The Roman Pantheon

One of the most impressive structures of Roman antiquity is the Pantheon (never to be confused with the Parthenon in Athens). Its supervising architect was the Roman emperor Hadrian. It was built between A.D. 118 and 128 and dedicated to all Roman gods (in Greek, *pan* = all, *theon* = of the gods). Its elevation, the term for a representation of a facade, is shown in Figure 2.42. The entrance hall is a large portico in the Greek Corinthian style. The portico fronts a large cylindrical structure that is capped by a hemispherical dome. The exterior of the cylindrical structure consists of flat Roman brick, carefully laid out, row upon row. The dome of the Pantheon was regarded to be a shape of ideal perfection that conveyed both beauty and power. Plate 5 shows the spectacular

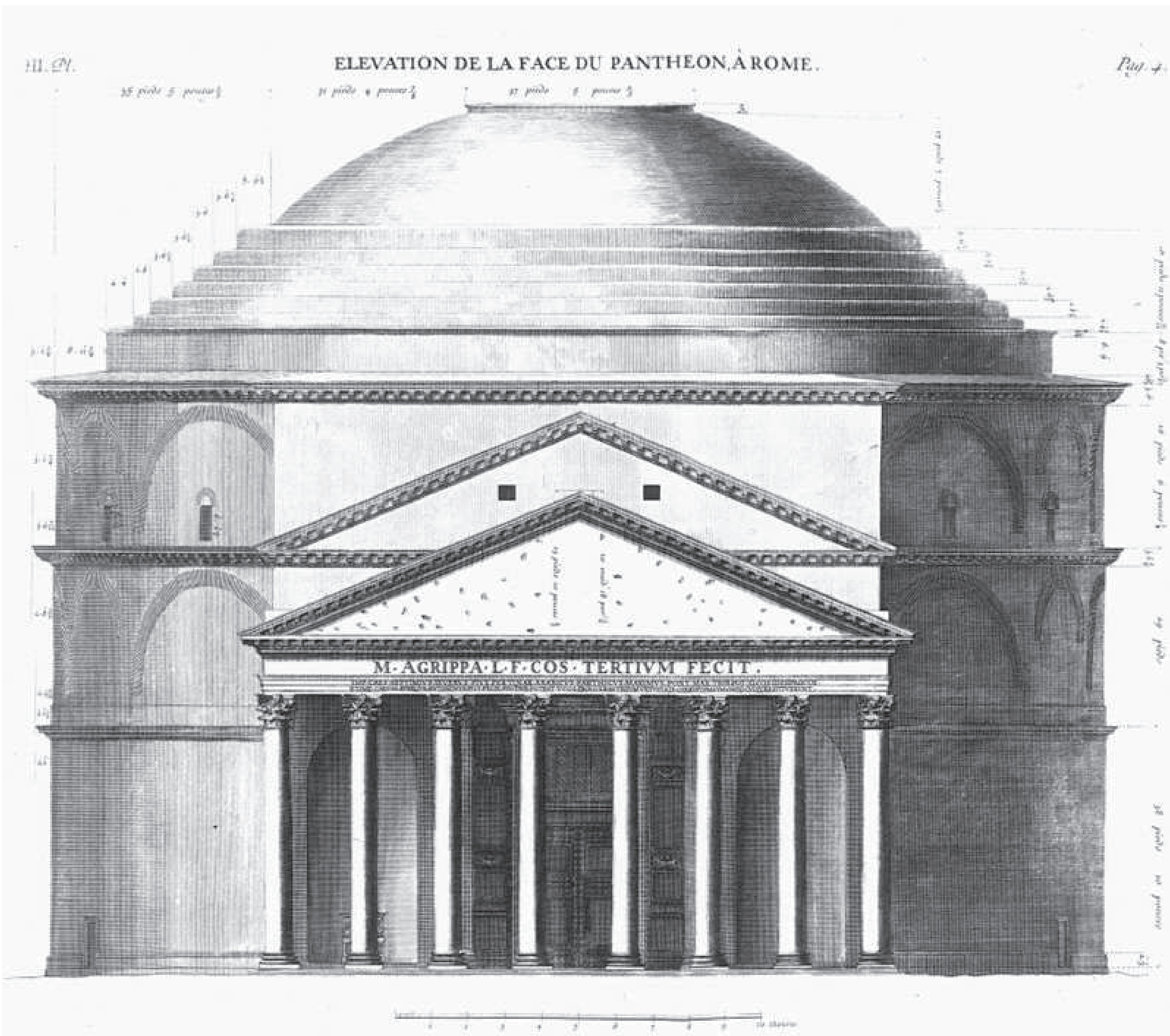


Figure 2.42. Antoine Desgodetz, engraving of the elevation of the Pantheon. From *Les edifices antiques de Rome*, Claude-Antoine Joubert, Paris, 1779

