# The Slice Group in Rubik's Cube

DAVID HECKER
RANAN BANERJI
*Saint Joseph's University*
*Philadelphia, PA 19131*

The different maneuvers on Rubik's cube can be thought of as a set of transformations forming a subgroup of the group of all permutations of the cube's 54 facelets. In this paper, we shall do a complete study of the small subgroup of this group generated by turning only the center layers of the cube. This is known as the **slice group**. Our purpose in this study is to use the cube to illustrate several fundamental group theoretic techniques.

Before an accurate statement of the problem can be made, we must introduce some notation, most of which is standard in the cube literature [**4**]. We shall describe all maneuvers as if they were carried out with respect to the cube held in a fixed position, referring to the six faces by the "colors" $F$, $B$, $R$, $L$, $U$, and $D$ for front, back, right, left, up, and down. The cube will be assumed to start with all six faces having solid colors, known as the **clean state** of the cube (see FIGURE 1). A clockwise 90° turn of any of these six faces (clockwise looking at the face "from outside") will be denoted by $T_F, T_B$, etc. (see FIGURE 2). We shall also talk about a 90° clockwise turn of the entire cube looking at the corresponding face from outside and denote these by $C_F, C_B$, etc. Every possible maneuver $M$ can be written as a finite sequence of these fundamental moves, hence these moves generate the group of all possible transformations of the cube. The inverse of any move $M$, denoted by $M^{-1}$, is the maneuver required to undo the effect of the move $M$. Note that the actual twists of the cube needed to accomplish this will not be unique. To avoid this problem, when speaking of a move or maneuver on the cube, we refer only to its effective **permutation** of the cube's 54 facelets, and not to the actual twists required. Two different sequences of twists having the same effect are considered to be the same move. Repeating any move a number of times will be expressed with the usual exponential notation.

In addition to referring to the cube's facelets, we shall also refer to the 27 subcubes of the cube,

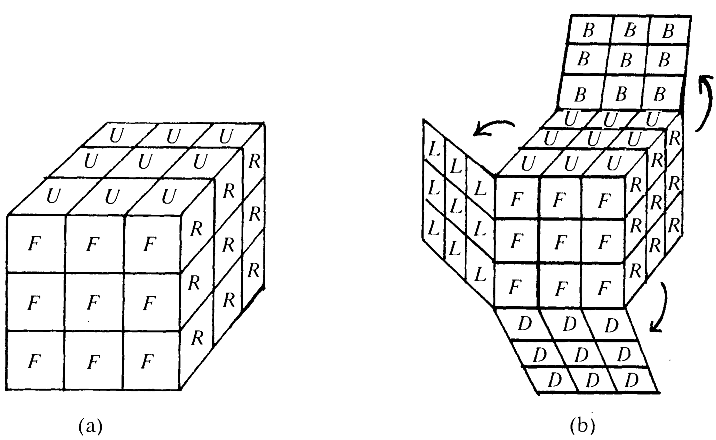



FIGURE 1. **(a) Rubik's cube in the clean state. (b) Rubik's cube in the clean state with hidden sides displayed.**



This content downloaded from 65.206.22.38 on Mon, 19 May 2014 12:55:43 PM
All use subject to JSTOR Terms and Conditions

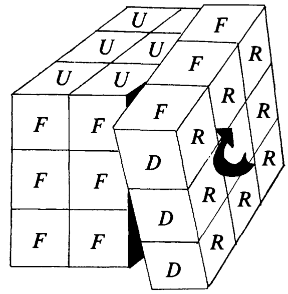


FIGURE 2. **The move $T_R$.**

often called **cubies** in the literature [**4**]. One of these cubies lies in the interior of the cube and cannot be seen. Eight others appear on the corners of the cube; these each have 3 facelets showing. Twelve are edge cubies with 2 facelets showing, and the remaining six lie at the center of each face and have only one facelet visible. The cubies are important since the facelets on each cubie stay on that cubie so that a large number of the 54! permutations in the group of all permutations of the 54 facelets are prohibited. Also, every move takes corner cubies to corner positions, edge cubies to edge positions, and center cubies to center positions, thus further limiting the possible permutations. The position of any facelet or cubie when the cube is in the clean state is called the **home position** of that facelet or cubie. The clean state is completely characterized by all facelets being in their home position. We think of this position as representing the identity permutation on the cube.

