Mississippi Delta

Do you remember making cubes from paper in kindergarten? There is an analogous construction for a hypercube from eight cubes, but what are the rules for "folding" them together? Observations from our kindergarten problem can help answer this question.

2:00-2:15

HONEY, WHERE SHOULD WE SIT?

Brian Hahn

St. Norbert College – Wisconsin Delta

A classic calculus problem involves determining where to sit on a level floor in order to maximize the viewing angle for a picture on a wall. We will discuss interesting generalizations using calculus and geometry.

2:20-2:35

DOES YOUR VOTE COUNT?

Adam Christman

St. Norbert College – Wisconsin Delta

In closely contested elections, what are the chances that any one vote cast will affect the outcome of an election? We will extend the question to consider effects of various sized blocks of voters.

#### Canyon Room (Millennium Hotel)

1:00 - 3:00 PM

1:00 - 1:15

### HOW IMPORTANT IS CONSTRUCTIVIST METHODOLOGY IN MATH EDUCATION? Antoinette Border Hood College

What methods denote one a constructivist teacher versus a traditional teacher? How do students learn mathematics so that they retain and can recall information? What are the implications of reform in math education? These are some of the questions I addressed in my research on constructivist teaching versus traditional teaching.

1:20 - 1:35

THE HISTORY OF AMERICAN MATHEMATICS PRIOR TO 1875

Dylan Burton

Hendrix College

European mathematics experienced tremendous growth during the 18th and 19th Centuries, but little is heard about mathematics or mathematicians in America during this time. Although little original research was produced, a few individuals were able to lay the foundation for Americans to excel in mathematics.

1:40 - 1:55

"WHICH CAME FIRST: e OR NATURAL LOG?"

Christy Sue Crouch

Sam Houston State University

John Napier is credited with inventing logarithms in the early seventeenth century. However, evidence suggests that logarithms were in use as early as 6th century AD. We will discuss the development of natural logarithms and the number e apart from Napier's invention. Expect some history and oddities of mathematical development.

2:00 - 2:15

A VISUAL DEMONSTRATION OF THE FUNDAMENTAL THEOREM OF CALCULUS

Emanuel A. Lazar

Yeshiva University

The Fundamental Theorem of Calculus asserts that differentiation and integration are inverse operators (save a constant). Here we provide a visual demonstration of this fact by showing that doing differentiation "backwards" leads to the same formula as doing integration "forwards".

2:20 - 2:35

THREE HUNDRED FIFTY YEARS OF PROVING FERMAT'S LAST THEOREM
Lindsay Hardy
Sam Houston State University

In this talk, we will discuss Fermat's Last Theorem. In particular, we will look at special cases proved between the mid 1600s, the time that Fermat proposed the Last Theorem, and the early 1990s when Andrew Wiles proved it. We will also discuss contributions that led to the proof of certain cases. This talk will focus more on the history of the proof of Fermat's Last Theorem than on the mathematics behind it.

2:40 - 2:55



CENTER OF ART
Natalie Puckett
Grand Junction (CO) Central High School/Mesa State College

A primitive society believed in geocentric astronomy. Their artwork was based on this belief, where all designs can be viewed as a graph on nine vertices. How many ways can we draw any number of lines using these nine vertices to create all the possible artwork designs?

Trail Ridge Room (Millennium Hotel)

1:00 P.M. - 2:35 P.M.

1:00-1:15

### BIPOLAR PLOTS Peter Horn Hendrix College – Arkansas Beta

The idea behind bipolar plots is to extend the traditional polar coordinate system. Complexities arise from simple functions and simplicities arise from "complex" functions. This is a very accessible talk; familiarity with polar coordinates and a little trigonometry is helpful. Graphs are essential in this talk.

1:20-1:35

# CENTERS OF N-FOLD TENSOR PRODUCTS OF GRAPHS Sarah Bendall Randolph-Macon College — Virginia Iota

I will present several formulas that we found for computing the eccentricity, radius, and center of *n*-fold tensor products of graphs.

1:40-1:55

# BUSEMANN POINTS OF INFINITE GRAPHS Adam Winchester University of Nevada at Las Vegas – Nevada Beta

Busemann points are the points in the metric boundary of a metric space that are limits of geodesic rays. We answer a question posed by Marc Rieffel by giving an if and only if condition for the existence of non-Busemann points of graph metrics.

2:00-2:15

# KNOTS IN THE CUBIC LATTICE Marta Kobiela Texas A & M University – Texas Eta

Knots in a cubic lattice are placed in a 3D grid. I illustrate theorems concerning the composition of these knots, finding better upper bounds for the stick numbers of composed knots in relation to those of the original knots. Other theorems discuss composition of trefoil knots and figure-eight knots.

