MAA MATHFEST

August 6-9, 2014

Invited Paper Session Abstracts

The Eyes Have It: Mathematical Modeling of the Retina

Saturday, August 9, 1:00 p.m. - 2:50 p.m., Hilton Portland, Plaza Level, Pavilion West

Models of the retina are crucial in understanding various retinal diseases and abnormalities that contribute to blindness such as myopia, glaucoma, retinitis pigmentosa, and others. In this session speakers will present mathematical models of retinal detachment, retinal blood flow, and melanopsin activation and inactivation. Utilizing a diverse set of mathematical techniques, analysis, and computer simulations from dynamical systems, numerical analysis, and stochastic processes these models investigate complex retinal process including elevated ocular pressure and forces from retinal adhesion, retinal pigment epithelium pumps, and retinal elasticity leading to retinal detachment, alterations in ocular curvature caused by a reduction retinal blood flow, and the chemical reaction associated with non-image forming process in the retina.

Erika Camacho, *Massachusetts Institute of Technology and Arizona State University*

Mechanical Models for Exudative Retinal Detachments

1:00 p.m. - 1:20 p.m.

We present a model of the mechanical and fluid forces associated with exudative retinal detachments where the retinal photoreceptor cells separate typically from the underlying retinal pigment epithelium (RPE). By computing the total fluid volume flow arising from transretinal, vascular, and retinal pigment epithelium (RPE) pump currents, we determine the conditions under which the subretinal fluid pressure exceeds the maximum yield stress holding the retina and RPE together, giving rise to an irreversible, extended retinal delamination. We also investigate localized, blister-like retinal detachments by balancing mechanical tension in the retina with both the retina-RPE adhesion energy and the hydraulic pressure jump across the retina. For detachments induced by traction forces, we find a critical radius beyond which the blister is unstable to growth. Growth of a detached blister can also be driven by inflamed tissue within which e.g., the hydraulic conductivities of the retina or choroid increase, the RPE pumps fail, or the adhesion properties change.

Thomas Chou, *Department of Biomathematics, UCLA*

New Paradigms in Retinal Blood Flow Simulation

1:30 p.m. - 1:50 p.m.

Clinical studies show that ocular hemodynamics play an important role in many ocular diseases, such as glaucoma and myopia. Blood flow in the retina is a multi-scale phenomena; on the largest scale blood flows through arterioles and venules and on the smallest scale single red blood cells move through the capillaries. We developed a model of the retinal vasculature as a layered porous medium on a curved surface where the geometry of the larger vessels is described by network and fractal models. The solution of the resulting system of partial differential equations on a curved surface is a challenging problem beyond the particular application to ophthalmology. To be able to conduct clinically relevant studies with patient specific and population-based data we mathematical modelers work together with eye specialists. We will also give a short overview of current research on retinal blood flow modeling.

Andrea Dziubek, *Mathematics Department, SUNY Institute of Technology*

Analytical Mechanics and Evolution of a Detaching Retina

2:00 p.m. - 2:20 p.m.

A mechanics based mathematical model for retinal detachment is developed, incorporating an energy based criterion for propagation. Retinas with and without central tears are considered and contraction of the vitreous and extension of its fibrils, along with a pressure difference across the retina, are taken as the stimuli for detachment propagation. Treatment for the affliction by mechanical banding is also modeled. In addition to the equations of motion, boundary and matching conditions, the variational formulation yields the self consistent energy release rate that governs detachment, and formulae for critical stress and critical deflections that provide a rational basis for measuring critical parameters. Exact analytical solutions are established for axisymmetric detachment of retinas with and without tears, and numerical simulations are performed based on these solutions. The results yield characteristic behavior, including threshold levels and stability of detachment, "dimpling" of the detaching retina, the effects of changes in material and geometric parameters, and the influence of the presence and size of the retinal tear on detachment propagation. The model predicts that once detachment ensues it does so in an unstable manner and is extensive in scope. This is in agreement with clinical observation. The predicted presence of "dimples" in the detaching retina appear to correspond to clinically observed phenomena as well. Results also suggest that, under appropriate conditions, the presence and size of a retinal tear or hole can have a "stabilizing" effect with regard to detachment propagation. Results for banding as a mechanism to close the detached area suggest the influence of the parameters of the band on its effectiveness with regard to closure and minimizing induced ocular pressure.

William J. Bottega, *Department of Mechanical and Aerospace Engineering, Rutgers University*

Stochastic Modeling of Melanopsin Activation and Deactivation

2:30 p.m. - 2:50 p.m.

Vertebrate melanopsin is a photo-pigment recently identified to be involved in retinal non-image forming processes such as circadian rhythm coordination and light/dark sensing. A stochastic model using Gillespie’s Algorithm was constructed based on invertebrate rhodopsin cascades and is presented as a proposed mechanism for the vertebrate melanopsin cascade.

Systems involving single light flashes and multiple light flashes were simulated and compared to experimental data. Results from the constructed model capture many of the key characteristics of the activation/deactivation process and sensitivity analysis indicates several reactions that are thought to strongly govern the kinematics of the cascade.

Christina Hamlet, *Center for Computational Science, Tulane University*