CUPM (Committee on the Undergraduate Program in Mathematics): Yesterday and Today

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Background

How is it decided what mathematical accomplishments are required for a bachelor's degree or to major in mathematics?

* How many credits should be required of mathematics majors?

* Should all majors take differential equations and/or discrete mathematics (a "phrase" not in use in CUPM's early days)?

* Should all mathematics majors take a geometry course? Should they take a course in both linear algebra and modern algebra?

* In what way should mathematics majors be exposed to the applicability of mathematics?

* Should there be tracks with different requirements for students with different mathematical/career goals?

* Should everyone with a bachelor's degree have had some mathematics and if there are to be service courses in mathematics for other disciplines or a General Education mathematics course what should these courses be?

The demography of higher education in America has changed relatively rapidly. In 2010-2011, 17,182 bachelor's degrees were awarded in mathematics and statistics, compared with 24,801 for 1970-1971, according to the National Center for Education Statistics. The comparable numbers for total number of bachelor's degrees awarded were 839,730 and 1,715,913, making clear the dramatic increase in the number of students obtaining a college education. What about degrees in computer and information science, a young field in 1970-1971 and a mature one by 2010-2011? The answer is 2,388 and 38,476. The number of students who major in computer science seems quite sensitive to economic conditions but this is a lot less true for mathematics. There is also the issue of the discrepancy between the number of students who come to college intending to major in mathematics but wind up instead majoring in another subject or dropping out of college all together. Many colleges find their mathematics majors not among those who arrived planning to major in mathematics but because their programs attract students to mathematics either through a course they take for another major (e.g. Calculus for engineering majors) or because they discover the appeal of mathematics from having taken a General Education mathematics course.

In the 1950's many students took a course in theory of equations and analytic geometry as part of completing a mathematics major. Today, Theory of Equations has all but disappeared as a course but some of the content of that course now appears spread out in other courses ranging from Calculus to linear algebra to abstract algebra. Similarly, material in the old Analytical Geometry courses is treated in Pre-Calculus or Calculus. Like history, which keeps adding more and more "events" to understand and put in context, mathematics continues to accrue more and more theorems, definitions, and ideas. While there are fashions in mathematics just as there are in clothes, the range of mathematical ideas that are finding applications in a widening panoply of areas is continually expanding. This creates problems about what requirements should be made of mathematics majors.

These issues can be looked at from a historical point of view or a policy point of view. While not so many years ago 128 credits were required to achieve a bachelor's degree, many schools have reduced this number to 120 credits. Students have different kinds of talents, employment plans and economic situations and the schools they go to vary from very large to very small, and include highly competitive schools and less competitive ones, as well as schools which can be private or public.

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Typically, the requirements for a particular major are set by the members of a department, with the approval of the governance structure of the college. Colleges which are part of a larger university structure may need to get the "university" to approve changes in their requirements. Where do mathematics department faculty, say at a new 4-year college, look to for what to require of mathematics majors and what responsibilities do mathematics faculty at existing colleges have to make sure that the major requirements of their departments meet the needs of the students they currently have?

One place faculty and administrators turn to for insights into the issues of the kind raised above is CUPM. CUPM is an acronym for the Committee on the Undergraduate Program in Mathematics, a standing committee of the MAA, now celebrating its 100th anniversary. What follows is a brief account of CUPM through the eyes of someone who served two three-year terms on CUPM (during the time that its 2004 "Handbook" was produced) and two terms as Chairperson of MAD, the Mathematics Across the Disciplines Subcommittee of CUPM, one of CUPM's standing committees. While I write from a personal viewpoint, I have tried to use a "scholarly" and "historical" perspective.