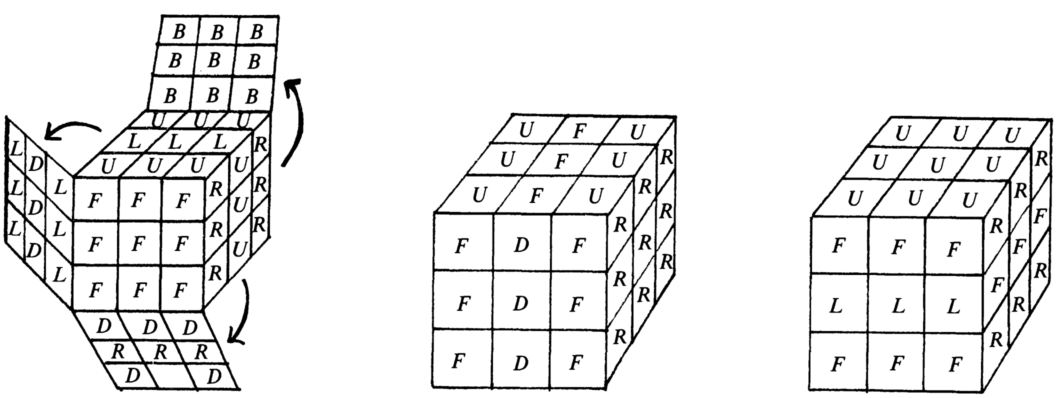
## The slice group

The **slice group** is the group of transformations of the cube generated by the following three move sequences:

$$T_B T_F^{-1} C_B^{-1}, \quad T_L T_R^{-1} C_L^{-1}, \quad T_U T_D^{-1} C_U^{-1}. \tag{1}$$

These are illustrated in FIGURE 3. One can think of them in two ways. First, consider them as turning two opposite faces "parallelly," one clockwise and one counterclockwise, then turning the whole cube to return those two faces to their original positions. Alternately, one can consider the overall effect, which is to turn the center slice of the cube clockwise 90° when looking at the $F$, $R$, and $D$ faces respectively.

Our discussion will be from this latter point of view. We shall denote the three sequences in (1) by the single letter $F$, $R$, and $D$, which will stand for turns of the corresponding center slice. We



$F: T_B T_F^{-1} C_B^{-1}$ $\qquad$ $R: T_L T_R^{-1} C_L^{-1}$ $\qquad$ $D: T_U T_D^{-1} C_U^{-1}$

FIGURE 3. **The three moves which generate the slice group.**



This content downloaded from 65.206.22.38 on Mon, 19 May 2014 12:55:43 PM
All use subject to JSTOR Terms and Conditions

have thus given these letters two meanings, as we have also used them to represent colors of the faces. The meaning in a given instance should be clear from the context.

To discover the structure of the slice group $G$ of permutations generated by the moves $F$, $R$, and $D$, we analyze the actions of slice moves on different sets of cubies, thus constructing homomorphisms from $G$ to known groups.

Recall that if $H$ is any group of permutations on a set $X$, and if $Y$ is a subset of $X$ such that for every $h \in H$ and $y \in Y$, we have $h(y) \in Y$, then $Y$ is said to be **closed** under the action of $H$. When we have such a closed subset, a homomorphism is induced from $H$ into the group of permutations of $Y$: simply map $h$ in $H$ to the permutation it induces on the set $Y$. In our case, $G$ is a group of permutations on the set of cubies and facelets. We will find subsets of cubies that are closed under $G$, thus inducing homomorphisms of the type just described.

