# **Calculus I in Community Colleges:** Findings from the National CSPCC Study

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Understanding how colleges manage to keep students in the Calculus I track is an issue of national importance and the impetus behind a national study of Calculus I in the United States sponsored by the Mathematical Association of America (MAA) and funded by the National Science Foundation (DRL REESE #0910240). The study titled Characteristics of Successful Programs in College Calculus I (CSPCC) took place between 2009 and 2015 and included a large-scale survey of Calculus I programs and case studies of programs identified as successful based on survey results. The authors of this article comprised the research team for the associate degree-granting institutions in the CSPCC study. We shall refer to these as *two-year colleges*.

The purpose of this article is to present the major findings that we believe are most interesting and helpful to faculty teaching Calculus I and to administrators in making decisions about allocating resources for maximum impact. Additional publications and information gathered from the study can be found at the CSPCC website<sup>1</sup>. There is a MAA Notes Volume in progress related to the study.

### **Study Background**

In fall term of 2010, the research group administered a largescale, web-based survey to a stratified sample<sup>2</sup> of colleges and universities (over 500 instructors and 14,000 students) to identify factors that are correlated with success in Calculus I. Initial surveys were completed by the department chair or Calculus I coordinator. Calculus I faculty completed presurveys (start of fall term) and postsurveys (end of the term).

 http://www.maa.org/programs/faculty-and-departments/curriculum-development-resources/ characteristics-of-successful-programs-in-college-calculus

2 Stratified sample based on highest mathematics degree awarded at the institution: AA – associate's, BA – bachelor's, MA – master's, PhD – doctorate. Separate research teams were formed for each sector. Dr. Vilma Mesa (UM) led the two-year college research team.

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Calculus I students completed presurveys around the third week of class and postsurveys at the end of the term. A total of 207 two-year colleges were sent surveys, with a response rate of 19% (n = 40).

Next, beginning fall term of 2012, we conducted case studies of four two-year colleges identified as having successful Calculus I programs based on an analysis of survey data that included the proportion of students taking Calculus II and gains in students' confidence, interest, and enjoyment in mathematics (See Hsu, Mesa, & the Calculus Collective, 2014). We shall refer to the case-study colleges as the selected colleges. During the site visits to the selected colleges, we interviewed 15 Calculus I instructors and 28 key personnel involved with Calculus I (e.g., department chairs, tutoring center staff), and we conducted 9 student focus groups with 150 students. We also observed 10 Calculus I classrooms. Lastly, we collected documents related to Calculus I, including course syllabi and copies of exams, homework, and classroom tasks. All interviews were transcribed and analyzed using the qualitative analysis software HyperRESEARCH. In the next section, we provide background information on the selected colleges. Following this, we present four major findings drawn from several rounds of analysis. For each major finding, we briefly describe the data source and methods of analysis used, and we cite the related publication or conference paper. Unpublished conference papers are available upon request.

## **Background Information on the Selected TYCs**

Table 1 shows key characteristics of the two-year colleges (TYC) selected for case study.

In terms of structure, three of the selected colleges had a stand-alone mathematics department, while the mathematics program in TYC C was embedded within a larger science division. Because of this difference, we use the term *program* rather than *department* where appropriate (e.g., program-wide policies rather than departmental policies). Further, two-year college faculty members generally hold the rank of *instructor* and will be referred to as such.

Table 1: Characteristics of the two-year	colleges (TYCs)	selected for	case study
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College	US Region	FTE <sup>ª</sup> (2010)	FT:PT⁵	Number Calc I sections/term	Class Size
TYC A	Southeast	4,300	7:10	2	30-35
TYC B	Midwest	9,500	9:20	3 to 4	30
TYC C	West	2,800	7:0	1	30 (52°)
TYC D	Southeast	12,500	35:30	10	30 (15 <sup>d</sup> )

a. Full-Time Equivalency student enrollment from IPEDS (rounded to protect anonymity).

b. FT:PT is the ratio of full-time to part-time instructors in the mathematics department or program. The ratios include the actual number of FT and PT faculty in the program.

c. Number of students in the day of the observation, clearly exceeding official class caps.

d. For honors sections.

Three of the four programs ran on a 16-week semester with roughly 250 minutes of instruction per week spread over two class periods. The fourth program ran on a 14.5-week semester and met for 150 minutes per week on a 2- or 3-class per week schedule with an additional 75-minute mandated computer lab, used variously by instructors for computer activities or extended lecture. In each of the selected colleges, instructors were expected to adhere to common learning objectives for Calculus I that, with minor variations, included standard outcomes related to differential and integral calculus. While the course contained some theory, there were no learning outcomes related to mathematical proofs.

