HOW TO MAKE TIME: THE RELATIONSHIPS BETWEEN CONCERNS ABOUT COVERAGE, MATERIAL COVERED, INSTRUCTIONAL PRACTICES, AND STUDENT SUCCESS IN COLLEGE CALCULUS

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This report draws on data collected by the Characteristics of Successful Programs in College Calculus project in order to investigate issues around coverage and pacing. This includes identifying what topics are being taught in Calculus I, determining the extent to which instructors and departments feel pressure to cover a set amount of material, and investigating possible relationships between concerns over coverage, instructional practices, and the nature of the material covered at five institutions selected for having successful Calculus programs.

Key Words: Calculus, Coverage, Teaching Practices, Pacing

The Characteristics of Successful Programs in College Calculus (CSPCC) project is a large empirical study designed to investigate Calculus I across the United States. While Calculus I is offered at nearly every college and university across the nation, and taken by approximately 300,000 students every fall, prior to CSPCC very little data had been collected about what happens in Calculus I (Bressoud et al., 2013). The primary focus of the CSPCC project is to identify factors that contribute to student success and understand how these factors are leveraged within highly successful programs. However, in addition to addressing these primary research goals, the CSPCC project has also amassed a wealth of data on the nature of Calculus I courses across the nation. In this report, we aim to draw on the CSPCC data in order to investigate issues around coverage and pacing. This includes investigating what topics are being taught in Calculus I and determining the extent to which instructors and departments feel pressure to cover a set amount of material. Further, because concerns over coverage are often cited as reasons to not implement reform-oriented instructional practices (Christou et al., 2004; Johnson et al., 2013; McDuffie & Graeber, 2003; Wagner, Speer, & Rossa, 2007; Wu, 1999) we will investigate relationships between teaching methods and concerns over coverage.

Theoretical Background

Students are citing poor instruction in their mathematics and science courses, with calculus instruction and curriculum often singled out, as a contributing reason for why they are discontinuing in STEM fields (Rasmussen & Ellis, 2013; Seymour, 2006; Thompson et al., 2007). Some specific problems with their learning experiences that students identified include: courses that were over-stuffed with material; pacing that inhibited comprehension and reflection; not including applications or conceptual discussions; and "faculty modes of teaching that suggested that they took little responsibility for student learning" (Seymour, 2006, p. 4). Thus, as reported by students, shallow treatments of large amounts of material and unresponsive teaching strategies are contributing to their reasons for leaving STEM majors.

The response from teachers seems to be that pressure to cover a set amount of material precludes efforts to adopt reform-oriented teaching strategies. For instance, in a case study of two mathematicians trying to implement reform curriculum in mathematics courses for preservice teachers, McDuffie and Graeber (2003) identified a number of institutional norms and policies that curtailed the mathematicians' efforts. As stated by one of the mathematicians:

> If you've got courses that link together, as most of the math curriculum does…there's an expectation that a certain amount of material be covered… And so you're fighting this constant battle…It means that you're limited on how much time you can spend to do real constructivist activities where the depth of knowledge is really greater (McDuffie & Graeber, 2003, p. 336).

Wu (1999) echoed this sentiment. In an op-ed reaction to mathematics education reform, he proposed that, "if the amount of material to be covered in a course can be greatly reduced … and students are expected to spend 8 years in college… then we can all safely abandon the lecture format and engage in a wholesale application of the guide-on-the-side philosophy" (p. 4). As examples of deliberate reduction in the material to be covered, Wu offers the textbooks *Calculus* by Hughes-Hallet et al. and *Calculus Concepts* by La Torre et al.

Taken as a whole, these reports from teachers and students suggest that 1) calculus courses are overburdened with content, and 2) in order to cover such large amounts of material teachers cannot implement reform-oriented instruction. In this study, we draw on the CSPCC data to investigate the validity of these claims using data collected at 197 research universities across the nation, including five institutions that have been selected for having particularly successful[1] Calculus I programs. Specifically, we investigate the following question: *In the PhD granting institutions with successful calculus programs, what is the relationship between concerns about coverage, instructional practices, and the nature of the material covered?*

Embedded in this question are issues regarding the expectations of students and faculty. These expectations relate to who is responsible for learning, where learning occurs, and how much material is reasonable to cover. Theoretically, we see these types of expectations as part of the didactical contract (Brousseau, 1997). The notion of didactical contract refers to the set of reciprocal expectations and obligations between the instructor and the students, most of which are implicitly formed through patterns of interaction. For example, at the secondary school level students do not expect to have to cover large amounts of material on their own at home. Much of learning therefore occurs in class and students and their teacher are mutually responsible for learning. At the university level, however, these expectations and obligations may shift – the amount of material covered increases, instructors tend to lecture more compared to secondary school teachers, and instructors expect students to learn more on their own at home. Students are often left feeling that their calculus course is overstuffed and taught in an uninspiring and unresponsive manner (Seymour, 2006). It is precisely these aspects of the didactical contract that we aim to unpack at institutions with more successful calculus programs.