Returning to the cube, first notice that every corner cubie is left fixed, or unmoved, by each slice move, thus the slice group induces the identity permutation on the set of corner cubies and their facelets. This is convenient, since the colors on these corner cubies act as benchmarks in case we forget which face is front, back, etc.

Next, slice moves always move center cubies to center positions, hence the set of six center cubies is closed under the action of $G$. We denote by $\tau$ the homomorphism this induces from $G$ into $S_6$, the group of permutations on six objects (in this case, the center cubies). The image $\tau(G)$ in $S_6$ ignores the action of $G$ on all cubies except the centers. By examining the cube, one notes that each slice move has the same effect on the centers as some rigid motion of the entire cube. For example, the move $F$ takes centers to the same positions that $C_F$ does. Therefore, $\tau$ maps $G$ into $T$, the group of rigid motions (rotations) of the cube (a subgroup of $S_6$). The map $\tau$ is actually onto $T$ since $T$ is generated by $C_F$, $C_R$, and $C_D$, and $\tau(F)=C_F$, $\tau(R)=C_R$, and $\tau(D)=C_D$. It is well known that $T$ is isomorphic to $S_4$, and thus has 24 elements. (The isomorphism with $S_4$ is unimportant in what follows. The interested reader can construct the isomorphism by thinking of $T$ as acting on the set of 4 diagonal line segments linking opposing corners of a rigid cube.)

The twelve edge cubies can be partitioned into three subsets of four each, $E_F$, $E_R$, and $E_D$, where each set contains the edge cubies on the center slice parallel to the $F$, $R$, and $D$ faces, respectively. By manipulating the cube, one notices that the generators $F$, $R$, and $D$ (and hence every slice move in $G$) take edge cubies in these sets to edge cubies in the same set. Hence, as above, we get homomorphisms $\phi_F$, $\phi_R$, and $\phi_D$, each mapping $G$ into the group of permutations of $E_F$, $E_R$, and $E_D$, respectively. Again, by examining the cube, one notices that $F$ induces a 4-cycle on the elements of $E_F$, while $R$ and $D$ each induce the identity permutation on $E_F$. Hence $\phi_F(F)$ is a 4-cycle, $\phi_F(R)=\phi_F(D)=id$, and we see that $\phi_F(G)$ is generated by $\phi_F(F)$ and is isomorphic to $Z_4$. For any element $g$ of $G$, $\phi_F(g)$ is the number of times mod 4 that $F$ appears in any sequence of moves generating $g$, with $F^{-1}$ counted as $-1$ (or 3 since $F^{-1}=F^3$). A similar analysis can be made of the homomorphisms $\phi_R$ and $\phi_D$.

We thus have four homomorphisms of $G$: one onto $T$ and three onto $Z_4$. By combining the four homomorphisms, one obtains a homomorphism from $G$ into $T \times Z_4 \times Z_4 \times Z_4$ given by

$$g \to (\tau(g), \phi_D(g), \phi_R(g), \phi_F(g)). \tag{2}$$

The kernel of this homomorphism is clearly the intersection of the kernels of the four homomorphisms defining it, which is the set of elements in $G$ leaving all center cubies and all elements of $E_D$, $E_R$, and $E_F$ fixed. Such a permutation must leave the cube in its clean state, and is thus the identity. Therefore, the homomorphism is injective and $G$ is isomorphic to its image, a subgroup of $T \times Z_4 \times Z_4 \times Z_4$.