2:20-2:35

"NICE SURFACE!" AND OTHER WAYS TO COMPLEMENT YOUR KNOT Jennifer Novak Texas A & M University -- Texas Eta

The complements of hyperbolic knots often contain surfaces with interesting geometric properties. However, identifying knots with these "totally geodesic surfaces" has proven difficult. We will explore extensions of manifolds, termed "orbifolds," with unique geometric structures that can generate these special knots.

#### Flagstaff Room (Millennium Hotel)

3:15 - 5:15 PM

3:15 - 3:30

### A READING RAINBOW Becky Grove Youngstown State University

Everything in our world is full of color. With so many colors from which to choose, it seems impossible to know which colors are going to look the best. I will help narrow down your color palette by showing why certain colors can not be used in a given situation.

3:35 - 3:50

Any Way You Slice IT!

Melanie Antos
St. Norbert College

When you have a peanut butter sandwich you want to have as much peanut butter on it as possible. We will investigate several ways of slicing bread and the amount of peanut butter that they allow for the resulting sandwich.

3:55 - 4:10

### NEW RESULTS IN MAGIC SQUARE ENUMERATION Moshe Cohen Binghamton University (SUNY)

Define a magic square to be a square matrix whose entries are non-negative integers and whose rows, columns, and main diagonals sum to the same number. We will prove structural results for the number of such squares as a function of the matrix size and the sum, giving some examples and showing similar results for magic hypercubes.

4:15 - 4:30

Press My Buttons: Strategies for Playing Circle Mania
Amanda Kirsch
Carthage College

There are 3-10 colored circles arranged in a ring. The circles rotate through designated color rotations. The goal of the game is to change all circles to the same color in the least amount of moves. We will explore some math behind the average moves involved in the game.

4:35 - 4:50

### Brocard-Ramanujan Diophantine Equation Daniel Baczkowski Miami (OH) University

We will investigate integers of the form n!+1 and discuss some related unsolved problems in number theory. In particular, we will discuss when n!+1 is a square.

4:55 - 5:10

### PROPERTIES OF HYPER-LOGARITHMIC AND HYPER-EXPONENTIAL FUNCTIONS Benjamin F. Mitchell Taos High School

Define  $\ln(n,x)$  and e(n,x) as composite functions obtained by n repetitions of the base function, e.g.  $\ln(3,x) = \ln(\ln(\ln(x)))$  and e(2,x) = e(e(x)). This paper develops formulae for asymptotes, n-th order derivatives and integrals of these rarely-studied functions. Unlike the base functions, the derivative and integral formulae exhibit interesting, complex coefficient patterns.

#### Sugarloaf Room (Millennium Hotel)

3:15 P.M. - 4:50 P.M.

3:15-3:30

MATH, SPRINGS, AND SQUISHY THINGS

Christopher Coffin

Mount Union College – Ohio Omicron

Jello, seat cushions, and helium balloons are all very normal objects, which can be difficult to simulate in a computer program. This talk will discuss the mathematics behind physical modeling of 3D deformable objects.

3:35-3:50

MAKING USE OF GEIGER COUNTERS

David Gohlke

Youngstown State University – Ohio Xi

This presentation is about work done for the Youngstown State University physics department. It includes results on cosmic rays, angular correlation of the decay of cobalt, and the mathematics of Poisson parameter estimation.

3:55-4:10

HOW TO PARALLEL PARK USING MULTIPLE PROCESSORS

Sarah Grove

Youngstown State University – Ohio Xi

The random filling of space with parked cars can be applied in the understanding of several other problems. To understand these problems better, I have examined the spatial distribution in the car parking problem. This has been done with serial and parallel methods.

4:15-4:30

CHAOS
Vincent Berardi
St. Peter's College – New Jersey Epsilon

An introduction to the origins, theory, and applications of chaos.

4:35-4:50

BIFURCATIONS OF THE HENON MAP **Derek Pope**Seton Hall University – New Jersey Delta

An overview of a classic example of a chaotic dynamical system, the Henon map, with special emphasis on creating a nearly complete bifurcation diagram for this map. Computer –generated images will be used to demonstrate the dynamics of this map for various parameters.

Canyon Room (Millennium Hotel)

3:15 - 5:15 PM

The last four speakers in this session are students of Steven Schliker and Edward Aboufadel in the REU program at Grand Valley State University.