CUPM was born as CUP (Committee on the Undergraduate Program) in the 1950's due to the energy of a group of mathematicians within the MAA to address some of the issues raised by changing tides in mathematics in America. World War II had drawn to an end. From America's perspective mathematical techniques (cryptography, linear programming, operations research, probability theory, etc.) had helped win the war. A "new breed" of students was entering America's colleges in part spurred by laws helping to ease the financial burden of getting a higher education by America's returning war veterans, and the increased aspirations of women, who during the war had taken on many jobs historically reserved for men. There was also the Supreme Court's desegregation decision, which was to set in motion attention to the rights of minorities in education at all levels.

CUPM, like the Mathematical Association of America which serves as an umbrella organization for its activities, builds primarily on "expert volunteers." At various times financial support for CUPM was provided by individual colleges and universities (by donating space or faculty time) and later the National Science Foundation and the Educational Advancement Foundation. In recent years this support has been used to fund the coordination of the creation of "national report" and support materials that address the curricular issues mentioned above and elaborated upon below. These reports in recent years have aimed to reexamine the major every 10 years. The 10-year figure mirrors the many colleges that require reaccreditation every 10 years. As part of the reaccreditation process (or independently) many colleges require mathematics departments to participate in a "self-study" at least once in a 10 year period.

Leadership for CUPM has been provided by those individuals who served as chairs of the committee itself, as well as two subcommittees that refer to CUPM in their titles: CRAFTY - CUPM Subcommittee on Curriculum and Renewal in the First Two Years and MAD - CUPM Subcommittee on Mathematics Across the Disciplines. However, CUPM has also worked very closely with COMET, the Committee on the Mathematical Education of Teachers. Though CRAFTY and MAD have CUPM tied to their names, members of MAD and CRAFTY are not members of CUPM. When CUPM meets (typically at the Joint Mathematics Meetings in January and Mathfest in August), all its meetings are open to the public and usually have reports about the activities of COMET, CRAFTY and MAD as part of its "business." While CUPM is a relatively large committee, its work is carried forward through many routes. These include ad hoc committees related to a huge range of curriculum matters. CUPM has over its lifetime sponsored "panels" dealing with liberal arts mathematics, applications of mathematics, what books should be in a college's library to support the mathematics major, teacher training, proper preparation for graduate school, etc. One constant issue, in part why CRAFTY exists, is what mathematics should students be seeing in their first two years of college? In CUPM's early days, there was the issue that rural high schools often had limited mathematical options so students arrived at elite colleges with very different backgrounds. To some extent, with the advent of the Advanced Placement system and the Standards movement (NCTM and Common Core State Standards), students have more opportunities to pursue advanced mathematics (4 years of high school mathematics and/or pre-calculus and Calculus) before they arrive in college.

One way to put CUPM in historical perspective is to look at those who chaired the Committee over the years. The terms of office listed are "approximate" in part because norms for when terms of service on MAA have begun or terminated have changed with time. The number of doctoral students listed was obtained from the Mathematical Genealogy web site, and while these numbers may not be fully accurate, they offer a guide for making comparisons.

Chair	Term	Ph.D from:	Number of doctoral students
Price	1959	Harvard	6
Buck	1959-1962	Harvard	15
Duren	1963-1965	Chicago	0
Anderson	1966-1967	U. Texas (Austin)	10
Boas	1968-1970	Harvard	13
Rosenberg	1971-1973	Chicago	20
Bushaw	1974-1976	Princeton	17
Lucas	1976-1978	Michigan	21
Bushaw	1979-1982	Princeton	17
Goldstein	1982-1988	Carnegie Mellon	27
Steen	1989-1992	MIT	0
Leitzel	1992-1995	Indiana	0
Tucker	1995-1998	Dartmouth	1
Berger	1999-2001	Cal Tech	5
Pollatsek	2001-2004	Michigan	0
Bressoud	2005-2008	Temple	0
Schumacher	2008-2011	U. Texas (Austin)	0
Siegel	2011-	Rochester	0