Next, we notice that each generator of the slice group acts as the identity on two sets of edge cubies, as a 4-cycle on one set of edge cubies, and as a 4-cycle inside $T$, the group of motions of the centers. Therefore, overall, each generator acts as a product of two disjoint 4-cycles, yielding an even permutation. It follows that $G$ is isomorphic to a subgroup of the *even* permutations in $T \times Z_4 \times Z_4 \times Z_4$. We will show that the image of $G$ contains all such even permutations. (Note



This content downloaded from 65.206.22.38 on Mon, 19 May 2014 12:55:43 PM
All use subject to JSTOR Terms and Conditions

that parity in $T \times Z_4 \times Z_4 \times Z_4$ is inherited from $S_{18}$, the group of all permutations of the 18 center and edge cubies, of which $T \times Z_4 \times Z_4 \times Z_4$ is a subgroup.)

Using (2), $G$ can be identified as a subgroup of $T \times Z_4 \times Z_4 \times Z_4$, and $\tau$ is the projection onto the first coordinate of the product. Hence, $\ker \tau$ can only contain elements of the form $(id, a, b, c)$ in $T \times Z_4 \times Z_4 \times Z_4$, with $a + b + c$ even for the reasons discussed above. The following moves generate the indicated elements:

| | | |
|---|---|---|
| (i) | $RF^{-1}DF$ | $(id, 1, 1, 0)$ |
| (ii) | $RDFD^{-1}$ | $(id, 0, 1, 1)$ |
| (iii) | $FDR^{-1}D^{-1}FD^{-1}RD$ | $(id, 0, 0, 2)$. |

Remember that the last three components of these elements refer to the cyclic permutation of the edge cubies in the $D$, $R$, and $F$ planes, respectively. It can be shown that the three moves (i), (ii), (iii) generate every element of the form $(id, a, b, c)$ with $a + b + c$ even. The procedure for doing this is similar to that of expressing the vector $(a, b, c)$ as a linear combination of (1,1,0), (0,1,1), and (0,0,2) as done in linear algebra, only here we work with integers and do arithmetic mod 4. Therefore, $\ker \tau$ is precisely the set of elements of this form, and thus has 32 elements. $\operatorname{Im} \tau$ has 24 elements since $\tau$ is onto $T$. Hence $G$ has $24 \times 32 = 768$ elements.. But the set of even permutations in $T \times Z_4 \times Z_4 \times Z_4$ also has 768 elements, so $G$ must equal this set.

Let us use this analysis of the structure of $G$ to solve a cube that has been mixed up through slice moves. For example, consider the cube oriented as in FIGURE 4. (Should the reader wish to follow this discussion on an actual cube, the move $R^2FR^2F^{-1}DF^2R^{-1}$ will put a cube in the clean state into this position.) The first requirement is that we be able to recognize the position's expression as an element of $T \times Z_4 \times Z_4 \times Z_4$. One finds the first coordinate by writing down the permutation in $S_6$ of the center cubies. For the other three coordinates, one examines the edge cubies in $E_D$, $E_R$, and $E_F$, respectively, and counts the number of $D$, $R$, and $F$ moves that were required to put the cubies in their current position. The result of this computation for FIGURE 4 is $([(F, L, U)(B, R, D)], 1, 3, 2)$.

To solve the cube, first make slice moves mimicking the rigid motions of the cube to bring the center cubies to their home positions. At most two 90° or 180° turns will always be sufficient to do this. In our example, the move $F^{-1}$ puts the right and left centers in their home position (see FIGURE 5(a)). The move $R$ will then return the remaining centers to their home position, as in FIGURE 5(b). The cube is now in the position $(id, 1, 0, 1)$. The purpose of this maneuver is to put the cube's positional representation in $\ker \tau$. The advantage here is that $\ker \tau$, having only 32