3:15 - 3:30

THREADING OUR WAY THROUGH A 3 DIMENSIONAL TORUS

Therese Hand

Missouri Western State College

Hamilton paths in a 2-dimensional torus are completely known. In this presentation, preliminary results for considering Hamilton paths in a 3-dimensional torus will be discussed. Of particular interest is the  $r \times s \times t$  torus where r, s, and t are set-wise relatively prime, but not pairwise relatively prime.

3:35 - 3:50

HAAR WAVELETS AND THE ORTHOGONAL DECOMPOSITION THEOREM
Nathan Atkinson
Mesa State College

Let V be a vector space and W a finite dimensional subspace of V. The orthogonal decomposition theorem guarantees that any vector  $v \in V$  can be expressed as  $v = w + w^{\perp}$ , where  $w \in W$  and  $w^{\perp} \in W^{\perp}$ . In this talk we discuss the orthogonal decomposition theorem and apply it to the Haar wavelet approximation of  $L_2[0,1]$  functions. Maple procedures will be used to illustrate the wavelet approximations. Two different wavelet bases are considered, along with a discussion of error and thresholding.

3:55 - 4:10

Breaking A.I. Puzzles Using Wavelet Analysis
Julia Olsen & Jesse Windle
Elmhurst College & University of Nebraska, Lincoln

CAPTCHAs are a tool to prevent spam companies from signing up for multiple e-mail accounts. The goal of our research is to create a computer program that uses wavelet analysis to "break" visual CAPTCHAs. The computer program will filter the CAPTCHA signal and then recognize the characters within the signal.

4:15 - 4:30

THE GEOMETRY OF THE HAUSDORFF METRIC
Christopher Bay
Truman State University

If X is a complete metric space, then the collection of all non-empty compact subsets of X is denoted H(X). The Hausdorff metric h provides a way to measure distances between the elements of H(X) and generates the complete metric space (H(X), h). Taking X to be  $\mathbb{R}^n$ , circles and lines are defined and their geometric structure is explored.

4:35 - 4:50

RATIO VECTORS OF POLYNOMIALS WITH DISTINCT REAL ZEROES

Jennifer Miller & Ben Vugteveen

Bucknell University & Grand Valley State University

We will present some recently published results that give us key insight concerning the global distribution of critical numbers of such polynomials. In addition, we will share some specific families of polynomials with distinct real zeros and corresponding results from this summer's research.

4:55 - 5:10

MATRIX SYMMETRY AND LIE GROUPS

Justin Hogg

University of Pittsburgh

The set of  $n \times n$  skew-symmetric matrices forms the Lie algebra which generates SO(n), the rotations of  $\mathbb{R}^n$ . What other types of matrix symmetry produce Lie algebras? In particular, we look at the set of skew "counter-symmetric" matrices, a Lie algebra that corresponds to a Lie group with hyperbolic properties.

Trail Ridge Room (Millennium Hotel)

3:15 P.M. - 5:10 P.M.

3:15-3:30

### FUN WITH TRIANGLES Holly Thomson St. Norbert College – Wisconsin

Take a triangle and draw segments from the vertices to determined points on the opposite sides. These segments form a triangle inside the original triangle. Taking inspiration from a 2001 Putnam Exam problem, we will investigate how the sides and area of this inner triangle are related to those of the original.

3:35-3:50

RELATIVE CONSISTENCY OF GEOMETRIES

Tristan Hauser

St. Michael's College – Vermont Alpha

A look at an argument verifying the statement: If Euclidean geometry is consistent then hyperbolic geometry is consistent. The purpose will be to demonstrate techniques of the proof as well as express the meaning and interest of such a statement and its implications.

3:55-4:10

THE ISOPERIMETRIC INEQUALITY

James Jessup

Seton Hall University – New Jersey Delta

The Isoperimetric Problem is one that has been pondered since the days of the Greeks. The problem is to find the shape that covers the largest area with any simple closed curve of a given perimeter. Several proofs will be discussed, including a technique called Steiner symmetrization.

4:15-4:30

PICK'S THEOREM
Steve Stanislav
Youngstown State University — Ohio Xi

In this talk we present a classical theorem due to Pick. Pick's Theorem tells us how to calculate the area of a polygon based on the number of elementary triangles within the polygon. The proof uses Euler's formula.

4:35-4:50

AN OVERVIEW OF MOLECULAR AND CRYSTALLOGRAPHIC SYMMETRIES

Angela Brown

Sam Houston State University – Texas Epsilon

While certain molecules have self-symmetries, other molecules do not have this property, but attain symmetries within a crystal structure. We will briefly introduce molecular symmetries, and then discuss crystallographic symmetries in both planar and three-dimensional settings. We will include a discussion of the symmetry groups of different molecular compounds.

