

Contents

Preface	xi
1 Introduction to inverse problems	1
1.1 Two integral equations	2
1.2 A forward problem/inverse problem pair	7
1.2.1 Forward versus inverse problems	10
2 Well-posed, ill-posed, and inverse problems	15
2.1 Well-posed problems	16
2.2 Ill-posed problems, part 1	18
2.2.1 The uniqueness condition and the minimum-norm solution	18
2.2.2 The existence condition and least-squares solutions	21
2.3 Ill-posed problems, part 2	27
2.3.1 Stability and the generalized inverse	27
2.3.2 Inverse problems	32
2.3.3 History of the concept of inverse problems	33
2.3.4 An example: medical imaging and computed tomography	34
2.4 Discretized inverse problems	37
2.4.1 Ill-conditioned matrix-vector equations	37
2.4.2 Discretization of inverse problems	42
3 Tikhonov regularization	49
3.1 Existence of the Tikhonov solution	52
3.2 Convergence of Tikhonov regularization	60

3.3	Convergence for noisy data	70
3.4	Rates of convergence	73
3.4.1	Examples	79
3.5	Parameter choice rules	84
3.5.1	The discrepancy principle	87
3.5.2	The L-curve method	95
3.6	Converse results	98
3.6.1	Preliminary results	100
3.6.2	The convergence of Tikhonov regularization can be arbitrarily slow	102
3.6.3	Tikhonov regularization can fail to converge if the regularization parameter is not chosen properly	103
3.6.4	Converse results about the rate of convergence . .	114
3.6.5	The rate of convergence in the discrepancy principle	118
3.6.6	Summary	121
4	Compact operators and the singular value expansion	123
4.1	Finite-dimensional problems and the singular value decomposition	123
4.1.1	An example with a symmetric matrix	124
4.1.2	The SVD of a matrix	132
4.1.3	Using the SVD to solve a discrete inverse problem	134
4.2	Compact operators	137
4.3	The spectral theorem for a compact self-adjoint operator	143
4.3.1	Orthogonal sets in Hilbert space	145
4.3.2	Examples of complete orthonormal sets	151
4.3.3	The spectral theorem	156
4.4	The singular value expansion of a compact operator . . .	163
4.4.1	Integral operators of the first kind	170
4.4.2	Integral operators of the second kind	180
4.5	The generalized inverse in terms of the SVE	182
4.5.1	Interpretation of the series representation of T^\dagger .	188
4.6	Tikhonov regularization in terms of the SVE	197

4.7	Convergence of Tikhonov regularization via the SVE . . .	202
4.7.1	Rates of convergence	206
4.7.2	Another converse result	208
4.8	General regularization methods	210
4.8.1	Functions of T^*T	210
4.8.2	An abstract regularization method	213
4.8.3	Rates of convergence	215
4.8.4	Asymptotic regularization	219
5	Tikhonov regularization with seminorms	225
5.1	Densely defined operators	226
5.1.1	The adjoint of a closed densely defined operator .	229
5.1.2	The graph norm and L^*L	231
5.1.3	An example: the derivative operator	236
5.2	Best approximate solutions using seminorms	239
5.3	Tikhonov regularization with seminorms	254
5.4	Convergence for exact data	257
5.5	Rates of convergence	264
	Epilogue	271
A	Basic Hilbert space theory	279
A.1	Hilbert spaces and linear operators	279
A.2	Subspaces of Hilbert space	284
A.3	Linear functionals	288
A.4	The adjoint of a linear operator	290
A.5	The open mapping theorem and the uniform boundedness principle	294
A.6	Weak sequential convergence in Hilbert space	301
B	Sobolev spaces	307
B.1	Sobolev spaces on an interval	307
B.2	Sobolev spaces in higher dimensions	313
	Bibliography	317
	Index	319
	About the Author	321