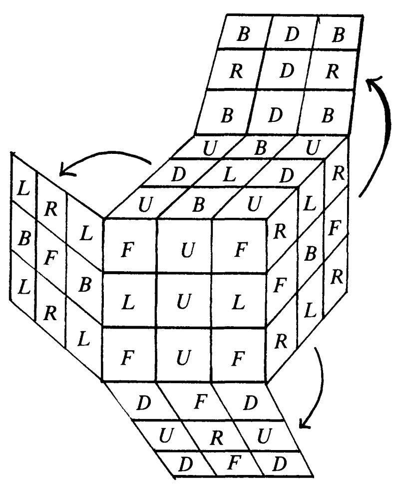



FIGURE 4. **The cube in position $([(F, L, U)(B, R, D)], 1, 3, 2)$.**



This content downloaded from 65.206.22.38 on Mon, 19 May 2014 12:55:43 PM
All use subject to JSTOR Terms and Conditions

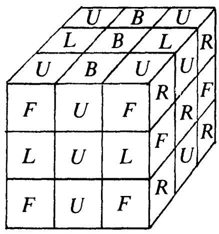



FIGURE 5 (a)

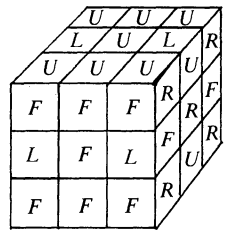



FIGURE 5 (b)

elements, is a much simpler group to work with than $G$. To complete the solution, we now use the generators for $\ker\tau$ given by equations (i), (ii), and (iii) to produce the inverse element of the current cube position so that the cube can be restored to the clean state. The inverse of $(id,1,0,1)$ is $(id,3,0,3)$, which can be obtained by move (i) 3 times followed by (ii) and then (iii). Denoting this sequence of moves by $M$ we write $M=\text{(i)}^3\text{(ii)(iii)}$. Hence, starting from FIGURE 4, we can make the following sequence of moves to restore the cube:

$$F^{-1}RM=F^{-1}R(RF^{-1}DF)(RF^{-1}DF)(RF^{-1}DF)(RDFD^{-1})(FDR^{-1}D^{-1}FD^{-1}RD).$$

This is definitely not the shortest possible solution, but it works. One method of shortening the answer is to use the fact that $\text{(i)}^3=\text{(i)}^{-1}=F^{-1}D^{-1}FR^{-1}$; this yields the solution

$$F^{-1}R(F^{-1}D^{-1}FR^{-1})(RDFD^{-1})(FDR^{-1}D^{-1}FD^{-1}RD). \tag{3}$$

Note that $R^{-1}R$ appears in this expression and can be cancelled out; however, very little else can be done to shorten this process without a whole new approach to the problem.

The method used here is typical of cube-solving strategies. One finds a sequence of nested subgroups of the group of allowable positions; in our case,

$$G\supset\ker\tau\supset 1.$$

Then, for each position in the sequence, one finds moves for a representative in each coset of the smaller subgroup in the larger. One calculates the cube's position, then performs the moves corresponding to the known coset representative of the position's inverse to reduce the cube's state to one in the next subgroup down the sequence. In our example, the first such reduction was made by performing the move $F^{-1}R$, a representative of the coset over $\ker\tau$ of the inverse of the cube's original position. All the published cube solutions (see references) use this strategy, although different solutions use different subgroup series. For example, Taylor and Rylands [**8**] solve a slice group problem by utilizing the series

$$G\supset\ker\Theta\supset 1$$

where $\Theta$ is the projection of $G$ onto $Z_4\times Z_4\times Z_4$. This solution first puts all edge cubies in their home positions to obtain a "spot pattern." Then generators of $\ker\Theta$ are given to solve that pattern.

In the case of the slice group, more efficient solutions could be found by an exhaustive search (a good exercise for those with a background in computer programming) since there are only 768 possible positions. This, however, is an unrealistic approach for solving a generally scrambled cube, since there are about $10^{18}$ possible different cube positions [**1**]. The series of subgroups approach seems to be the only one to generate practical algorithms for solving larger cube problems. For further information on this approach, and some interesting subgroup series used for larger cube problems, we refer the reader to Frey and Singmaster [**4**].