The list of CUPM chairs is noteworthy for the prestige of the institutions from they received their doctoral degrees, the presence of several MAA presidents, book authors and editors, and careers that bridged an interest in teaching, research in mathematics, and active participation in issues involving mathematics education. However, it is also clear that in recent years there is a trend for CUPM chairs to have careers at highly selective liberal arts colleges which are neither the institutions that produce the largest numbers of mathematics majors nor the schools that grant doctorate degrees in mathematics. While some mathematics departments have sizable numbers of mathematics majors who go on to get a doctorate degree (for 2011-2012 there were about 1640 doctorate degrees awarded in the mathematical sciences), most do not. Furthermore, the reasons for getting a doctorate have changed. While in the early history of CUPM it was assumed that those who were going on to get a doctorate degree were seeking an education that involved training for a career of research in mathematics, this is no longer altogether true. Many mathematics majors who plan to get a doctorate degree do so in order to be able to have careers in college (and university) teaching rather than to become mathematics researchers per se.

CUPM, like its "parent" MAA, aims for a grassroots democratic structure with attempts to represent many "interests" ranging from geographic interests, size of school, public vs. private schools, etc. Though CUPM is a creature of MAA, it serves the mathematics community and the country in that training and skills that mathematics majors acquire provide workers for businesses and for researchers in a wide range of subjects. Mathematics majors not only go on to get higher degrees in mathematics but also in business (MBA), law, computer science, medicine, economics, statistics, physics, biology, bioinformatics, and many other fields. As a consequence, CUPM draws on mathematicians who are identified more with other professional societies than they are with the MAA. These societies include AMS, SIAM, and NCTM. Some members of MAA committees are "representatives" of these other professional societies in mathematics. CUPM attempts to get input from all of these professional societies as a way to broaden the legitimacy of the reports it prepares. In recent years this has included CUPM sponsored special sessions at national and regional meetings and focus groups to discuss CUPM issues.

Range of CUPM activities

In its earlier days, as is true today, CUPM worked through the activities of many committees, panels, and reports. CUPM also did significant amounts of what today would be called "curriculum development." One of the earliest "problems" that CUPM wanted to address was that at one time the beginning requirements of mathematics majors were not that different from those for anyone seeking a college degree. In many cases, if there was a General Education Requirement in mathematics, that requirement was a Calculus Course. Calculus was also an entry point for nearly all majors in the sciences and engineering. Since some students arrived at college from "weak" high schools, students had very erratic preparation for Calculus. In light of this CUPM worked on a book called Universal Mathematics. The title reflects the hope that one course could be designed to meet the initial mathematics needs of all incoming students. To try to see what options might work, a chart was prepared where students were classified into 6 categories by preparation in mathematics when they left high school: Less than two years; two years; 2 1/2 years; 3 years; 3 1/2 years; 4 years. The goals of the students were classified as: liberal arts and sciences, terminal course or foundation for mathematics and science, foundation for social science, engineering, education, business, pharmacy, fine arts. Intriguingly, the career options today are not that far from those considered in the 1950s except for singling out pharmacy as a special domain.

The full title of the book was: Universal Mathematics: A Book of Experimental Text Materials and there were two parts, each a separate volume.

Here are the different chapters to give some idea of the content:

Part I: (1954)

Author team: F. Bowers, R. Bradt, C. Capel, W. Duren, G. Price, W. Scott

Introduction
Chapter I Coordinate Systems
Chapter II Scientific Measurement
Chapter III Functions
Chapter IV Limits
Chapter V Derivatives
Chapter VI Integrals
Chapter VII The Logarithm and Exponential Functions
Chapter VIII Summary
Appendix I

Part II (1958)

Elementary Mathematics of Sets with Applications

Author team: R. Bradt, C. Capel, R. Davis, W. Duren, B. Jones, J. Kemeny, H. Kuhn, E. McShane, D. Morrison, G. Price, A. Putnam, W. Scott, A. Tucker

Introduction
Chapter I. Sets and Subsets
Chapter II. Natural Numbers
Chapter III. Laws of Counting
Chapter IV. Applications
    13. Decision-making Bodies
    14. A Power Index
    15. A Set Operation in Genetics
    16. Applications in Contract Bridge
    17. Switching Networks
Chapter V. Probability

Chapter VI. Mathematics Systems
Chapter VII. Algebraic Structures
- 26. The Domain of Integers
- 27. Divisibility in Integral Domains
- 28. Fields
- 29. Systems of Linear Equations

(Some section titles for chapters are given above to convey more of the flavor of what the book contained.)