4:55-5:10

CRYSTALLOGRAPHIC FRACTAL TILINGS

Maria Salcedo

Youngstown State University – Ohio Xi

In this talk I will discuss the seventeen crystallographic groups and show some representative patterns that I created which have fractal boundaries and tile the plane. Fractals were generated through an iterative process using Visual Basic.

This research was conducted while I was enrolled in the McNair scholarship program at Akron University.

Flagstaff Room (Millennium Hotel)

1:15 - 3:15 PM

The last five speakers in this session are students of Colin C. Adams in an REU program at Williams College.

1:15 - 1:30

# A GENERALIZATION OF THE AREA PRINCIPLE Kristina Lund Grand Valley State University

Come discover how the volume of a special tetrahedron can be used to generalize the area principle to spherical and hyperbolic geometry. We use this tool to generalize certain cyclic product relations, such as Menelaus' and Ceva's theorem,

to polygons in these geometries.

1:35 - 1:50

THE GEOMETRY OF SURFACES
Hanna Bennett
Carleton College

In subsequent talks we will discuss 3-manifolds, the 3-dimensional analog to surfaces. It is easier to understand these objects if first we explore the basic ideas in lower dimensions. We will discuss the geometries found on surfaces and look at some properties and definitions associated with hyperbolic geometry.

1:55 - 2:10

An Introduction to Hyperbolic 3-Manifolds
Michael Jennings
Cornell University

We discuss 3-manifolds and the different geometries certain 3-manifolds inherit. We then explain the relationship between 3-manifolds and knots, concluding with a discussion of surfaces that exist in knot complements, in particular, totally geodesic surfaces in hyperbolic knot complements.

2:15 - 2:30

BEYOND MANIFOLDS: A FLEXIBLE INTRODUCTION TO RIGID ORBIFOLDS

Christopher Davis

Stanford University

Building on preceding talks, we introduce orbifolds, which allow us to produce certain totally geodesic surfaces. Alternatively, a process called "doubling" allows us to show that large classes of surfaces cannot be totally geodesic. We will explore various ways to implement these methods.

2:35 - 2:50

LET BIGONS BE BIGONS:
A FURTHER EXPLORATION OF HYPERBOLIC STRUCTURE IN KNOT COMPLEMENTS
Nick Perry
Williams College

In previous talks, we have discussed geometric means to find totally geodesic surfaces, beautiful structures which exist within hyperbolic 3-manifolds. Now, we look more specifically at knot complements, further refining our search for these surfaces and developing new tools to classify and understand the knots themselves.

2:55 - 3:10

A BOULDER APPROACH TO TOTALLY GEODESIC SURFACES

Eric Schoenfeld

Williams College

Building on the previous four talks, we discuss new methods of identifying totally geodesic surfaces in knot complements. As an application of these methods, we will also show a collection of knot complements each of which realizes the bound in the Six Theorem.

Sugarloaf Room (Millennium Hotel)

1:15 P.M. - 3:10 P.M.

1:15-1:30

#### MATHEMATICAL FREEDOM **Chris Jones** Youngstown State University - Ohio Xi

In this talk we will begin with the definition of "freeness" as it pertains to group theory and other objects in abstract algebra. From there, we will investigate other times mathematical freedom occurs, such as in category theory and topology.

1:35-150

THERE ARE NO SUBGROUPS OF  $A_4$  OF ORDER SIX Gina M. Grisola Mount Union College - Ohio Omicron

Many math problems have multiple solutions, some better than others. The alternating group  $A_4$  provides a counterexample to the converse of Lagrange's Theorem because it has no subgroup of order six. I will explain several proofs which show there is no possible way to obtain a subgroup of  $A_4$  of order six.

1:55-2:10

#### SPACE GROUP INVESTIGATIONS WITH MAGMA Nicholas Ciotola Benedictine University - Illinois Iota

I will present a talk on space groups. My object has been to identify certain space groups by investigating the matrix group GL(n, Z) and its finite subgroups. These subgroups were generated with the aid of the computer algebra system Magma. Really, folks, it is fairly interesting!

This research was funded by the Howard Hughes Medical Institute as part of a summer research program at Benedictine University under the direction of Dr. Donald Taylor and Dr. Lisa Townsley.

2:15-2:30

#### A MATRICIAL ALGORITHM FOR POLYNOMIAL REFINEMENT **Emily King** Texas A & M University - Texas Eta

This project provides a methodology for obtaining sets of polynomial coefficients, refinement coefficients or integral displacements for finite polynomial refinement given the other two sets. Transforming the refinement equation of an nth degree polynomial into vectors in R" utilizing a common basis yields a solvable system of matrices and vectors. Patterns that emerge in this system provide a foundation for many proofs involving refinement of polynomials.