## The oriented slice group

Some of the cubes on the market today have symbols or designs pasted on the facelets instead of solid colors. Some even have pictures, and one must unscramble all six pictures to solve the cube. In solving a scrambled cube of this type, one might notice that some of the center facelets



This content downloaded from 65.206.22.38 on Mon, 19 May 2014 12:55:43 PM
All use subject to JSTOR Terms and Conditions

have been rotated with respect to the other designs on the faces on which they lie. Move sequences are known [8] that yield rotations of individual center cubies. We will study the rotations which are possible using slice moves.

To analyze the effect of slice moves on center cubies, we shall redefine the identity transformation of the cube to include fixed orientation of the centers. This, of course, redefines the term "clean state" for the cube as well (see FIGURE 6). We then get a larger group $G^*$ of possible cube positions, the **oriented slice group**. If one does not have a cube with designs or pictures, one can experiment with $G^*$ by pasting pieces of mailing labels over the center and upper right hand corner of each face, and then drawing arrows on the labels, both in the same direction, to signify orientation. The arrows on the corners provide benchmarks of orientation since the corner facelets remain fixed under slice moves. In what follows, we will assume that the arrows have been drawn on the cube in the directions indicated in FIGURE 6.

To describe the position of the cube after some slice moves, one must first describe the positions of all the center facelets and edge cubies with some element of $G$, then describe the orientation of each of the six center facelets. We represent the orientation of a given center as an element of $Z_4$ in the following way. One finds the center facelet in question on the cube, then compares the direction of the arrow pasted on it with the direction of the arrow on the corner cubie of the face on which the center facelet currently resides. The element in $Z_4$ counts the number of 90° clockwise rotations the center has made from the standard direction noted on the corner cubie. Using one copy of $Z_4$ for each center facelet, the cube's position can be represented as an element of the set $G \times Z_4 \times Z_4 \times Z_4 \times Z_4 \times Z_4 \times Z_4 = G \times (Z_4)^6$, where the last six coordinates represent the orientation of the $U$, $D$, $F$, $B$, $R$, and $L$ colored centers, respectively. The representations of the three generators of the oriented slice group, using this notation are given in TABLE 1.

It is important to note that we have yet to define a group operation on the set $G \times (Z_4)^6$. We will write the representation of a slice move in $G \times (Z_4)^6$ as $(\rho,(A,B,C),(a,b,c,d,e,f))$, where $\rho$ is in $T$, a subgroup of $S_6$, $(A,B,C)$ is in $Z_4 \times Z_4 \times Z_4$, and $(a,b,c,d,e,f)$ is in $(Z_4)^6$. Note that $(\rho,(A,B,C))$ comprises the $G$ component of the slice move. Then given two slice moves,

$$g = (\rho,(A,B,C),(a,b,c,d,e,f))$$

and

$$h = (\sigma,(X,Y,Z),(u,v,w,x,y,z)),$$

we define their composition (the group operation in $G \times (Z_4)^6$) to be

$$hg = \big(\sigma\rho,((A,B,C)+(X,Y,Z)),((a,b,c,d,e,f)+\rho^{-1}(u,v,w,x,y,z))\big)$$

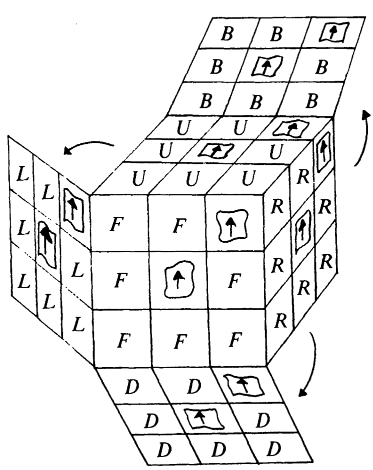


FIGURE 6. **An oriented cube in the clean state.**



This content downloaded from 65.206.22.38 on Mon, 19 May 2014 12:55:43 PM
All use subject to JSTOR Terms and Conditions

| ←G→ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Move | Permutation of Center Facelets | Cycles of Edge Planes | | | Rotations of Center Facelets | | | | | | |
| | | $E_D$ | $E_R$ | $E_F$ | $U$ | $D$ | $F$ | $B$ | $R$ | $L$ |
| $D$ | $(F,R,B,L)$ | 1 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 |
| $R$ | $(U,B,D,F)$ | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $F$ | $(U,R,D,L)$ | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 1 |

