By the beginning of the 12th century, the Romanesque form was gradually giving way to the Gothic style. The word “Gothic” was first used in the Italian Renaissance as a negative term for all art and architecture of the Middle Ages, suggesting that it was of the quality of the work of the barbarian Goths. Today the term Gothic Age refers to the period of art and architecture immediately following the Romanesque. It is regarded to be an era of outstanding artistic achievement.

The most easily recognized feature of the Gothic style is the pointed or Gothic arch. Figure 3.16 shows one common design. Simply take a segment $AB$ and draw two circular arcs of the same radius, one centered at $A$ and the other at $B$. To draw the outer perimeter of the arch, keep the two centers but increase the radii of the arcs. A look at Figure 3.17 tells us that as structural device, the Gothic arch has advantages and disadvantages when compared to the semicircular arch. To span the same space, the Gothic arch needs to be higher. However, when it comes to transferring loads, the Gothic arch transfers them with a smaller horizontal component and a larger vertical component. This means that the outward thrusts generated by Gothic arches are less and therefore more easily contained. The fact that the vertical components are larger, means that the materials from which it is constructed are under greater compression. In this regard, however, we know that stone, masonry, and concrete stand up to compression very well.

We’ll now take a look in quantitative terms at the outward thrust that a Gothic arch generates. The diagram of Figure 3.18a is the simplified model of a Gothic arch that we will consider. Our
study will assume that the total load $L$ that the arch supports acts at the very top and that this load includes the weight of the slanting elements of the arch. The angle that these elements make with the horizontal is $\alpha$ in each case. The slanting parts of the arch transfer the load downward. Can we compute, or at least estimate, the horizontal component $H$ of the thrust that is generated?

![Figure 3.18. Horizontal thrust generated by an arch](image-url)

We will answer an equivalent problem instead. If we were to reinforce the arch by adding a horizontal beam as shown in Figure 3.18b, what would the pull $T$ by this beam have to be to make the arch stable? This is the horizontal force $H$ that we are looking for. The question about the pull $T$ has transformed the problem into the context of Figures 3.14 and 3.15 of Section 3C. Applying the analysis there, tells us that

$$H = T = \frac{L}{2\tan\alpha}.$$  

Suppose, for example, that $L = 5000$ pounds. Let’s take $\alpha$ successively equal to $30^\circ$, $45^\circ$, $60^\circ$, and finally, $75^\circ$, to see how $H$ depends on the steepness of the arch. Because $\tan 30^\circ = 0.58$, $\tan 45^\circ = 1.00$, $\tan 60^\circ = 1.73$, and $\tan 75^\circ = 3.73$, we find that the corresponding horizontal forces $H$ are

$$4330, 2500, 1433, \text{ and } 670$$

pounds respectively. As Figure 3.17 already suggested in qualitative terms, the horizontal thrust $H$ decreases dramatically as the steepness of the arch increases.

Our analysis also points to the important role played by the materials used in the components of a structure. If an arch is firmly braced as shown in Figure 3.18b, then it may be able to contain on its own the outward thrust produced by the loads that it supports. An arch made of a strong material such as high grade steel or reinforced concrete may be able to respond in the same way. However,
an arch made of stone, masonry, and ordinary concrete offers much less internal resistance. Such an arch will be able to support massive loads only if the horizontal thrusts that the loads generate are counteracted by components of the structure acting against the sides of the arch.

The most important structural element of Gothic architecture is the ribbed vault. To understand the Gothic ribbed vault, return first to the ceiling of the basilica of Vezély in Color Plate 11. Recall that the nave is segmented into bays, and that the ceiling of each bay is a groin vault obtained by the intersection of two cylindrical barrel vaults. Each of these groin vaults is bounded by two striped semicircular arches. Have a careful look at Color Plate 11 and notice that the two cylindrical surfaces of each groin vault give rise to two intersecting circular arcs. They are the dashed curves highlighted in Figure 3.19. Figure 3.20a is an abstract and reoriented version of the groin vault of Figure 3.19. The two barrel vaults, one coded in black and the other in gray, intersect perpendicularly. The two

![Figure 3.19. Groin vault at Vezély](image)

![Figure 3.20. From the groin vault to the Gothic ribbed vault](image)
dashed lines represent the circular arcs at which they intersect. (The fact that one of the arcs is
drawn as a straight line is a consequence of the chosen orientation.) Now modify Figure 3.20a as
follows. Replace each of the two cylindrical surfaces by a curved surface that has pointed Gothic
arcs as vertical cross sections. With this change, the semicircular arcs at the boundaries of the
two barrel vaults (depicted in black and gray in Figure 3.20a) are transformed into the two pairs
of Gothic arcs depicted in Figure 3.20b. The black dashed curves are transformed into Gothic arcs
as well. What has happened so far is only a small geometric change. But now comes the Gothic
architect’s golden idea: Build the two crossing Gothic arcs – at this point they are only geometric
curves – into structurally relevant, load bearing, Gothic arches. In combination with the Gothic
arches at the boundaries they form the interlocking grid of ribs shown in Figure 3.20b. This grid is
the structural skeleton of the Gothic ribbed vault.