2:35-2:50

#### TYPES OF BLOCK PRODUCTS Sara M. Rogala Elmhurst College – Illinois Iota

For my senior paper, I investigated a particular binary operation on strings (blocks) of numbers. The operation is called the block product. In the introduction I provide the audience with background information on the subject of block products. In particular, I make reference to two types of blocks, respectively, circular and cyclic. In the main portion of the presentation, I introduce the audience to the definition of semi-cyclic blocks. I then show that the block product of two circular blocks is circular and the block product of two cyclic blocks is semi-cyclic

2:55-3:10

### ALGEBRAIC STRUCTURES AND THE LONG-TERM BEHAVIOR OF DISCRETE DYNAMICAL SYSTEMS Brenda Russo

Salisbury University - Maryland Zeta

Discrete dynamical systems known as cellular automata will be presented. We study the long-term behavior of these structures when generated over group and monoid alphabets. This research relates algebraic properties of these generating alphabets, such as those of automorphisms and subgroups, to the long-term dynamics of cellular automata.

#### Canyon Room (Millennium Hotel)

1:15 - 3:15 PM

1:15 - 1:30

### PERIODIC ORBITS IN TRIANGULAR AIR HOCKEY Andrew Baxter & Steve Weaver Millersville University

We investigate periodic orbits on frictionless triangular surfaces. When equilateral, there are infinitely many families of periodic orbits; when acute, there is a period 6 family; when isosceles, there is a period 4 family. Orbits with odd period greater than 3 have only been found on obtuse triangles.

1:35 - 1:50

### A CONJECTURE ON INTRINSICALLY LINKED GRAPHS Ryan Ottman California State University-Chico

I will discuss the conjecture mentioned in Adams' *The Knot Book* that removing any vertex from an intrinsically knotted graph results in an intrinsically linked graph. Then I will discuss intrinsic linking with an unused vertex and suggest ways to prove the conjecture or special cases of the conjecture.

1:55 - 2:10

### Modeling Type II Topoisomerases Using Computational Knot Theory Diana Nguyen University of California-Berkeley

Type II topoisomerases are enzymes, existing in all organisms, that perform strand passages to reduce DNA entanglement. We use computational knot theory to model their action by simulating random strand passage on a polygonal knot in the simple cubic lattice. We aim to understand how the enzyme unknots knotted DNA.

2:15 - 2:30

# Almost-Regular Stick Numbers of Knots Jessica Tyrus Benedictine University

The  $(\alpha, \epsilon, \eta)$ -almost regular stick number of a knot K is the minimal number of sticks needed to construct K, with lengths within  $\epsilon > 0$  of 1 and angles within  $\eta > 0$  of  $\alpha$ . For specific values of  $\alpha, \epsilon$ , and  $\eta$  we determine lower and upper bounds for many knots.

2:35 - 2:50

### EVOLVING IDEAL KNOTS, PART I Amelia Reeves University of Georgia

Knot theory, a branch of topology, has been a rich source of problems for undergraduates. In this introductory talk, we consider the intuitive problem of minimizing the rope length of knots with thickness. Our approach is to use a genetic algorithm to explore knot space. Our implementation will be discussed.

2:55 - 3:10

EVOLVING IDEAL KNOTS, PART II

Michael Piatek

Duquesne University

We present the intitial results achieved with the genetic algorithm, as well as a comparison to existing methods of minimizing rope length. Issues of performance and scalability will also be discussed.

#### Trail Ridge Room (Millennium Hotel)

1:15 P.M. - 3:10 P.M.

1:15-1:30

### WHY ARE THERE ONLY FIVE REGULAR POLYHEDRA? Luke Oeding

Franklin and Marshall College - Pennsylvania Eta

A topological proof of the fact that there are at most five regular polyhedra is given. The proof uses Euler's formula for a sphere:

$$v + f - e = 2$$

where v, f and e are the numbers of vertices, faces, and edges, respectively.

1:35-1:50

### MEANS OF COMPLEX NUMBERS Barbara Sexton Sam Houston State University – Texas Epsilon

A great deal is known about the arithmetic mean, harmonic mean, and geometric mean of real numbers. In this talk we will examine the arithmetic mean, harmonic mean and geometric mean of complex numbers. In particular we will examine properties that we expect such means to possess.

1:55-2:10

### INFINITE PRODUCTS Hai He Hunter College – New York Beta

Like infinite series, infinite products have some very interesting properties, but the topic is seldom taught in a calculus class. We will investigate some of these properties.