TABLE 1

where additions are componentwise, mod4. The purpose of applying $\rho^{-1}$ before adding the last six components is to make sure the orientation changes are applied to the proper center facelets. The representation of a position in $G\times(Z_4)^6$ assumes that moves were made starting from the clean state. However, composing moves requires the second move to start from where the first one left off, so the center cubies are not starting from their home positions. Since $\rho$ indicates the centers' positions after the move $g$, applying $\rho^{-1}$ before adding permutes the orientation changes being made by $h$ so that they are applied to the proper facelets' coordinates. The resulting group structure is called a **semidirect product** of groups. We refer the reader to the literature [**5**] for more information on this topic.

It can be shown that $G^*$ can be represented as a subgroup of the set $G\times(Z_4)^6$ with the group operation described above. In fact, $G^*$ is the subgroup generated by the moves $R$, $D$, and $F$ whose representations are given in TABLE 1.

Let $\pi$: $G^*\to G$ be the homomorphism which projects $G^*$ onto $G$, that is, $\pi$ focuses on cubie position and ignores center facelet orientations. Then $\ker\pi$ is the set of slice moves that leave all cubies in their home position, but may rotate center facelets. Note that in $\ker\pi$, the composition of moves corresponds to ordinary addition in $(Z_4)^6$ since the $G$ component is the identity element. This greatly simplifies matters.

The following two moves, $\alpha$ and $\beta$, are in $\ker\pi$. Their last six coordinates in our representation are given.

$$\begin{aligned} \alpha &= RF^{-1}DFR^{-1}F^{-1}D^{-1}F \qquad (0,0,2,2,0,0)\\ \beta &= RF^{-1}DFD^{-1}FR^{-1}F^{-1} \qquad (3,1,0,0,3,1) \end{aligned} \tag{4}$$

One can construct similar moves by conjugating these with rigid motions of the cube (think of holding the cube with different faces forward and up and using the same twists) to obtain

$$\begin{aligned} \alpha^* &= C_U\alpha C_U^{-1} \qquad (0,0,0,0,2,2),\\ \beta^* &= C_U\beta C_U^{-1} \qquad (3,1,1,3,0,0),\\ \alpha' &= C_L\alpha C_L^{-1} \qquad (2,2,0,0,0,0),\\ \beta' &= C_L\beta C_L^{-1} \qquad (0,0,1,3,3,1). \end{aligned} \tag{5}$$

The six moves in (4) and (5) generate $\ker\pi$. We outline a proof of this, but leave the details to the reader.

First, show that for any element of $G^*$, the number of odd numbers in the last six coordinates is either 0 or 4. (This is true of the generators. Show that this property is preserved under composition in $G^*$. You will need other properties of the generators and the fact that opposite centers always remain opposite to each other.) Next, prove that within $\ker\pi$, the sum over coordinates corresponding to orientations of opposite faces is 0 mod 4. (This is only true in $\ker\pi$ and is more difficult to prove. Consider the generators $R$, $D$, and $F$ without the final rigid motion given in their definition, and remember that in $\ker\pi$, all centers end up in their home position.) Then count and find that there are 32 six-tuples having these two properties, and check that all 32 can be generated using $\alpha$, $\alpha^*$, $\beta$, $\beta^*$, $\alpha'$ and $\beta'$.



