REVIEWS

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Calculus: Modeling and Application, second edition. By David A. Smith and Lawrence C. Moore. Mathematical Association of America, Washington, D.C., 2010. http://maa.pinnaclecart.com. Price: $35.00. ISBN 978-1-6144-610-1.

Reviewed by Betty Mayfield

Sit back. Relax. Close your eyes. Now imagine that you are picking up a calculus textbook. Feel its mighty heft. Open it to see the formulas inside the front cover, the table of integrals at the back of the book, the solutions to odd-numbered exercises. Scan the table of contents for the familiar topics, in the familiar order. Flip open to a page at random and admire the colorful graphics, the theorems helpfully boxed and shaded, the examples carefully worked out. Note the many exercises you may assign to your students. Resist the urge to call the university bookstore and ask how much they are charging this semester for this impressive tome.

Now imagine a completely different kind of text—different both in content and delivery. In fact, you are reading it on your iPad. That book might be David Smith and Lang Moore's *Calculus: Modeling and Application* (*CMA*), available through the MAA Bookstore as an electronic text. It is a direct descendent of *The Calculus Reader* [**2**], one of the more popular texts to grow out of the calculus reform movement of the 1990s [**1**], and it retains much of the flavor of that earlier book. Its emphasis is on the use of authentic real-world problems to motivate learning new mathematics; on the use of technology for graphing, computation and exploration; and on collaboration among students as they discover patterns and construct their own mathematical knowledge.

As an example of how this book is different from a standard calculus text, we examine its treatment of several standard calculus topics. In most texts, the section on the Product Rule begins with a statement of the answer, $\frac{d}{dt}(uv) = u \cdot \frac{dv}{dt} + v \cdot \frac{du}{dt}$, set off in a color-coded box. Then the student is shown several examples to convince him or her that the rule seems to work. This discussion is followed by a formal proof, and maybe a picture of why it makes sense. The text quickly goes on to state the Quotient Rule and a Power Rule for Negative Exponents for differentiation. There are homework problems of each of these types at the end of the section.

In *CMA*, the section on the Product Rule begins with a motivating real-world problem: In the 1980s, the U.S. population was growing (at a roughly exponential rate, though admittedly with a very small exponent), and per capita energy consumption was also growing (at a roughly linear rate), and so the total energy consumption, represented by the product of those two functions, was certainly growing as well—but at what rate? Students realize that they know how to differentiate each factor separately, but they are not sure if they know how to differentiate their product.

Instead of giving them the answer, the text launches into an Activity (a familiar sight to students of this text) in which they look at several examples of functions u, v, their product uv, and their derivatives. Not only do students quickly notice that the

http://dx.doi.org/10.4169/amer.math.monthly.120.09.862

derivative of a product is apparently not the product of the derivatives, they are led to conjecture a formula on their own. In my experience, after experimenting with four or five examples of simple functions, someone in the class always says, "Hey, what if we added those two? No wait, multiply those first..." and we really do "discover" the Product Rule together. (There will undoubtedly be students in your class who have taken calculus before, and some of them might actually remember the Product Rule correctly; you may have to ask them to keep quiet for a few minutes.) Then we can go back and answer the question about energy consumption.

In fact, the introduction to the derivative itself is unusual in *CMA*. There is no preliminary discussion of limit or continuity, and the word "secant" does not make an appearance until we reach trigonometric functions at the end of the semester. Instead, students are reminded of what they already know about slope and average rate of change and are introduced to the concept of a difference quotient, all in the context of the motion of a falling body. Then the text shifts to a discussion of local linearity: Zoom in really close on the graph of a parabola; what does it look like? How can we use that information to define the instantaneous speed of a falling body at a certain time? We agree that we can get a pretty good idea by computing the difference quotient using a tiny value of Δt— say, 0.001. After students compute this nearly-instantaneous rate for several values of t, and are convinced that we can do so for any value, they begin to understand that we have defined a new *function* of t, an instantaneous speed function. And we can graph it. Using a computer algebra system, students graph the function $f(t) = k \cdot t^2$, along with its almost-instantaneous speed function, for several values of k, and then try to write down a formula for the speed in each case. By the end of the activity, whether they realize it or not, students have just discovered the derivative of the quadratic function $f(t) = k \cdot t^2$. Eventually students do verify these results by evaluating the difference quotient algebraically and taking the "limiting value" (the word "limit" is not used until we reach definite integrals and their limits of integration many chapters later) of the result, and a formal definition of the derivative appears—in the requisite box. But we return over and over to this graphical/numerical approach of approximating a derivative with a difference quotient and trying to guess what the resulting function is, whenever we are confronted with a function whose derivative we do not know.