Research methodology

In order to answer our research question, we draw on data collected in the two phases of the CSPCC project. The first phase of the CSPCC study involved surveys sent to a stratified random sample of students and their instructors at the beginning and the end of Calculus I. These surveys were designed to gain an overview of the various calculus programs nationwide, and to determine which institutions had more successful calculus programs. Here success was defined by a combination of student variables: persistence in Calculus as marked by stated intention to

[1] Our measures of success are outlined in the "Research methodology" section.

take Calculus II; affective changes, including enjoyment of math, confidence in mathematical ability, and interest to continue studying math; and passing rates. In the second phase of this project, surveys were analyzed in order to select four or five successful schools of each type (community college, Bachelor's granting, Master's granting, and Doctoral granting). We then conducted three-day site visits at each of the 18 institutions selected, during which we: interviewed students, instructors, and administrators; observed classes; and collected exams, course materials, and homework.

To understand departmental[2] *concerns about coverage*, we drew on instructors' agreement levels to the following survey prompts: When teaching my Calculus class, I (a) had enough time during class to help students understand difficult ideas, and (b) felt pressured to go through material quickly to cover all the required topics. To understand the departmental *instructional practices*, we drew on instructors' reports or the frequency of 8 instructional activities: (a) show students how to work specific problems; (b) have students work with one another; (c) hold a whole-class discussion; (d) have students give presentations; (e) have students work individually on problems or tasks; (f) lecture; (g) ask questions; and (h) ask students to explain their thinking. For both sets of questions, instructors were prompted to provide a response ranging from 1 to 6 on a Likert scale, with 1 meaning "not at all" and 6 meaning "very often". Descriptive and correlational analyses were conducted on these questions, with results discussed below.

There were 238 instructors who answered the above questions, 50 of who came from one of the five selected Doctoral granting institutions: Western Religious University (WRU), Northern Tech (NT), University of West Coast State (UWCS), University of Northern State (UNS), New England Polytechnic Institute (NEPI). Table 1 provides a brief overview of these institutions.

Table 1. Summary of selected institutions

Doctoral Institution	**Instructors with survey responses**	**Term length (weeks)**	**Text Used**
Western Religious University (WRU)	3	15	*Single Variable Calculus: Early Transcendentals* by Stewart
Northern Tech (NT)	7	14	*Calculus, Single and Multivariable (Fifth Edition)* by Hughes-Hallett, et al.
University of West Coast State (UWCS)	4	11	*Calculus: Early Transcendentals* by Jon Rogawski
University of Northern State (UNS)	30	15	*Calculus, Single and Multivariable (Fifth Edition)* by Hughes-Hallett, et al.
New England Polytechnic Institute (NEPI)	6	7	*Calculus: Early Transcendentals (7th edition)* by Edwards and Penny

[2] For the purposes of this analysis, we consider instructor responses together as representative of department concerns about coverage and instructional practices. In later analyses we consider in depth the variation among instructors within departments.

To understand the *nature of the material covered*, course syllabi and the departmental course list of required sections to be covered were analyzed. A master list of section titles was sorted into five categories: function review, limits, derivatives, differentiation rules, applications of differentiation, and integrals. Equivalent section titles were then grouped together to better reflect commonalities between the topics. For instance, the sections entitled *The Derivative as a Rate of Change, Rates of Change*, and *The Derivate and Rates of Change* were condensed into one heading.

Results

To answer our research question, we first report on departmental concerns about coverage; departmental teaching practices; and, for the five selected institutions, the material intended to be covered. We then present on how each these are related to one another.

Departmental concerns about coverage

As shown in Table 2, there are no significant differences between how concerned the departments are about coverage. On average, instructors at both the selected and not selected institutions reported having enough time to help students understand difficult ideas with around 4/6 frequency, and reported feeling pressured to go through the material quickly to cover all the required topics around 3/6 frequency.