This content downloaded from 65.206.22.38 on Mon, 19 May 2014 12:55:43 PM
All use subject to JSTOR Terms and Conditions

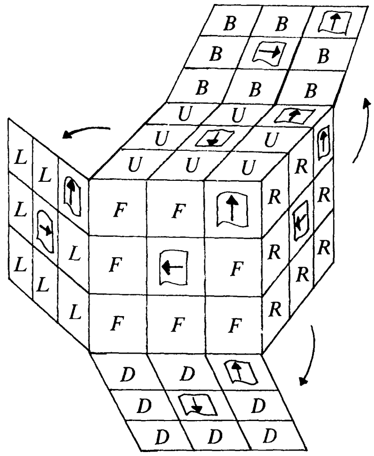



FIGURE 7. **The cube in position ($id$, (0,0,0), (2,2,3,1,3,1)).**

From the information above, we find that

$$|G^*| = |\ker \pi| \cdot |\operatorname{Im} \pi| = |\ker \pi| \cdot |G| = 32 \times 768 = 24576.$$

This is the total number of distinct possible positions of an oriented Rubik's cube scrambled by slice moves.

Although we do not have a simple description of which elements of $G \times (Z_4)^6$ are in $G^*$, we do have an algorithm for solving an oriented cube problem by using a series of subgroups. If we let $\tau'$: $G^* \to T$ be the composition $G^* \xrightarrow{\pi} G \xrightarrow{\tau} T$, then the series

$$G^* \supset \ker \tau' \supset \ker \pi \supset 1$$

provides our solution. The passage from $G^*$ to $\ker \pi$ through $\ker \tau'$ is precisely the method of solution given previously, which ignores orientation; that is, first get the centers in their home positions (pass to $\ker \tau'$), then move the edge cubies to their home positions leaving centers fixed (pass to $\ker \pi$) using the moves (i), (ii), and (iii). Now express the inverse of the current position (in $\ker \pi$) as a product of the generators $\alpha$, $\alpha^*$, $\alpha'$, $\beta$, $\beta^*$, and $\beta'$ and perform these moves to orient the center facelets, thus solving the cube. For example, suppose an oriented cube is put into the position of FIGURE 4 by making the move $R^2FR^2F^{-1}DF^2R^{-1}$ on a cube in the clean state. One solves this cube problem by first ignoring the orientation arrows and following the solution technique described for an unoriented cube. According to our previous calculations, this requires us to make the move given in formula (3). This will leave the cube in the position $(id, (0,0,0), (2,2,3,1,3,1))$, as shown in FIGURE 7. Since this position is in $\ker \pi$, its inverse is $(id, (0,0,0), (2,2,1,3,1,3))$, and this inverse can be expressed as a product of the known generators of $\ker \pi$. The composition $\alpha^*\alpha'\beta'$ is one of many that work. Making this move will restore the cube to the clean state.

One of us (Ranan Banerji) is supported part-time by the National Science Foundation under Grant MCS-821 7964 to Saint Joseph's University.

### References

[1] C. Bandelow, Inside Rubik's Cube and Beyond, Birkhauser, Boston, Mass., 1982.
[2] E. Berlekamp, J. H. Conway, and R. Guy, Winning Ways, Academic Press, 1982, pp. 760–768, 808–809.
[3] T. Davis, Teaching mathematics with Rubik's cube, Two-Year College Math J., 13 (1982) 178–185.
[4] A. H. Frey and D. Singmaster, Handbook of Cubik Math, Enslow Publishers, Hillside, N.J., 1982.
[5] S. MacLane and G. Birkhoff, Algebra, Macmillan Co., New York, 1967, p. 461.
[6] J. G. Nourse, The Simple Solution of Rubik's Cube, Bantam Books, New York, 1981.
[7] D. Singmaster, Notes on Rubik's Magic Cube, Enslow Publishers, Hillside, N.J., 1980.
[8] D. Taylor and L. Rylands, Cube Games, Holt, Rinehart, and Winston, New York, 1981.



This content downloaded from 65.206.22.38 on Mon, 19 May 2014 12:55:43 PM
All use subject to JSTOR Terms and Conditions