2:15-2:30

PARTITIONS OF {1, 2, ...,n} INTO SUBSETS OF EQUAL CARDINALITIES AND EQUAL SUMS

Andrew Anschutz

Ashland University — Ohio Rho

For what values of n is it possible to divide the integer set  $\{1, 2, ..., n\}$  into k subsets of equal cardinality such that the sum of the elements in each subset is a constant? This presentation will explore this question with different specifications of n and k.

2:35-2:50

A SURVEY OF (8, 8, 8, 1) RELATIVE DIFFERENCE SETS

Paul Carmany

Ashland University – Ohio Rho

It is known that  $C_4 \times C_4 \times C_4$  contains an (8, 8, 8, 1) relative difference set. Results of an exhaustive search for other such sets will be shared, along with an overview of additional topics dealing with relative difference sets.

This research was conducted in collaboration with Nick Bauer as part of an REU program at Central Michigan University under the direction of Dr. Ken Smith.

2:55-3:10

DIFFERENCE SETS WITHOUT SQUARES

Joel Lepak

Youngstown State University - Ohio Xi

A question I'm sure many of you have wondered about before is: How many numbers can I choose from 1 to n such that no pair of them differs by a square number? Let's say we can choose K(n) numbers in that range. If you're anything like me, you certainly hoped that

$$\lim_{n\to\infty}\frac{K(n)}{n}=0$$

Luckily this turns out to be true! As if that wasn't enough, a bound on the order of K(n) can (and will!) be established.

Flagstaff Room (Millennium Hotel)

3:30 - 5:30 PM

The first two speakers in this session are students of Frank Morgan in the SMALL program at Williams College. The last four speakers are students of Susan Loepp in an REU at Williams College.

3:30 - 3:45

### Double Bubbles in Other Universes Neil Hoffman Williams College

The recently proved Double Bubble Conjecture says that the familiar double soap bubble is the least-area way to enclose and separate two regions of prescribed volumes in  $\mathbb{R}^3$ . We report on extensions to other three-dimensional universes.

3:50 - 4:05

Double Bubbles in Surfaces
Stephen Moseley
Williams College

Double bubbles in surfaces seek the least-perimeter way to enclose and separate two regions of prescribed areas. We report on known results and current work.

4:10 - 4:25

COMMUTATIVE ALGEBRA, PART I: REVIEW OF COMMUTATIVE RINGS

Jordan Rodu

Williams College

We begin by reviewing some basic concepts in commutative ring theory. Topics include Noetherian rings, maximal ideals, prime ideals, associated prime ideals, local rings, localization of rings, and completion of rings.

4:30 - 4:45

COMMUTATIVE ALGEBRA II: COMPLETIONS OF DOMAINS WITH LOCAL GENERIC FORMAL FIBER

David Jensen

Williams College

Let T be a complete local ring and P a prime ideal of T. We present necessary and sufficient conditions for T to be the completion of a local integral domain A such that  $P \cap A = \{0\}$  and if Q is a prime ideal of T with  $Q \not\subseteq P$  then  $Q \cap A \neq \{0\}$ .

4:50 - 5:05

COMMUTATIVE ALGEBRA III: LOCAL RINGS WITH CONTROLLED FORMAL FIBERS

Ariana Dundon

Pomona College

Given a complete local ring T and a prime ideal  $\mathfrak{q}$  of T, we ask if there exists a local integral domain A whose completion is T such that we may control the inverse image of  $\mathfrak{q} \cap A$  under the map  $\operatorname{Spec} T \longrightarrow \operatorname{Spec} A$ . (Spec T denotes the set of prime ideals of T.)

5:10 - 5:25

COMMUTATIVE ALGEBRA IV: FURTHER RESULTS ON FORMAL FIBERS

John Provine

Harvard College

Given a complete local ring T and a finite set of prime ideals  $\mathfrak{q}_1, \ldots, \mathfrak{q}_n$  of T, we ask if there exists a local integral domain A whose completion is T such that we may control the inverse images of all the ideals  $\mathfrak{q}_1 \cap A, \ldots, \mathfrak{q}_n \cap A$  under the map  $\operatorname{Spec} T \longrightarrow \operatorname{Spec} A$ .

#### Canyon Room (Millennium Hotel)

3:30 - 5:30 PM

3:30 - 3:45

# GETTING FUZZY: AN INTRODUCTION TO FUZZY LOGIC Cathalain Tobin Florida Southern College

In 1965 Lofti Zadeh challenged the idea of binary logic and introduced a new form of multivariable logic, coined "fuzzy logic." This talk discusses the basic mathematical principles in fuzzy logic, and presents an application to a traffic signal which uses fuzzy logic techniques, and provides evidence that fuzzy logic is more efficient than a traditional traffic controller. 3:50 - 4:05

### A COMPETING POPULATION MODEL FOR MOSQUITOES Nicholas McClure College of St. Benedict/St. John's University

We built a differential equation model of competing mosquito populations that incorporates logistic growth, the mosquito life cycle stages and seasonal influences. A parameter sensitivity analysis suggests that the modified birth, survival and death rates would best enable a modified population to win under competition against another population.