Plate 5. Giovanni Paolo Panini, The Interior of the Pantheon, 1737.
Oil on canvas. Detroit Museum of Art

interior. Its circular floor has a diameter of 142 feet and the interior of the dome rises about 142 feet above the floor at its highest point. At the top is a circular opening, or oculus (in Latin, *oculus* = eye), with a diameter of about 24 feet to let in light and air. Other than the entrance, the oculus is the sole source of natural light for the interior. Notice the circular arrays of rectangular indentations, or coffers, in the ceiling. The attention to detail in the composition of the interior with its framed statues and sets of fluted Corinthian columns and pilasters (the rectangular vertical elements flanking pairs of columns) in Plate 5 stands in contrast to the Greeks' greater focus on external space and form. Before the Romans could build the massive dome of the Pantheon, they had to have some understanding of the structural challenges that a dome presents and they had to respond to them.

The shell of a dome is its structural part. Classically, shells of domes are made of masonry or concrete. Figure 2.43 shows how the shell can be thought of as a composite of arches obtained by slicing the dome vertically through its central axis. These arches, just like those studied earlier, generate outward horizontal forces. Because the arches of the shell taper as they rise, they weigh

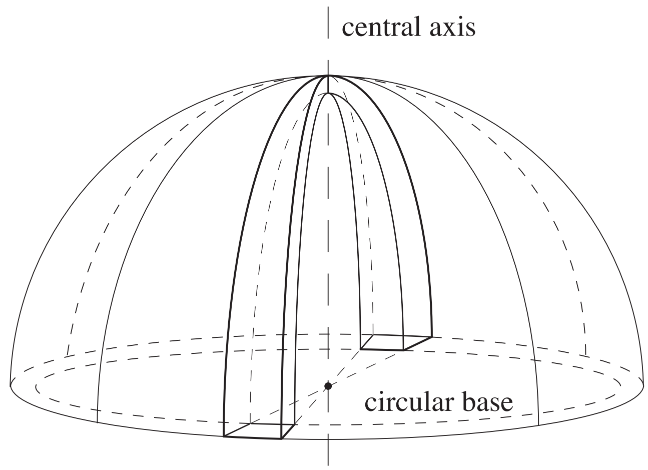


Figure 2.43. The shell of a dome as a composite of arches

less comparatively, so that the magnitudes of these horizontal forces are less than before. However, unlike the situation of the aqueduct of Figures 2.33 and 2.34, there is only the base of the dome and the internal resistance of the shell to contain them. The vertical cross sections of a dome through its central vertical axis are called meridians and the horizontal circular cross sections are called hoops. They are depicted in Figure 2.44. A hoop is a ring-shaped, horizontal slice of the shell. Two forces are at work on the hoops. One is the push of the base of the dome propagated up along the rigid shell. The other is the weight of the dome above the hoop pushing down. Toward the top, the weight is not a factor and the push of the structure from below dominates. This puts the hoop under compression, in the same way that the keystone of an arch is under compression. But farther down, the weight of the shell above exceeds the upward push from below. The outward push of the horizontal components of this excess force puts the hoops (those from the base to about two thirds of the way up the shell) under tension. See Figure 2.44. This tension is called hoop stress. The First Principle of Structural Architecture tells us that unless the shell is able to resist this tension, the shell will expand along the the hoops, so that cracks will develop along the meridians (that can