Table 2. Departmental reports of concern for coverage at selected and not selected institutions.

When teaching my Calculus class, I: *(1=Not at all; 6=Very often)*	**Not Selected**	**Selected**
had enough time during class to help students understand difficult ideas.	4.19 (1.31)	4.42 (1.14)
felt pressured to go through material quickly to cover all the required topics.	3.06 (1.46)	3.33 (1.35)

Note.* = $p \leq .10$, ** = $p \leq .05$, *** = $p \leq .001$; Std. dev. is in parentheses.

Departmental instructional practices

As shown in Table 3, there are significant differences between the reported instructional practices of the instructors at the selected and not selected institutions. Specifically, instructors at the five selected institutions report higher frequencies of having students work with one another, holding a whole-class discussion, having students give presentations, and asking students to explain their thinking.

Table 3. Instructor reports of instructional activity at selected and not selected institutions.

During class time, how frequently did you: *(1=Not at all; 6=Very often)*	**Not Selected**	**Selected**
show students how to work specific problems?	5.14 (1.12)	5.22 (.89)
have students work with one another? ***	2.71 (1.65)	4.28 (1.84)
hold a whole-class discussion? **	2.68 (1.56)	3.32 (1.66)
have students give presentations? ***	1.47 (.91)	2.35 (1.74)

have students work individually on problems or tasks?	2.82 (1.60)	3.18 (1.66)
lecture?	5.25 (1.20)	5.12 (1.17)
ask questions?	5.15 (1.08)	5.08 (1.09)
ask students to explain their thinking?**	3.77 (1.50)	4.30 (1.42)

Note.* = p ≤ .10, ** = p ≤ .05, *** = p ≤ .001; Std. dev. in parentheses.

Nature of material covered at selected institutions

Analysis of the common syllabi from the five selected institutions identified six areas that were included in at least one of the Calculus I programs: Function Review, Limits, Derivatives, Differentiation Rules, Differentiation Applications, and Integrals. Only two of the schools, UWCS and UNS, covered sections in all six areas. Table 4 shows the number of sections in each area that the five schools included in their Calculus I course as well as their pace (number of topics per week). Notice that WRU did not include any review sections, NT did not include any sections on limits, and NEP did not cover any sections on integration.

Table 4. Nature of material covered at selected institutions

Topic covered	**Institution**				
	WRU	**NT**	**UWCS**	**UNS**	**NEPI**
Function Review	0	6	3	6	5
Limits	4	0	8	2	4
Derivatives	2	4	3	6	1
Differentiation rules	4	7	7	7	5
Differentiation Applications	7	4	8	7	13
Integrals	6	8	6	5	0
Total	***23***	***29***	***35***	***33***	***28***
Pacing (Topics per weeks in term)	**1.53**	**2.07**	**3.18**	**2.20**	**4.00**

Differences were also found within the main areas. In total, syllabi from the five schools included 84 different sections. However, only 7 topics were common to at least four of the five schools. These topics were: Limits and Continuity, Differentiation Rules (power, sum, product, quotient, exponential, chain, trigonometric), Related Rates, Max/Min/Optimization, Optimization and Modeling, Linear Approximations, and The Fundamental Theorem of Calculus. Additionally, there was variability among the sections that defined the derivative, both in terms of the number of sections covered and in terms of the topics. In this area sections names included: The derivative as the slope of a tangent line, The derivative as a rate of change, Derivative at a point, Derivative Function, and Definition of the Derivative. Finally, the schools varied greatly in the pace at which they went through sections, ranging from 1.53 sections per week at WRU to 4 sections per week at NEPI.

Relationship between concerns about coverage and nature of the material covered at the selected institutions.

In order to understand the relationship between departmental concerns and the nature of the material covered at the selected institutions, we first conducted correlation analysis between instructors' responses to the two questions regarding concerns about coverage and the intended pacing as determined in the syllabi analysis. There is no correlation between (a) departmental reports of having enough time during class to help students understand difficult ideas and intended pacing, r(48) = .053, $p = .713$; or between (b) feeling pressured to go through material quickly to cover all the required topics and intended pacing, r(48) = .070, $p = .632$.

