Part II

Historical Projects in Discrete Mathematics and Computer Science

Introduction

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Introduction

A course in discrete mathematics is a relatively recent addition, within the last 30 or 40 years, to the modern American undergraduate curriculum, born out of a need to instruct computer science majors in algorithmic thought. The roots of discrete mathematics, however, are as old as mathematics itself, with the notion of counting a discrete operation, usually cited as the first mathematical development in ancient cultures. By contrast, a course in finite mathematics is sometimes presented as a fast-paced news reel of facts and formulae, often memorized by the students, with the text offering only passing mention of the motivating problems and original work that eventually found resolution in the modern concepts of induction, recursion and algorithm. This chapter focuses on the pedagogy of historical projects, which offer excerpts from original sources, place the material in context, and provide direction to the subject matter.

Each historical project is centered around a publication of mathematical significance, such as Blaise Pascal's "Treatise on the Arithmetical Triangle" [2, vol. 30] from the 1650s or Alan Turing's 1936 paper "On Computable Numbers with an Application to the Entscheidungsproblem" [3]. The projects are designed to introduce or provide supplementary material for topics in the curriculum, such as induction in a discrete mathematics course, or compilers and computability for a computer science course. Each project provides a discussion of the historical exigency of the piece, a few biographical comments about the author, excerpts from the original work, and a sequence of questions to help the student appreciate the source. The main pedagogical idea is to teach and learn certain course topics from the primary historical source, thus recovering motivation for studying the material.

For use in the classroom, allow several weeks per project with one or two projects per course. The time spent working on a project may vary from two to six weeks, depending on the depth and breadth in which the material is covered. Some instructors may wish not to assign all parts of a project, while others may wish to add further questions. The source files for the projects may be downloaded from the web resource [1] and edited. Before assignment, the instructor should work through all details of a project, not only for acquaintance with the historical source, but to determine which questions best suit the course material. Pay particular attention to the language of the historical author, and its mathematical content. Be prepared to examine a quote from the source in class and discuss its meaning, either verbally or with present-day formulas. Student solutions may be collected in installments or only once upon completing the project. In either event, monitor student progress on what for them is a lengthy assignment. Each project should count for a significant portion of the course grade (about 20%) and may take the place of an in-class examination. Begin early in the course with a discussion of the relevance of the historical piece, its relation to the course curriculum, and how modern textbook techniques owe their development to problems often posed centuries earlier. Each project offers a "Notes for the instructor" for more detailed information about the level, content and implementation of the project.

The topics covered by the projects include naive set theory, mathematical induction, binary arithmetic, computability, graph theory, and the combinatorics of the Catalan numbers. They range in level from beginning undergraduate courses in discrete mathematics to advanced undergraduate courses in logic, graph theory, and computer science. Most are independent of each other, although a few projects build on the same source or present closely related topics via different sources. Each historical source is one which either solved an outstanding problem, inaugurated a mathematical technique, or offered a novel point of view on existing material.

The next page is a list of instructions to students concerning "How to Work on Your Project." A good student solution should reflect the benefits afforded by close attention to these points. The instructor may wish to photocopy the following instructions for distribution to the entire class.

After completion of a course using historical projects, students write the following about the benefits of history:

"See how the concepts developed and understand the process."

"Learn the roots of what you've come to believe in."

"Appropriate question building."

"Helps with English-math conversion."

"It leads me to my own discoveries."

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Bibliography

[1] Bezhanishvili, G., Leung, H., Lodder, J., Pengelley, D., Ranjan, D., "Teaching Discrete Mathematics via Primary Historical Sources," `www.math.nmsu.edu/hist_projects/`

[2] Pascal, B., "Treatise on the Arithmetical Triangle," in *Great Books of the Western World*, Mortimer Adler (editor), Encyclopædia Britannica, Inc., Chicago, 1991.

[3] Turing, A. M., "On Computable Numbers with an Application to the Entscheidungsproblem," *Proceedings of the London Mathematical Society* **42** (1936), 230–265.

How to Work on Your Project: What is Expected

Here are some words of advice and encouragement for completing the historical project. This is a significant course requirement, in which you will examine a key historical episode in the development of a mathematical topic. The historical presentation is often more verbal than modern textbook formulations, which carries the advantage that no technical knowledge is assumed a priori. On the other hand, you must carefully read the historical author's words and be prepared to experiment with your own calculations to verify or amplify the historical source. Here are some specific instructions.

1. Start today. This is a major assignment, requiring more work than most homework problems from a textbook. Work on the project every day before it is due.
2. Read the entire project to see what is involved. From what time period is the historical piece? What modern topics have evolved from the source?
3. Next read the project very carefully and make a list of any unfamiliar words or concepts. Do you understand the language of the historical author? If not, ask your instructor.
4. Feel free to seek additional historical or mathematical information about the source from either the library or the Internet. Always question the validity of Internet sources.
5. Begin answering questions in the project. Use complete sentences along with equations, tables, or diagrams to support your work. Some questions will be rather simple, while others require detailed solutions. Identify the difficult questions early and seek help from your instructor if you don't know how to proceed.
6. Seek feedback from your instructor even if you feel that you have answered the questions correctly, since you may have misinterpreted a passage.
7. The final solution to the project should be a written paper in which you address all the issues raised in the project. Use complete sentences along with modern equations to support your claims. Discuss any connections from the historical source with present-day techniques that you may have encountered. How does the historical source differ from the textbook presentation?