The ribbed vault was critical to the development of Gothic architecture. It gave builders new
possibilities of design and construction and came to dominate medieval architecture. In a ribbed
vault, the ribs are the primary structural members. The spaces between them are structurally less
relevant and can be filled with thinner masonry materials. So the ribbed vault is lighter and hence
structurally more resilient than the more massive barrel and groin vaults. It was also easier to
construct and it made a number of architectural innovations possible. Building a vault requires
centering (the temporary wooden structure that we have already encountered in Section 2E in the
context of the Roman arch) that supports the masonry until the shell of the vault is completed and
the mortar has set. In the construction of a groin vault, the ceiling of an entire bay must be

Figure 3.21. Ribbed vaults over several bays in the ceiling of a Gothic cathedral
supported in this way. The construction of a ribbed vault is simpler. Builders laid two intersecting diagonal arches across the bay and supported them with light centering high on the nave walls. As part of an interlocking grid of ribbed arches this divided the vault into smaller triangular cells. One such arrangement is shown in Figure 3.21. After the ribs had set, these triangular cells were filled with masonry or concrete. Again, no extensive centering was required.

The configuration of the ribs in the ceiling of the nave and transept determine to a large extent the rest of the structure of a Gothic church. The points of convergence of the ribs determine the placement of the supporting vertical columns that run down the sides of the nave. As shown in Figure 3.22, these columns flow into thicker piers towards the bottom. The final piece of the structural puzzle has to do with the outward forces that the vaulted ceiling and especially the ribs generate. See Figures 3.14 and 3.18 and the discussions that explain them. Since these thrusts are concentrated at the points where the ribs converge and along the columns that support them, they can be countered by flying buttresses. Figure 3.23 shows how these half-arches push against the wall of the nave from the outside along the pressure lines determined by the ribs of the vaults.
and the supporting columns. The fact that the walls between the columns are not critical to the structural integrity of a Gothic church means that they can be punctured to provide large areas for windows. Large, finely crafted, stained glass windows (often magically illuminated by the light from the outside) are characteristic features of Gothic architecture. The delicate masonry work that frames the glass panels of these windows is known as *tracery*. The rose window depicted in Figure 3.24 provides an artistic example.

The structural scheme of a Gothic cathedral starts from the ribbed vault and flows logically
downward and outward. The ribbed vault made it possible for Gothic master architects to build churches with soaring interiors that were higher, more elegant, and more delicate than earlier, sturdier, and more massive Romanesque churches.

A number of splendid Gothic cathedrals were built in France, England, Germany, and Austria. Regarded by architectural historians to be one of the finest is the Cathédral de Notre Dame in the town of Chartres, an important center of pilgrimage about 50 miles from Paris. Construction of the cathedral began in the middle of the 12th century, but a fire late in the century destroyed much of what had been built (and much of the town). The cathedral was rebuilt within 25 years (a short time for Gothic cathedrals) and by the year 1220 it was finished. The ribbed vaults, supporting columns, flying buttresses, and rose window of Figures 3.21, 3.22, 3.23, and 3.24 are those of the Cathedral of Chartres. Plate 13 shows a comprehensive view. Flying buttresses run along the sides of the nave and around the apse. The pattern is interrupted by the transept, braced not by
Plate 13. The Cathedral of Notre Dame, Chartres, France. View from the south

Plate 14. The rose window on the north side of the transept of the Cathedral of Notre Dame, Chartres, France
flying buttresses but by small towers and thicker walls. The spires at the entrance act as piers against the pressure generated by the segments of the vaults near the entrance. The two spires are very different. One is 349 feet high and dates from the 12th century. It is in plain early Gothic style. The other is 377 feet tall and was built in the 16th century. It is in French high Gothic style. The cathedral is famous for its stained glass windows, many in beautiful blue hues dating from the 13th century. Plate 14 depicts the rose window of Figure 3.24 viewed from the interior. In the same way that colored tiles with intricate designs are central to the artistry of Islam and delicate golden icons and mosaics are the hallmark of Byzantine art, stained glass windows in splendid hues exemplify the art of the Gothic Age.

We have seen that in its essence, Gothic architecture is geometry executed in stone. The upcoming discussion will tell us how very literally true this was.

Problems

Problem 1. Suppose the truss in Figure 3.14 is an abstract model of a rib segment of the vault of a Gothic cathedral. What structural feature of the cathedral plays the role of the tie-beam connecting $A$ and $B$? Assume that the load $L$ is 10,000 pounds. Take $\alpha = 25^\circ$, $\alpha = 50^\circ$, and $\alpha = 75^\circ$, and compute in each case the horizontal component of the outward thrust that the slanting beam generates at $A$ (or $B$).

Problem 2. Suppose that the load $L$ on the arch depicted in Figure 3.18 consists only of the combined weight of the two slanting members. Let $\alpha$ be the indicated angle, let $d$ be half the span
of the arch, let $h$ be the height, and let $l$ be the length of the slanting elements of the arch. Let $w$ be the weight per unit length of these elements. Explain why $L \approx 2w\sqrt{d^2 + h^2}$. Let $H$ be the horizontal thrust that the load generates and show that $H \approx wd\sqrt{1 + \frac{d^2}{h^2}}$. Now let $w$ and $d$ be fixed and discuss what happens to both $L$ and $H$ as $h$ varies.

**Problem 3.** Discuss the outward forces generated by a spire of the Cathedral of Chartres in light of the conclusion of Problem 2.

**Problem 4.** Look up the history of the French Gothic Cathedral of Beauvais and write a paragraph that explains the nature of and reasons for the structural failures.