The 15 Calculus I instructors we interviewed included 5 women and 10 men, all full-time, tenured, or tenure-track faculty. It is noteworthy that, at all four selected colleges, adjunct faculty do not teach Calculus I except under rare circumstances. The instructors interviewed held either a Ph.D. in mathematics (n = 5) or educational leadership (n = 1), or a master's degree in mathematics (n = 5) or computer science (n = 1). The remaining instructors held a M.A.T. (n = 3). The instructors ranged in experience, with two-thirds (n = 10) having more than 10 years of experience teaching math. Three had over 25 years of experience.

### **Major Findings**

Major finding 1: High-quality instruction was the main feature identified as contributing to successful Calculus I. In each of the selected colleges, high-quality instruction was cited as a major reason for program success. This finding emerged from two rounds of data analysis that began with identifying cross-cutting themes from key faculty, staff, and administrator interviews (Mesa, White, & Burn, 2014). Next, we further analyzed instructor interviews and student focus group data in order to develop a rich description of classroom instruction and to identify resources that supported instruction (Burn & Mesa, 2014).

Descriptions of "high-quality instruction" included instructor availability and approachability, abundant content knowledge, and high expectations for developing conceptual understanding alongside procedural competency/fluency. We found that instructors in the selected colleges mostly lectured but did so with substantial interactions in the form of questions and answers. We came to refer to this as *interactive lecture*, described by one instructor as

The impression I get is that, for most, when they're saying "lecturing" that just means that they are at the front of the classroom and they're presenting material, but at the same time they're using questioning techniques and getting feedback from the students as they go along. So it's not what you might picture like a big lecture hall where it's just a presentation.

Both faculty and students attributed high levels of student engagement and faculty-student interactions to small class size combined with instructors' interpersonal skills. Two-thirds (n = 10) of the instructors started class by fielding student questions and 12 provided students with time to work problems during class. More than half of the instructors (n = 8)described motivating students through their enthusiasm for the subject, establishing personal relationships and trust with their students, and using their personal skills to lighten the mood in class and to give students positive feedback. Instructors expressed pride in being able to provide additional support to their students. In one case, the instructor held additional office hours in the department's conference room to help students with assignments. Providing this extra student support may

### Figure 1: Sample problem from 10-minute quiz given at the beginning of class.

1. True or false. If true, explain why. If false, explain why or give an example to show it is false:

1. If  $\lim f(x) = L$  then f(a) = L.

2. A rational function has at least one vertical asymptote.

3. 
$$\lim_{x \to 5^{-}} \sqrt{x-5} = 0.$$
  
4. The function  $f(x) = \begin{cases} \frac{x^2 - 6x + 5}{x-5}, & x \neq 5 \\ 4 & x = 5 \end{cases}$  is discontinuous at  $x = 5.$ 

have been partly enabled by having tenured or tenure-track instructional staff.

The majority of instructors (n = 12, 80%) used technology during class to demonstrate or motivate Calculus I concepts. This usually was accomplished with a computer projector, which was standard classroom technology. The most popular technology for such demonstrations was the graphing calculator, which was the preferred technology for students as well. In addition, six instructors (40%) mentioned using more advanced software for classroom demonstrations, such as Maple, Geogebra, or Wolfram Alpha. In two programs, students had access to a computer lab where they could explore and work on Calculus I problems that required technology.

Nearly half of the instructors (n = 7) created their own problem sets or worksheets that had the common purpose of providing opportunities for students to reinforce, integrate, or apply Calculus I concepts. One instructor described assigning additional graded problems "because the [text] book doesn't have it and I think it's good." Another instructor described the homework as "compartmentalized" and that the worksheets were an opportunity to combine topics to help students "put together what they've already had."

Lastly, both faculty and students at the selected colleges expressed their belief that Calculus I examinations were fair. Students indicated not being surprised by the exams, and most instructors (n = 10, 75%) stated that students had been exposed to problems that would be on exams either through homework or through the problem sets or worksheets. Further, most of these instructors claimed they had a "no surprises on exams" policy or did not believe in throwing students "curve balls" on exams. However, this did not translate into exams that were routine or easy. For example, one instructor asked students an "in your own words" question in the exam ("explain in your own words the meaning of derivative") and another instructor gave gave conceptually complex true/false statements in his 10-minute quizzes at the beginning of class (see Figure 1).