In a later chapter, after working with left- and right-hand sums to approximate the area under a curve, and noting their inherent limitations, students are led to consider two improvements on them: using the midpoint of each interval instead of either endpoint, or taking the average of the two sums. Thus they are introduced to the Midpoint Rule and the Trapezoidal Rule in a natural way. Simpson's Rule is presented as a weighted average of those two new approximations—something I had never seen before.

Those easily shocked will have many provocations: The definition of a continuous function does not appear until Chapter 10; the Quotient Rule is hidden in a homework exercise and may be skipped entirely; the name Riemann does not appear in the index. Instructors will search in vain for the standard related rates problems (the conical pile of sand, the sliding ladder, the shadow of the man who is doomed to spend eternity walking away from that light pole). Instead, there is an Air Traffic Control project involving velocities and three-dimensional distances which students are expected to solve in groups, working with minimal guidance and using their knowledge of derivatives. There are no solids of revolution whose volumes must be found by shells or discs. But students compute the balance point of a pool cue and, later, the centers of mass of several regions in the plane. (In our classes, students cut those regions out of cardboard, and we make a class mobile using their computed balance points.) Students become adept at fitting models to data, often using semilog and log-log plots to determine what kind of function may work best. We should take the title of this book seriously. To help them decide which topics to include—and which to omit—from this text, the authors consulted with faculty from our "partner disciplines" and asked them how and when they really used calculus in their fields. The answers they got drove the content of this book.

Another unique feature of this text is the fact that it is only available in electronic form, in either a computer or tablet version. This is not a static pdf file which happens to be available on a computer; it is truly electronic, truly interactive. As students read the text, they can click on live buttons that lead them to pictures, graphs, a calculator, and to computer activities where they can explore and experiment. The computer version requires the use of the Firefox browser and one of the computer algebra systems Maple, Mathematica, or Mathcad for most of the activities. It can be downloaded and then used without internet access. The tablet version runs on either the Safari or Firefox browser and uses interactive Sage worksheets; it does require internet access. The price is right for this text: students pay $35 for one-year electronic access from the time of purchase.

There are ample homework exercises; in fact, for each section there are two levels of assignments—Exercises (the more customary mechanical/rote problems, reinforcing the material in the text) and Problems (deeper, richer, requiring more exploration and thought). There are no answers to odd-numbered problems at the end of the text, but almost all of the Exercises are available through the CMA Library in WeBWorK, the open-source online homework system now managed by the MAA (`http://webwork.maa.org/`). In that system, students are given immediate feedback and, if the instructor allows, may go back and try the problem again; faculty have no homework to grade but are given detailed information about students' performance. Our experience here at Hood College has been that students will exhibit a remarkable level of perseverance in trying to solve a problem correctly and to receive the computer's affirmation—a trait we have never particularly observed when assigning pencil-and-paper assignments.

Although this text is designed to be used in a class—and a rather unconventional one at that—its features make it appropriate to be used in many settings (with or without group work, with or without a computer algebra system, with or without the WeBWorK exercises), and even for independent study. The activities and checkpoints (all answers are in the text), the exercises and problems, and even the projects could be completed by a student working alone.

Lots of information is available about this text on the MAA website, including sample chapters. If you are ready to make a change in the way you and your students experience calculus, this just might be the book for you.

You can open your eyes now.

REFERENCES

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