Overview

The title of this book contains the word "introduction" for two reasons. First, there is so much that could be covered that is not included here, not only mathematics, but programming, art, and design as well. The text is not comprehensive in any of these realms.

Second, it requires minimal prerequisites, as discussed in the following section. It starts at the beginning, so that most college students could take a course based on this book and be able to do well. This book can serve as just about anyone's introduction to the mathematics of computer graphics.

Prerequisites

Most of this book can be used with only a background of high school algebra and trigonometry. Beginning with the end of Chapter 10, knowledge of differential calculus is important, and in several chapters thereafter, summation notation is used. The reader does not need to know linear algebra. The modest matrix algebra required will be covered along the way, beginning in earnest in Chapter 5.

Readers who have already studied linear algebra will breeze through Chapter 5. Instructors teaching a course in which the students have already had linear algebra may not want to dedicate any in-class time to Chapter 5, and will therefore have more time for the more challenging chapters at the end of the text.

Part III covers vector-valued functions as paths of motion in space. It introduces this topic from scratch, so readers need not have seen it before. But those who have will find the material easier. Similarly, Part IV covers two-dimensional surfaces in three-dimensional space, starting from scratch, but readers experienced with that material will find it easier.

For mathematically curious readers, not only are the final chapters of the text more challenging, but each chapter also includes some exercises that assume more mathematical sophistication than the rest of the book does. These appear in a subsection entitled "Mathematical thinking" within each chapter's exercises. For example, the final chapters of the text have some optional exercises that require multivariable calculus. Instructors using this book in a mathematics course may find such exercises useful.

The reader also needs no background in computer programming, nor will any be covered here. Though most of the suggested projects involve describing three-dimensional scenes in a computer language, there are two reasons why this does not count as programming.

First, the intricacies of programming come about when combining loops, conditional expressions, recursion, and other control structures. All the projects in this book can be done without any such structures. The code required is more like HTML than a more complex language; it does not require

writing any algorithms. Second, the software in which the coding will take place comes with dozens of code examples ready to be inserted, so that the user does not need to memorize any complex syntax. See POV Break A (page 29) for more information.

But for readers that have a programming background, each chapter includes optional exercises that leverage it. Such exercises appear in a subsection entitled "Programming" within each chapter's exercises. Instructors using this book in a computer science course may find such exercises useful.

Because of the minimal prerequisites, this book could be used for a wide variety of courses, from a mathematics elective (in which the author uses it) to the freshman liberal arts mathematics course common at many schools. Furthermore, it is accessible to many recreational readers.

All required software can be obtained for free, and later chapters specify where to obtain it and provide instructions for how to use it. The only exception is that the text does not cover how to use a spreadsheet, which some exercises require. Microsoft Excel or any free alternative will work for the modest needs of those exercises.

Part I: Affine space

The first part of the book covers how to construct a scene full of objects. By its end, the reader will have completed the first computer projects, using the algebra and geometry of affine transformations. Mathematical topics in this part include

- coordinate spaces in two and three dimensions,
- transformations of such coordinate systems,
- how to represent such transformations using matrices,
- and the centrality of matrix algebra to computer graphics hardware.

POV Breaks in this part enable the reader to install the software, edit and render scenes, and arrange objects using affine transformations.

Part II: Ray tracing

This part of the book answers the question that naturally arises from doing the first computer projects: how does the computer render a scene? More algebra and geometry are introduced to answer the question, but not only for that purpose; they also prepare for animation in Part III. Mathematical topics in this part include

- lines in three dimensions,
- mathematical models of sight, light, and color,
- intersections of lines with a variety of three-dimensional figures,
- and a bit of constructive solid geometry.

POV Breaks in this part enable the reader to construct his or her own three-dimensional objects using simple shapes (such as spheres, boxes, and cones), mathematical operations (such as union, intersection, and difference), and a knowledge of color vectors. In a more introductory course, the instructor may consider omitting Chapter 7.

The optional programming exercises in this part of the book guide the reader through the development of a simple ray-tracing engine. An example implementation appears on the website for this book [10].

Part III: Animation

This part covers how to animate a scene, thereby enabling the reader to achieve the book's goal, making a computer-generated movie. Mathematical topics in this part include

- vector-valued functions as paths through space,
- polynomials (especially Bézier curves and Bernstein polynomials),
- derivatives of vector-valued functions,
- types of continuity, and their real-world impacts on animation.

POV Breaks in this part enable the reader to create animated scenes using POV, and then assemble them into a short movie using other free software.

Part IV: Modeling

This part of the book covers advanced techniques for creating more realistic models of three-dimensional shapes. It is optional (or bonus) material, included for three reasons. First, the reader who learns to build his or her own objects in Part II may wonder how to go beyond the limits of the introductory techniques taught there. Second, much of the research being done in the computer graphics industry today is on advances in modeling, because digital artists constantly use the types of models discussed in this part of the book. Finally, although Parts I through III are sufficient to fill a semester-long introductory course, a more advanced course requires more material. Part IV provides that material.

These are the most mathematically challenging chapters, and cover

- parametric surfaces, including Bézier surfaces,
- using such surfaces for more advanced modeling,
- and meshes and subdivisions thereof, for creating and smoothing realistic shapes.

Although the main goal of the book is the movie project in Part III, instructors who assign that project and then go on to cover Part IV will give their students more tools to use in constructing their final movie and more time to perfect it before the end of the course.

Appendix

The solutions to a subset of the book's exercises appear in the appendix. Exercises marked with $**$ in the book have solutions in the back, and those marked with $*$ have either a hint or a partial solution. The text aims to contain enough solutions and hints to assist struggling learners (especially recreational readers) without taking away too many problems from instructors who wish to assign them for homework.