Major finding 2: Cognitive orientation of assigned problems varied by type of coursework and by instructor. An analysis of the problems instructors assigned to their students revealed that the *cognitive orientation*<sup>3</sup> of problems varied by type of coursework (e.g., homework versus exams) and by instructor (White & Mesa, 2014). For this analysis we collected all coursework assigned by five of six Calculus I instructors in one of the selected colleges. The coursework collected included graded and ungraded worksheets, bookwork/webwork<sup>4</sup>, and exams. There were too few problems from quizzes to include quizzes in the analysis. The combined coursework analyzed contained 4,905 different problems, and each problem was categorized as either a simple procedure, complex procedure, or rich task. Table 2 shows these categories along with verbal descriptions and an illustrative example. In creating these categories we drew from Anderson, et al.'s (2001) revised Bloom's taxonomy (Bloom, et al., 1956) and an analysis of final examination problems conducted by Tallman and Carlson (2012).

The analysis revealed several interesting findings. First, there were statistically significant differences in the cognitive orientation of problems included on different types of coursework, as shown in Table 3. More specifically, over half

3 *Cognitive orientation* refers to the potential cognitive demand of a particular task while acknowledging that cognitive demand necessarily depends on student and classroom characteristics.

4 Webwork refers to web-based homework, distributed by the textbook publisher.

#### Table 2: Three categories of cognitive orientation used in analysis of problems contained on Calculus I coursework<sup>a</sup>

Cognitive Orientation	Description	Sample problem		
Simple Procedure	Task requires recollection from memory or prompts student to apply a one-step procedure	Find the derivative of $f(x) = \sin(2x)$ at $x = \pi/4$ .		
Complex Procedure	Task requires student to recognize what procedure(s) to apply or may require more than one nontrivial step. Conceptual knowledge plays a plausible role in the venture.	At what value of t will $g(t) = t^2 \cos(t)$ attain a minimum?		
Rich Task	Task requires students to make interpretations and inferences, apply conceptual understanding in addition to procedural fluency, critically analyze a mathematical claim, or create new examples/counterexamples.	Provide an example of a function so that the right-hand Riemann sum with 5 partitions will be off of the true integral value by more than 200%.		

Table 3: Frequency and percent of tasks assigned by coursework and by task orientation.

	Simple Procedure		Complex Procedure		Rich Task		Total	
	N	%	N	%	N	%	N	%
Bookwork <sup>a</sup>	2,104	54%	811	21%	977	25%	3,892	100%
Worksheet	288	54%	52	10%	198	37%	538	100%
Exam	191	40%	53	11%	231	49%	475	100%
Total⁵	2,583	53%	916	19%	1,406	29%	4,905	100%

a Includes Webwork.

b Excludes Quizzes.

the problems assigned in bookwork/webwork and worksheets were simple procedures (54%). In contrast, exams contained proportionally more rich tasks (49%) than bookwork/webwork (25%) or worksheets (37%).

Second, we found statistically significant differences between instructors in the cognitive orientation of problems assigned. For example, irrespective of the fact that Calculus I instructors in this college used a common textbook, the percentage of rich tasks instructors assigned for bookwork/webwork ranged from a low of 15% to a high of 31%. There was even more variation in other types of coursework. Focusing on exams, the percentage of problems that were simple procedures ranged by instructor from 21% to 56% (mean of 42%) while the percentage of problems that were rich tasks ranged from 26% to 68% (mean of 47%)<sup>5</sup>. To illustrate this point, consider two instructors, Bob and David (both pseudonyms). Bob's exam problems comprised 54% simple procedures, 20% complex procedures, and 26% rich tasks. In contrast, David's exam problems comprised 21% simple procedures, 11% complex procedures, and 68% rich tasks.

# Major finding 3: Student were supported into and through Calculus I.

We found that Calculus I programs in the selected colleges supported students into and through Calculus I. This finding emerged as a common theme from the analysis of key

5 For the six instructors studied, the percentage of simple procedure problems on exams was 21%, 38%, 40%, 40%, 54%, and 56%. The percentage of rich task problem on exams was 26%, 34%, 47%, 49%, 56%, and 68%.

interviews (Mesa, White, & Burn, 2014). The selected colleges supported students initially by attending to effectively placing students into Calculus I. Three of four selected colleges had mandatory placement. In the two smaller programs, department chairs reported that virtually all of their Calculus I students took precalculus at the same institution, rather than placing directly into Calculus I via a placement test. We came to refer to this as "coursing in," which was described as very effective. We note that on the national survey, 66% of twoyear college students reported coursing into Calculus I.