Loosely speaking, Part I covers material from what today would be called college algebra and pre-calculus as well as differential and integral calculus, while Part II has topics from discrete mathematics, linear algebra, and modern algebra. These books were very ambitious even by the standards of today and were not very successful in the sense that they were not used widely.

The author team for the second part has authors whose names are more familiar today than those of the first part. H. Kuhn is Harold Kuhn, who just died in July, 2014 and is famous for his work in game theory at Princeton. He also is well known for the Kuhn-Tucker Theorem. Albert Tucker (at one time Chair of Mathematics at Princeton) was also involved with Universal Mathematics as an author.

Another remarkable CUPM effort in curriculum was published as CUPM Reports 16-18, in 1967, under the title: CUPM Geometry Conference.

Report 16: Lectures by Branko Grünbaum and Victor Klee
Convexity and Applications

Report 17: Lectures by A. M. Gleason and Norman Steenrod
Geometry in Other Subjects

Report 18: Lectures by H.S.M. Coxeter and others
Geometric Transformation Groups and Other Topics.

These volumes were an attempt by prominent geometers of the time to offer curriculum alternatives for the undergraduate courses in geometry as well as thoughts about the role of geometry in pre-college education. The issue of what role geometry should play is still a complex one, in part, because many mathematics majors become middle school or high school mathematics teachers. Traditionally geometry has played a big role in K-12 mathematics.

From the very beginning of CUPM there were discussions about the issue of the role of CUPM and MAA with regard to writing and publishing "texts" rather than "reports." Currently MAA has a very elaborate book publishing program which involves both enrichment materials, texts, and reports. CUPM no longer is actively engaged in "writing" projects other than the reports it generates, recently at 10-year intervals. The 2004 CUPM report was "delivered" in print form (to department chairs (and others)) but is also available online in pdf format. Large amounts of material were generated for the report that in prior years might have appeared in print form, but by 2004 were made available only online.

What are some of the major issues that CUPM has had to address that were not among the first challenges it had to face?

a. Tremendous growth in theoretical and applied mathematics

b. The digital revolution

c. The changing nature of the students arriving for higher education

d. In 1954 the Supreme Court handed down its famous Brown vs. Topeka Board of Education decision that addressed the problem of segregated schools in America. One response to this was "affirmative action," where students whose "track records" were not necessarily the very best were offered opportunities that otherwise might not have been available to them. Over a period of time this decision affected the education of minorities, women and the poor, which changed the profile of who came to college in America and required accommodating the needs of these students. In very recent years new Supreme Court decisions are affecting affirmative action.

e. Changes in the delivery system of higher education - online courses and MOOCs.

f. Theorems, unlike the theories of science, do not change with time.

While some parts of mathematics may come and go in interest and popularity, theorems stay theorems. For example, at the turn of the nineteenth century invariant theory occupied the minds of many of the great mathematicians of the time (Sylvester, Gordon, and Hilbert) but has little current place in research or the undergraduate curriculum. Quaternions were another topic that was widely explored and co-evolved with the theory of vectors. When CUPM was founded, linear algebra was not a widely required course for mathematics majors. Today, there would be very few mathematics majors (or for that matter those who take a few mathematics courses) who would not see some of the ideas of linear algebra.