4:10 - 4:25

### SEPARABLE PREFERENCES AND ADMISSIBLE CHARACTERS Micah TerHaar Grand Valley State University

In situations when multiple decisions are made simultaneously, it is often the case that an individual's preferred outcome on one or more of the decisions depends on the outcome of the others. We will formalize this notion of interdependence and study its relationship to similar ideas from economics.

4:30 - 4:45

### A STATISTICAL VIEW OF THE DEATH PENALTY IN TEXAS Katherine Magouirk Sam Houston State University

In this talk, we will examine the death penalty in Texas from a statistical point of view. In particular, we will examine how the time between the offense and the execution has changed since the death penalty was reinstated in 1974. We will also consider race and ethnicity.

4:50 - 5:05

A STUDY ON THE INCREASING TENDENCY OF HOME RUNS TO BE HIT IN MAJOR LEAGUE BASEBALL

Eddie Policastro

Pepperdine University

With the recent increase in the number of home runs in Major League Baseball, the purpose of my project is to use regression analysis on the rise in home runs per player to investigate each year's probability that a player breaks Barry Bonds' single-season record.

5:10 - 5:25

### CONTINUOUS APPROXIMATION METHODS FOR NOISY DATA Mihaela Guberovic Virginia Military Institute

In NMR-spectroscopy and gas chromatography, data errors are assumed to follow zero-mean Gaussian distributions with standard deviations known to increase with time. We consider subdividing the data into k subsets, each with constant-error standard deviation, computing a cubic smoothing spline on each subinterval and then connecting the k splines to achieve a smooth global approximation.

Sugarloaf Room (Millennium Hotel)

3:30 - 5:30 PM

The last five speakers in this sessions are students of Satyan Devadoss in an REU at Williams College.

3:30 - 3:45

### VERY DISCRETE DYNAMICAL SYSTEMS Rebecca Wolf College of St. Benedict/St. John's University

The chaotic behavior of discrete dynamical systems, such as that associated with the logistic difference equation, is fairly well understood. However, what happens when this equation is modified to become very discrete? In particular, we will consider the floor logistic map: Floor $[4x(1-\frac{x}{M})]$  where  $M=3,\,4,\,5...$  and discuss the dynamics of this map of the set of integers from 0 to M onto itself, including computational results and theorems.

3:50 - 4:05

### CONFIGURATION SPACES I: WHEN PARTICLES COLLIDE Suzanne Armstrong Williams College

Configuration spaces describe different ways particles can move on a manifold. We will use group theory, geometry, topology, and combinatorics to understand the structure of these spaces.

4:10 - 4:25

### CONFIGURATION SPACES II: COXETER GROUPS Michael Carr Emory University

Coxeter groups are generated by reflections in space. We will introduce Coxeter groups and diagrams and show how to derive information about configuration spaces.

4:30 - 4:45

### CONFIGURATION SPACES III: SHAVING POLYHEDRA Ananda Leininger MIT

Compactification of Coxeter complexes leads to truncations of simplices in their fundamental domains. We will explore the polytopes created by compactifying these spaces, where particles in the configuration spaces are allowed to collide.

4:50 - 5:05

# CONFIGURATION SPACES IV: GEOMETRIC PROPERTIES Eric Engler Williams College

We will discuss some amazing geometric properties of polytopes and configuration spaces derived from Coxeter complexes.

5:10 - 5:25

### CONFIGURATION SPACES V: TOPOLOGICAL PROPERTIES Michael Manapat University of California-Berkeley

We will discuss some topological properties of compactifications of configuration spaces.

