# Introduction

For some twenty years now, the MAA Notes Series has published secondary materials for undergraduate mathematics courses, such as projects that can be used in teaching calculus. These publications reflect the interests of instructors, providing a means of sharing innovative ideas for teaching calculus, linear algebra, differential equations, statistics, geometry, and abstract algebra. With this book, discrete mathematics joins the list. This collection includes nineteen classroom-tested projects, eleven additional projects based on historical sources, three expository articles considering discrete mathematics topics in more depth, and two articles focused on pedagogy especially related to discrete mathematics.

Why is discrete mathematics only now the subject of such a collection? One possible reason is that, unlike concepts taught in the course itself, discrete mathematics is not well-defined. While there are controversies on how to teach calculus, there is relative unanimity on what topics a calculus course should address. On the other hand, a survey of discrete mathematics courses around the country shows a variety of different topics being covered, goals being sought, and students being served. Does the course cover circuit design and the tools for algorithm analysis? How much logic is covered? Graph theory? Combinatorics? Is this the course where students first learn to write proofs? Are the students mathematics majors, computer science majors, or is the course offered for a general education requirement?

This book does not address those questions. The projects and articles here reflect the wide breadth of topics taught in the diverse discrete mathematics courses offered in universities, colleges, and (increasingly) high schools. I hope that every instructor of discrete mathematics will find projects and articles relevant to the topics of his or her course, and also learn more about other topics, with the realization that those are covered in someone else's course.

The timing of this collection also follows from a number of related activities in the professional organizations. There have recently been workshops and many sessions devoted to discrete mathematics in conferences of the Mathematical Association of America, the Association for Computing Machinery, and the Institute of Electrical and Electronics Engineers. I participated in several of these workshops and sessions, initially with the idea of writing a textbook. But as I discovered the innovative and thoughtful work of so many colleagues, I decided to redirect my creative impulse into putting together this collection. The call went out for classroom-tested projects and articles addressing advanced discrete mathematics topics and teaching issues related to the course. I received submissions from faculty in mathematics departments and computer science departments, from a high school teacher, from new instructors to experts in the field. I am grateful that a wide variety of instructors were willing to provide their work for this volume.

Most of the responses were classroom-tested projects, which vary widely in difficulty and are sometimes distinguished from the advanced articles only by the inclusion of exercises (and solutions!). Some are means of introducing a topic, such as graph theory, strong induction, and motivation for clearly written proofs. Some extend common topics, such as the Towers of Hanoi, the Josephus problem, and Euler's formula. Some discuss applications, such as chemistry, bioinformatics, information storage, and typesetting. Some use technology, including graphing calculators and programming. Some use manipulatives, including integer rods, strings on a pegboard, and pipes from the hardware store. In addition to exercises and solutions in all of these projects, some include open-ended questions and some extension questions suitable for student research. The format for each is a summary, notes to the instructor, references, student worksheets (often mixed with explanatory handouts), and solutions. The classroom projects are ordered topically, following the order of Susanna Epp's *Discrete Mathematics with Applications*.

I am pleased to be able to include eleven of the historical modules developed through New Mexico State University, a continuation of their long-standing program of teaching mathematics from original sources. These too vary from introductory to advanced topics, including combinatorics, set theory, logic, and graph theory. The projects are arranged chronologically by their primary source, allowing readers to follow the historical development of certain discrete mathematics topics. The initial two related projects, though, do not fit into that structure, going from Leibniz to von Neumann in the first, and from Shannon to the abacus in the second. A combined introduction explores how these projects can be used in the classroom, and each article includes exercises in conjunction with the source material, references, and additional notes to the instructor.

The five expository articles examine discrete mathematics content beyond the level of a first course or discuss teaching issues specific to the course. But like many of the projects, some of the articles also defy easy categorization. For example, Shai Simonson's "A Rabbi, Three Sums, and Three Problems" uses fourteenth century mathematics as a springboard for counting problems as an example of the discovery method. To assist the reader in navigating such rich material, the table of contents is annotated with a summary of each project or article and, for the classroom projects, how many 50-minute periods an instructor might dedicate to the activity.

There are many people I want to thank. Bill Marion organized two summer Professional Enhancement Programs on discrete mathematics, where I met Peter Henderson and Susanna Epp; all three have been very helpful and encouraging over the long course of this book's creation. Jerry Lodder, another PREP participant, coordinated work on the historical projects. Michael Jones and Larry Thomas helped with sundry typesetting and graphics issues. Barbara Reynolds, Stephen Maurer, the entire MAA Notes editorial board, and the MAA publications staff have helped immensely in bringing this project to completion. My greatest appreciation goes to the authors of the projects and articles for their patience, creativity, and willingness to share their good work.