Another major change involves the Advanced Placement (AP) system, whereby students can get college credit or skip courses in college because of courses they take in high school, as well as the results they obtain on examinations related to these AP courses. For quite a long time many people saw the tremendous growth in students who took AP Calculus as a harbinger of growth in the number of students who would get degrees in the mathematical sciences. This growth has failed to materialize and one possible reason is that students preferred the "protective" environment of high school to complete the mathematics requirement that many colleges have, as well as the "general" prestige of taking Calculus as a way to promote admission to highly selective colleges.

While in the 1950's higher education was de facto "highly selective" in America, that is much less so than in the past. Community colleges, public state university systems, and scholarship and loan systems have all worked to make it possible for students who have limited financial means to come to colleges. However, in some cases such students have more trouble completing college because internal and external expectations for completing college are not the same as is the case for students who come from more affluent backgrounds.

At the time the 2004 CUPM "Curriculum Guide" was developed, one big issue that emerged was whether to have course descriptions for individual courses such as linear algebra, topology, geometry, discrete mathematics, etc. These were relatively well known courses for which there existed many texts with wide ranges of goals and student sophistication. However, the issue also arose about whether there should be course descriptions for rapidly emerging "new" courses and areas of mathematics. Examples of such included dynamical systems, game theory, social choice, network science, bioinformatics, computational biology, etc. In the end it was decided not to have any course descriptions. However, the next CUPM report will have course description information. Rather than argue that there was “one size fits all” for, say, modern algebra, the new report will have samples of different kinds of courses for different goals and different kinds of students.

Flash points of curricular discussion

Just as there need to be course descriptions for different niches that a "generic course title" (e.g. geometry) might fill, there are many hotly debated curriculum issues at the current time that engage CUPM. These discussions center upon not only the changing nature of mathematics and its applications, but the wide range of colleges that serve a higher education student body which has also changed rapidly. Some of these issues are:

* Should an "introduction to proofs" course be required of all mathematics majors?

While what separates mathematics dramatically from other disciplines is the concept of proof, relatively few mathematics majors will be proving theorems as part of their jobs or career plans. Should being able to prove theorems be an output skill of courses required of all mathematics majors?

* Should a mathematical modeling course be required of all mathematics majors?

Mathematical modeling is the branch of mathematics that investigates how to effectively use mathematics in fields outside of mathematics via constructing "mathematical models." A mathematical model might be a physical model used to trigger questions about the object it simplifies or represents, a collection of equations that capture information available and can be used to get insights, a diagram such as a graph that shows relationships between objects, or differential and integral equations.

The Common Core State Standards in Mathematics has a Practice Standard: Model with Mathematics. However, more generally there is the fact that mathematics has two faces, theoretical and applied, both of which must be part of the mathematics that any student sees when majoring in mathematics. How best to make this happen is the source of much debate.

* Special needs of future teachers

A major employer of college mathematics majors is the K-12 educational system. The needs of future teachers with regard to their mathematical training are somewhat different from the needs of those majors who plan careers in higher education, industry, or "government."

* Research experiences

To what extent should all mathematics majors have a "research" experience as an undergraduate?

* Technology

Technological tools ranging from word processors to slide projectors, symbolic algebra packages, etc., have changed the experience for all college students. How can technology be optimally used to promote mathematics?

* Data

While statistics is a part of mathematics, it has not always been put on an equal footing with other topics within mathematics because many large schools have separate programs in statistics. However, with the advent of dramatic increases in computing power, the need for students involved with mathematics to learn more about statistical and probabilistic issues has skyrocketed.

CUPM is one of the MAA's most visible and important activities - centering as it does on what the experience of majoring in mathematics should consist of. I encourage you to learn more about CUPM and to try to participate in its many outreach efforts in order to help students engage with mathematics as effectively as possible.

Acknowledgment

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Web site:

MAA maintains a special web page devoted to CUPM. At this site, various CUPM materials can be downloaded:

http://www.maa.org/programs/faculty-and-departments/curriculum-department-guidelines-recommendations/cupm