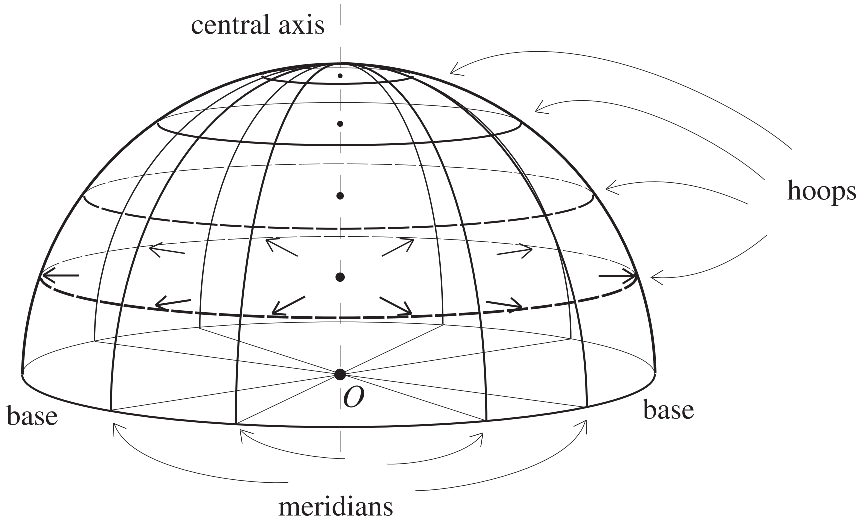


Figure 2.44. Hoop stress on the shell of a dome

lead to structural failure of the dome in extreme cases).

The Romans built the Pantheon out of concrete and intermittent courses of bricks. Concrete was a building material that was relatively easy to build with, but like brick or stone, it had little tensile strength. So it facilitated the construction of the Pantheon, but it had relatively little capacity to contain the hoop stress within the shell of the dome. However, concrete could be made lighter or heavier simply by adding lighter or heavier aggregate, meaning stone and masonry materials, to the mix. The Romans used this to advantage. By making concrete with the light volcanic rock pumice for the upper sections, they reduced both the weight of the dome and therefore the hoop stress. The concrete of the dome weighs about 81 pounds per cubic foot for most of the shell and about 100 pounds per cubic foot for the section of the shell above the supporting cylindrical wall. The circular oculus at the top not only supplies light and air, it reduces the dome's weight further. (Chapter 7, "Volumes of Spherical Domes," applies basic calculus to estimate the weight of the dome.) To contain the outward thrusts of the dome at it base, the Romans made the supporting cylindrical wall up to 20 feet thick with concrete that increased in weight from 100 pounds per cubic foot near the top of the wall to 115 pounds per cubic foot at the bottom. The aggregate of the concrete in the cylindrical wall includes the dense, resilient volcanic stone basalt. The rigid concrete shell propagates the push from the base upward. The inward component of this push counters the tensile stress on the hoops in much the same way as the hoops of a barrel keep its wooden slats (or staves) together. The cylindrical wall of the Pantheon rests on a substantial foundation. The concrete used in the foundation also contains basalt and weighs 140 pounds per cubic foot (close to the 150 pounds per cubic foot of standard modern concrete).

The coffering on the inside of the shell shown in Plate 5 is shallow, serves no structural purpose, and has essentially no impact on the weight of the dome. But the vertical and horizontal configuration of ribs that the coffers suggest does resemble the ribbed elements that would play an important structural role in later domes and vaults. The Romans often placed masonry and concrete masses on top of the lower, outer sections of arches and vaults. These masses were intended to increase the stability of such structures. The Romans may have intended for the step rings they built into the lower part of the dome of the Pantheon (see Figures 2.42 and 2.45) to serve such a function and to contain hoop stress. However, recent studies have indicated that the step rings seem to play no significant role in this regard. The step rings may also have been put in to facilitate the construction work on the shell. The dome was constructed with the use of centering. An elaborate forest of timbers reached upward from the floor of the Pantheon to support the growing shell until the construction was completed and the concrete had set.

Figure 2.45 depicts half of a central vertical section of the Pantheon. It shows the structure of the shell, a section of the cylindrical supporting wall, the oculus, and the step rings. The vectors flowing down the shell represent the downward transmission of the weight of the dome. Their horizontal components generate the hoop stress already discussed. The upward-pointing vectors

