Hints to Chapter Problems

7.34 Reduce the problem to minimizing \( \frac{1}{\tan \frac{A}{2}} + \frac{1}{\tan \frac{B}{2}} + \frac{1}{\tan \frac{C}{2}} \), where \( A, B, \) and \( C \) are the measures of the angles in a triangle.

**Coordination**

8.1 Choose a convenient coordinate system and calculate the coordinates of the midpoint of each diagonal.

8.2 Introduce a coordinate system as you would for a parallelogram.

8.3 Place \( C \) at the origin and let \( A \) and \( B \) lie on the coordinate axes.

8.4 If \( P, Q, R, \) and \( S \) are the midpoints of the original quadrilateral, prove that the midpoints of the diagonals of \( PQRS \) coincide.

8.5 (a) Find the coordinates of the midpoint of \( AB \) and use the distance formula.

(b) Use the formula for the distance from a point to a line.

8.6 Follow the method of Example 30.

8.7 Place \( A \) at the origin and give \( B \) coordinates \((2x, 2y)\). Calculate midpoints and use the distance formula.

8.8 Let \( A : (-a, 0) \) and \( C : (a, 0) \). Find the equations of the perpendicular bisectors of \( AB \) and \( BC \).

8.9 Choose \( A : (0, 0) \) and \( D : (5, 0) \). Use the equations for lines to find the coordinates of the relevant interior point.

8.10 Introduce a coordinate system, not necessarily Cartesian, such that \( A \) and \( D \) lie on the \( x \)-axis, and the \( y \)-axis passes through the midpoints of the bases.

8.11 Choose \( A : (-a, 0) \), \( B : (-b, 1) \), \( C : (b, 1) \), and \( D : (a, 0) \).

8.12 Position \( A \) at the origin and \( D \) on the \( x \)-axis.

8.13 Choose \( A : (-1, 0) \), and \( C : (1, 0) \). Let \( P : (s, t) \) be a point inside or on the boundary of the triangle.

8.14 First determine the area of the rectangle that circumscribes the triangle and whose sides are parallel to the coordinate axes. Another approach is to use the fact that \( \text{Area}(\triangle ABC) = \frac{1}{2}ab \sin C = \frac{1}{4}ab \sqrt{1 - \cos^2 C} \), the Cosine theorem, and the distance formula.

8.15 A point \((s_x, y_0)\) lies on or in the exterior of a circle \( x^2 + y^2 = R^2 \) if and only if \( s_x^2 + y_0^2 \geq R^2 \).

8.16 Introduce a (not necessarily Cartesian) coordinate system \( OXY \) such that \( E : (0, 0) \), \( A : (a, 0) \), \( B : (0, 1) \), \( C : (c, 0) \), \( a < 0 < c \). Then follow the approach used in the second proof of Theorem 8.8.

**Conics**

9.1 Follow the method of Example 39 or of Example 41.

9.2 Consider different combinations of signs of \( A, C, \) and \( H \). Consider the case when \( H = 0 \).

9.3 Complete the square.

9.4 Use the translation and rotation formulae.

9.5 Introduce a coordinate system and use the equation of a parabola.