### MAA Student Lectures

| 2003 | Arthur T. Benjamin        | The Art of Mental Calculation                                   |
|------|---------------------------|---|
| 2003 | Donna L. Beers            | What Drives Mathematics   |
|      |                           | and Where is Mathematics Driving Innovation?                    |
| 2002 | Colin Adams               | "Blown Away: What Knot to do When Sailing"                      |
|      |                           | by Sir Randolph "Skipper" Bacon III                             |
| 2002 | M. Elisabeth Pate-Cornell | Finding and Fixing Systems' Weaknesses:                         |
|      |                           | The Art and Science of Engineering Risk Analysis                |
| 2001 | Rhonda Hatcher            | Ranking College Football Teams                                  |
| 2001 | Ralph Keeney              | Building and Using Mathematical Models to Guide Decision Making |
| 2000 | Michael O'Fallon          | Attributable Risk Estimation:                                   |
|      |                           | A Tale of Mathematical/Statistical Modeling                     |
| 2000 | Thomas Banchoff           | Interactive Geometry on the Internet                            |
| 1999 | Edward G. Dunne           | Pianos and Continued Fractions                                  |
| 1999 | Dan Kalman                | A Square Pie for the Simpsons and Other Mathematical Diversions |
| 1998 | Ross Honsberger           | Some Mathematical Morsels                                       |
| 1998 | Roger Howe                | Some New and Old Results in Euclidean Geometry                  |
| 1997 | Aparna Higgins            | Demonic Graphs and Undergraduate Research                       |
| 1997 | Edward Schaefer           | When is an Integer the Product                                  |
|      |                           | of Two and Three Consecutive Integers?                          |
| 1996 | Kenneth Ross              | The Mathematics of Card Shuffling                               |
| 1996 | Richard Tapia             | Mathematics Education and National Concerns                     |
| 1995 | David Bressoud            | Cauchy, Abel, Dirichlet and the Birth of Real Analysis          |
| 1995 | William Dunham            | Newton's (Original) Method - or - Though This                   |
|      |                           | Be Method, Yet There is Madness                                 |
| 1994 | Gail Nelson               | What is Really in the Cantor Set?                               |
| 1994 | Brent Morris              | Magic Tricks, Card Shuffling                                    |
|      |                           | and Dynamic Computer Memories                                   |
| 1993 | Richard Guy               | The Unity of Combinatorics                                      |
| 1993 | Joseph Gallian            | Touring a Torus   |
| 1992 | Peter Hilton              | Another Look at Fibonacci and Lucas Numbers                     |
| 1992 | Caroline Mahoney          | Contemporary Problems in Graph Theory                           |
| 1991 | Lester Lange              | Desirable Scientific Habits of Mind Learned from George Polya   |
|      |                           |   |

### J. Suthlerland Frame Lectures

| 2003 | Robert L. Devaney       | Chaos Games and Fractal Images                                |
|------|-------------------------|---|
| 2002 | Frank Morgan            | Soap Bubbles: Open Problems                                   |
| 2001 | Thomas F. Banchoff      | Twice as Old, Again, and Other Found Problems                 |
| 2000 | John H. Ewing           | The Mathematics of Computers                                  |
| 1999 | V. Frederick Rickey     | The Creation of the Calculus: Who, What, When, Where, Why     |
| 1998 | Joseph A. Gallian       | Breaking Drivers' License Codes                               |
| 1997 | Philip D. Straffin, Jr. | Excursions in the Geometry of Voting                          |
| 1996 | J. Kevin Colligan       | Webs, Sieves and Money  |
| 1995 | Marjorie Senechal       | Tilings as Differential Gratings                              |
| 1994 | Colin Adams             | Cheating Your Way to the Knot Merit Badge                     |
| 1993 | George Andrews          | Ramanujan for Students  |
| 1992 | Underwood Dudley        | Angle Trisectors  |
| 1991 | Henry Pollack           | Some Mathematics of Baseball                                  |
| 1990 | Ronald L. Graham        | Combinatorics and Computers                                   |
| 1989 | Jean Cronin Scanlon     | Entrainment of Frequency                                      |
| 1988 | Doris Schattschneider   | You Too Can Tile the Conway Way                               |
| 1987 | Clayton W. Dodge        | Reflections of a Problems Editor                              |
| 1986 | Paul Halmos             | Problems I Cannot Solve                                       |
| 1985 | Ernst Snapper           | The Philosophy of Mathematics                                 |
| 1984 | John L. Kelley          | The Concept of Plane Area                                     |
| 1983 | Henry Alder             | How to Discover and Prove Theorems                            |
| 1982 | Israel Halperin         | The Changing Face of Mathematics                              |
| 1981 | E. P. Miles, Jr.        | The Beauties of Mathematics                                   |
| 1980 | Richard P. Askey        | Ramanujan and Some Extensions of the Gamma and Beta Functions |
| 1979 | H. Jerome Keisler       | Infinitesimals: Where They Come From and What They Can Do     |
| 1978 | Herbert E. Robbins      | The Statistics of Incidents and Accidents                     |
| 1977 | Ivan Niven              | Techniques of Solving Extremal Problems                       |
| 1976 | H. S. M. Coxeter        | The Pappus Configuration and Its Groups                       |
| 1975 | J. Sutherland Frame     | Matrix Functions: A Powerful Tool                             |

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