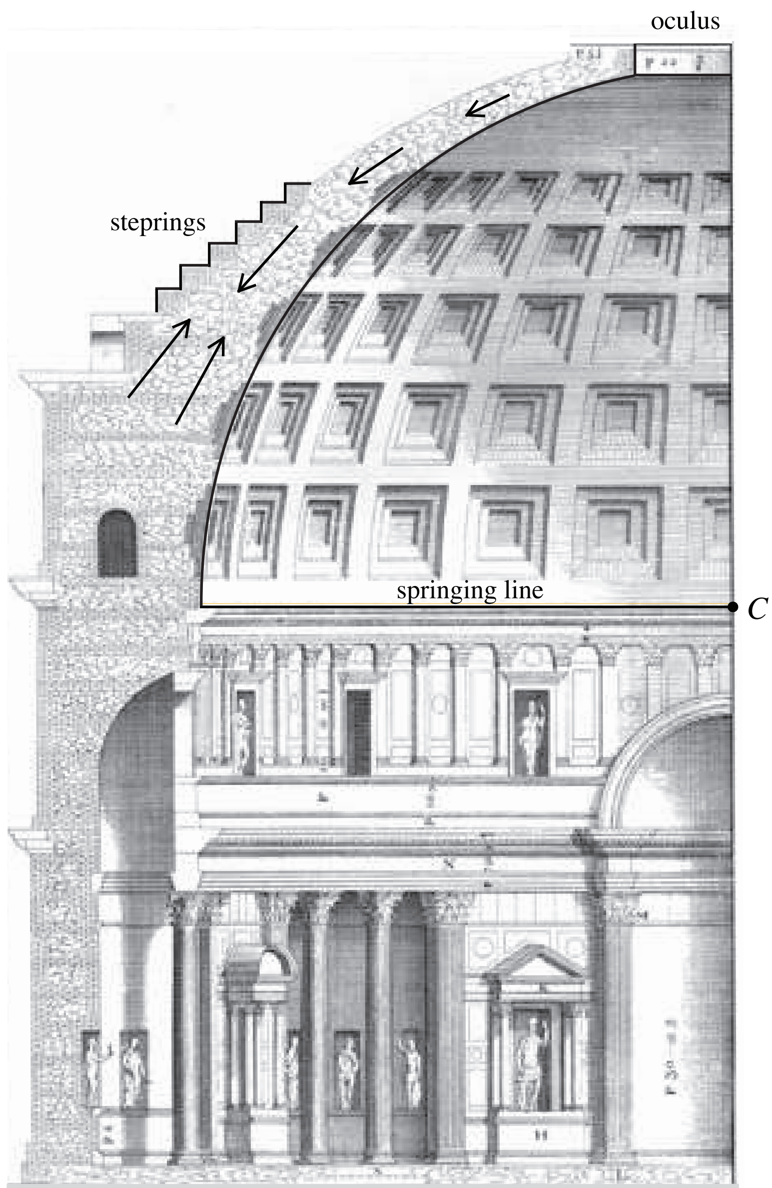


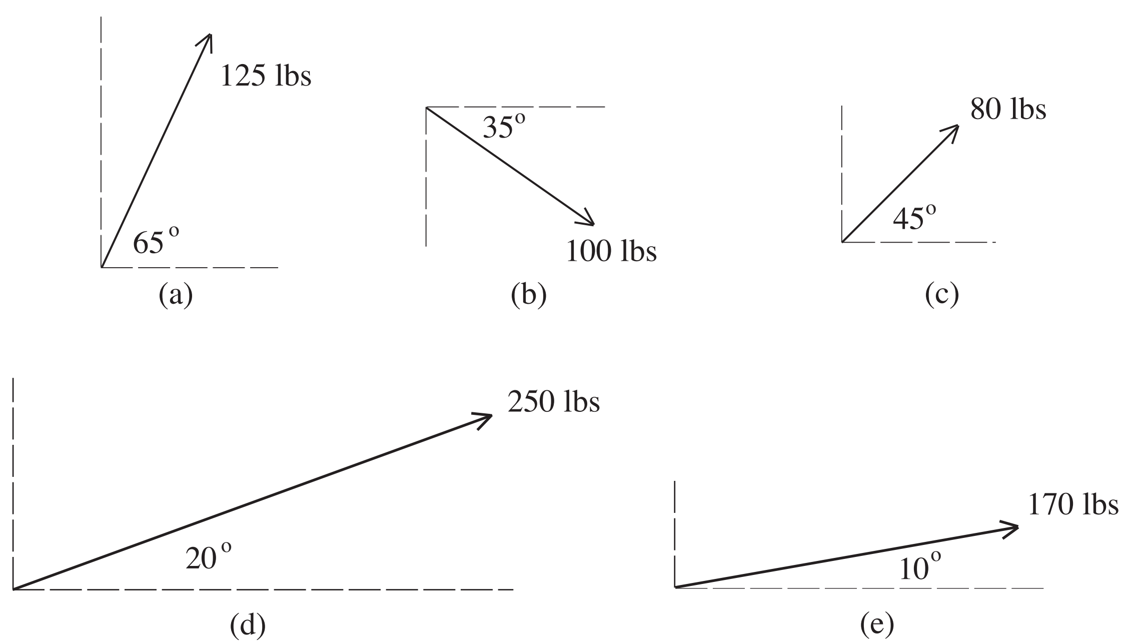
Figure 2.45. Section of the Pantheon from Andrea Palladio's *I Quattro Libri dell' Architettura* (*The Four Books of Architecture*), Venice, 1570