Next, academic support structures outside of class were identified as contributing to successful Calculus I programs. Some forms of academic support at the selected colleges were institutional, such as campus-wide learning centers. However, in the selected colleges, support available in the learning centers was mainly limited to courses below Calculus I. Thus, instructors and students often spoke of the importance of informal student study groups for best learning the difficult content of Calculus I. All of the programs had specific social opportunities within their math program (e.g., math clubs, math teams, math competitions). However, it was unclear the extent to which students participated in these offerings. This is in contrast to informal study groups described previously in which students in focus groups readily reported participating actively.

Further, in all four selected colleges, transfer to four yearinstitutions was mediated by state documents such as common course descriptions or articulation agreements (either between schools or coordinated by the state). Calculus I faculty often described using personal knowledge of transfer institutions to advise students and ease transfer. Lastly, we observed three types of data commonly used: (1) student learning outcomes at the college and department level; (2) student success at the college (e.g., pass rates, graduation rates); and (3) student success at transfer institutions. Data was used mainly for reporting or accountability purposes rather than for internal decision-making or improvement.

# Major finding 4: Organizational factors contributed to programs success.

The analysis of key interviews revealed several organizational factors that faculty and administrators believed contributed to the success of their programs (Mesa, White, & Burn, 2014). Most saliently, instructors enjoyed much latitude and freedom in teaching Calculus I. Faculty and administrators explicitly stated that they trusted their colleagues to do the best for their students. No instructors were subject to program-wide policies related to common assessments (e.g., common final), use of technology, or preferred pedagogy. In programs with multiple sections, some amount of loose coordination across sections

was sought through informal faculty collaboration as well as more formal Calculus I committees that provided coordination through common course outlines and/or common textbooks.

In all programs, instructors reflected on their instruction individually and with their colleagues, mainly through casual conversation and email. This peer communication appeared to be the main mechanism for professional development and support. There were several examples of on-campus professional development opportunities and some support for faculty to participate in conferences, but they were not widely utilized by the mathematics faculty members we interviewed.

### Implications and Recommendations

Taken together, these findings have several implications for program improvement in Calculus I. First, in the case of Calculus I learning in the two-year college setting, the instructor might be the most important resource for students. If students' interest in taking more mathematics hinges on the quality of their experiences in the classroom—that is, the everyday contact with a knowledgeable, caring, and supportive instructors, and other peers—how instructors are chosen to teach Calculus I is fundamental. Programs seeking improved student outcomes should assign instructors to Calculus I using selection criteria in addition to seniority and instructor preference.

The findings also suggest that instructors in the selected colleges attended both to students' academic and social needs. According to Tinto's (1975/1993) model of student departure, the more complex the network of relationships a student develops with the institution, academically and socially, the less likely it is that the student will drop out. Academic integration occurs mainly in the classroom mediated by the instructor and by classroom activities. Students' social integration can occur either within the classroom or through out-of-class support. Encouraging participation in informal study groups can be a simple strategy for Calculus I instructors to try out.

The findings further suggest topics for professional development of Calculus I instructors. Computer-generated visualizations and animations for Calculus I concepts were frequently used instructional tools. Thus, Calculus I instructors can benefit from professional development around computer algebra systems to develop these skills. Further, given that interactive lecture appears to be a stable pedagogy in Calculus I in two-year colleges, professional development could focus on effective classroom questioning techniques, understanding cognitive orientation of assigned problems, developing classroom activities, and fair assessment practices in mathematics. We remind the reader that both instructors and students in the selected programs believed course examinations were fair. Thus, Calculus I instructors inclined to include a high proportion of rich task problems on exams should ensure students receive adequate academic support around challenging, demanding work that will prepare them to succeed.

Lastly, the findings reveal that, regardless of common course objectives and textbooks and efforts to coordinate Calculus I, students will be held to different standards based on the cognitive orientation of problems assigned by instructors. Thus, our findings serve as a caution to curriculum committees or department chairs that there needs to be other mechanisms for aligning instructors' enactment of learning goals within a department to ensure high-quality learning across different sections of Calculus I. Faculty collaboration around outcomes assessment of key Calculus I concepts is one means for Calculus I instructors to to examine and mitigate problems that might arise from having different implementations of the curriculum across sections of the program. These findings replicate known aspects of good teaching and suggest the amplified importance of the instructor, informal study groups, and administrator trust in faculty in the two-year context. Collectively, the insights drawn from the CSPCC study provide new lenses for thinking about quality instruction and student success in Calculus I in two-year colleges.

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