We then looked in depth at the two schools with the largest difference in the number of sections covered per week: WRU with 23 sections included in their required section list to be covered in a 15-week term (1.53 topics per week) and NEPI with 28 sections in their departmental syllabi to be covered in a 7-week term (4 topics per week). When asked if they felt that they had enough class time to help their students understand difficult ideas, 2 of the 3 teachers from WRU responded that they *did not* feel like the had enough time (both answering with a 2 out of 6 on a Likert scale with 1 being not at all and 6 being very often). When answering the same question, only 1 of the 6 NEPI instructors gave a rating of 3 or less. Additionally, when asked if they felt pressured to go through material quickly to cover all the required topics, all 3 of the WRU replied with a score of 4 or more (again on a Likert Scale with 1 being not at all and 6 being very often). For the same question, 4 of the 6 NEPI replied with a score of 4 or more. These findings indicate that while instructors at NEPI (the institution with the quickest pace) felt pressured to go through the material quickly, they also felt like they had time to help their students understand difficult ideas. Conversely, instructors at WRU (with the slowest pace) felt both pressured to quickly cover the material and like they did not have enough time to help their students understand difficult topics.

Relationship between instructional practices and the nature of the material covered at the selected institutions.

To understand the relationship between departmental instructional practices and the nature of the material covered, we again first conducted correlational analyses between the eight reported instructional practices and the intended pacing at each of the five selected institution. Of the eight instructional practices, only one is correlated to pacing. There is a strong negative correlation between the frequency that students worked together and the intended pacing, r(48) = -.548, $p < .001$. This result implies that instructors who cover material quickly do not have students work in groups often. Indeed, instructors at the two institutions with the quickest pacing, NEPI (4 sections per week) and UWCS (3.18), reported that they rarely had students work in groups. However, at both UNS (2.2 section per week) and NT (2.07 sections per week) the majority of instructors reported that they often had students work in groups.

Relation between concerns about coverage and instructional practices at selected and not selected institutions.

Lastly, we looked at the relationship between reported departmental concerns about pacing and instructional practices. Again, we conducted correlational analyses between the two questions regarding concerns about coverage and the eight questions regarding instructional practices. Among the five selected institutions, there was a slight positive correlation between having enough time during class to help students understand difficult ideas and the frequency of the instructor asking the students questions, r(48) = .268, $p = .060$, and between feeling pressured to go through material quickly to cover all the required topics and the frequency of lecture, r(47) = .263, $p = .068$. These results suggest that, at the selected institutions, instructors who reported having enough time to help their students with difficult ideas often asked their students many questions during class, and those instructors who felt pressured to rush through material quickly tended to lecture more.

Among the not selected institutions, there was a strong correlation between having enough time during class to help students understand difficult ideas and the frequency of showing students how to work specific problems, r(186) = .296, $p < .01$, and a slight correlation between having enough time during class to help students understand difficult ideas and the frequency of having students give presentations, r(184) = .134, $p = .068$. Additionally, there were strong negative correlations between feeling pressured to go through material quickly to cover all the required topics and showing students how to work specific problems, r(182) = -.204, $p = .006$, and having students give presentations, r(180) = -.182, $p = .014$. These results indicate that, at the institutions not selected, instructors who reported having enough time to help their students with difficult ideas often showed their students how to work specific problems and had them give presentations. Further, instructors who felt pressured to rush through material quickly tended to infrequently do these activities.

Discussion

Given that these five institutions were selected based on student success (including persistence in Calculus, positive affective changes, and high pass rates), these results may suggest components of didactical contracts that support student success. For instance, between the selected and not selected institutions, there were no differences in the amount of time instructors felt like they had to help students through challenging material. However, there was a difference with how the instructors chose to use their time. When instructors report having enough time to help student understand difficult material, instructors at the selected institutions are more likely to use that time asking their students questions during class and instructors at the not selected institutions are more likely to use that time showing their students how to work specific problems and having them give presentations. Additionally, between the selected and not selected institutions, there were no differences in the amount of pressure that instructors felt to cover material (and in fact, when looking at the five selected institutions, there is no correlation between the reported concerns about coverage and the intended pacing of the course). There was, however, a difference in how instructors at the selected and not selected institutions chose to cover the material. Instructors at the selected institutions reported higher frequencies of having students work with one another, holding a whole-class discussion, having students give presentations, and asking students to explain their thinking. Thus, at these selected institutions it appears that part of the didactical contract between the instructors and their students involves covering material, sometimes large amounts, in ways that will involve and engage students in their learning.

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