represent the support of the shell from its cylindrical base. Their horizontal components counteract hoop stress. The arc that is highlighted lies on a circle of radius $\frac{1}{2} \cdot 142 = 71$ feet. Its center C is the point on the centering from which the builders of the Pantheon stretched ropes in all upward directions to guide the spherical shape of the shell during construction.

In spite of the efforts to control it, the hoop stress in the dome of the Pantheon did lead to extensive cracks along some meridians of the dome. The distribution of cracks generally corresponds to openings within the upper parts of the cylindrical wall (some of these are shown in Figures 2.42 and 2.45). These openings increase the hoop stress in the parts of the shell that rise near them. Nonetheless, the fact that the Pantheon has remained standing for almost 1900 years tells us how well the Romans succeeded. The Pantheon is one of the most important buildings in the history of architecture. Roman design and construction practices have been very influential. It would be hard to imagine today's architecture without arches, domes, and the use of concrete.

Problems

Problem 1. The vectors in Figure 2.57 represent forces of the indicated magnitudes and



directions. In each case, draw in the vectors that represent their horizontal and vertical components and compute the magnitudes of these components.

Problem 2. Each of the diagrams in Figure 2.58 represents two forces and their resultant. Identify the resultant in each case. The magnitudes of some of the forces (and some of the angles between

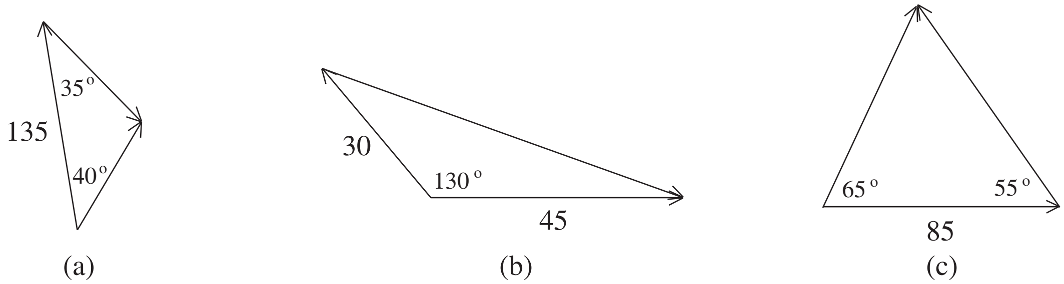


Figure 2.58

them) are given. Use the Law of Sines or the Law of Cosines to compute the magnitudes of the forces that are not specified.

Problem 3. Refer to Figure 2.59. The vector A represents the gravitational force acting on some object. The vectors B and C provide a decomposition of A into components. The vector D

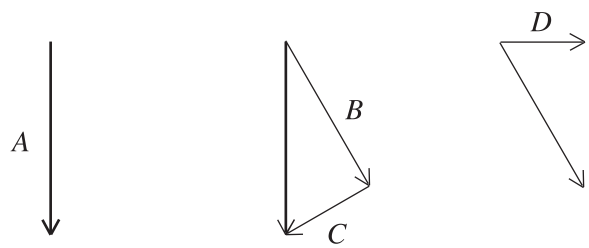


Figure 2.59

is the horizontal component of B. Figure 2.59 appears to show that the downward force A has a horizontal component (namely D). But this can't be since the original A acts vertically. Explain the apparent contradiction.