Talkative Eve

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This article first appeared in the April 1996 issue of Math Horizons.

Talkative Eve

This cryptarithm (or alphametic, as some puzzlers prefer to call them) is an old one of unknown origin, surely one of the best and, I hope, unfamiliar to most readers:

$$\frac{\text{EVE}}{\text{DID}} = .\text{TALKTALKTALK}\ldots$$

The same letters stand for the same digits, zero included. The fraction EVE/DID has been reduced to its lowest terms. Its decimal form has a repeating period of four digits. The solution is unique. To solve it, recall that the standard way to obtain the simplest fraction equivalent to a decimal of n repeating digits is to put the repeating period over n 9's and reduce the fraction to its lowest terms.

Three Squares

Using only elementary geometry (not even trigonometry), prove that angle C in Figure 1 equals the sum of angles A and B.

I am grateful to Lyber Katz for this charmingly simple problem. He writes that as a child he went to school in Moscow, where the problem was given to his fourth-grade geometry class for extra credit to those who solved it. "The number of blind alleys the problem leads to," he adds, "is extraordinary."

Red, White, and Blue Weights

Problems involving weights and balance scales have been popular during the past few decades. Here is an unusual one invented by Paul Curry, who is well known in conjuring circles as an amateur magician.

You have six weights. One pair is red, one pair white, one pair blue. In each pair, one weight is a trifle heavier than the other but otherwise appears to be exactly like its mate. The three heavier weights (one of each color) all weigh the same. This is also true of the three lighter weights.

In two separate weighings on a balance scale, how can you identify which is the heavier weight of each pair?

ANSWERS

Talkative Eve

As stated earlier, to obtain the simplest fraction equal to a decimal of n repeated digits, put the repeating period over n 9's and reduce to its lowest terms. In this instance TALK/9,999, reduced to its lowest terms, must equal EVE/DID. Consequently, DID is a factor of 9,999. Only three such factors fit DID: 101, 303, 909.

If DID = 101, then EVE/101 = TALK/9,999, and EVE = TALK/99. Rearranging terms, TALK = (99)(EVE). EVE cannot be 101 (since we assumed 101 to be DID), and anything larger than 101, when multiplied by 99, has a five-digit product. And so DID = 101 is ruled out.

If DID = 909, then EVE/909 = TALK/9,999, and EVE = TALK/11. Rearranging terms, TALK = (11)(EVE). In that case, the last digit of TALK would have to be E. Since it is not E, 909 is also ruled out. Only 303 remains as a possibility for DID. Because EVE must be smaller than 303, E is 1 or 2. Of the 14 possibilities (121, 141, … , 292) only 242 produces a decimal fitting .TALKTALK…, in which all the digits differ from those in EVE and DID.

The unique answer is 242/303 = .798679867986… If EVE/DID is not assumed to be in lowest terms, there is one other solution, 212/606 = .349834983498…, proving as Joseph Madachy has remarked, that EVE double-talked.

Three Squares

There are many ways to prove that angle C in the figure is the sum of angles A and B. Here is one (see Figure 2). Construct the squares indicated by red lines. Angle B equals angle D because they are corresponding angles of similar right triangles. Since angles A and D add to angle C, B can be substituted for D, and it follows immediately that C is the sum of A and B.

This little problem produced a flood of letters from readers who sent dozens of other proofs. Scores of correspondents avoided construction lines by making the diagonals equal to the square roots of 2, 5, and 10, then using ratios to find two similar triangles from which the desired proof would follow. Others generalized the problem in unusual ways.

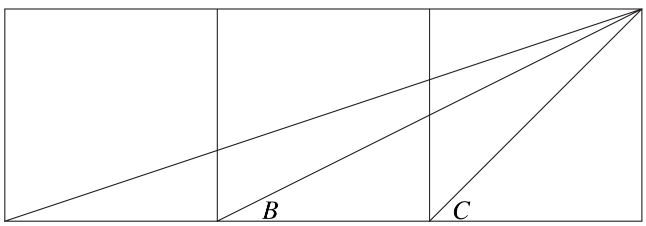

Figure 1. Prove that angle A plus angle B equals angle C.

Charles Trigg published 54 different proofs in the *Journal of Recreational Mathematics* Vol. 4, April 1971, pages 90–99. A proof using paper cutting, by Ali R. Amir-Moéz, appeared in the same journal, Vol. 5, Winter 1973, pages 8–9. For other proofs, see Roger North's contribution to *The Mathematical Gazette*, December 1973, pages 334–36, and its continuation in the same journal, October 1974, pages 212–15. For a generalization of the problem to a row of n squares, see Trigg's "Geometrical Proof of a Result of Lehmer's," in *The Fibonacci Quarterly*, Vol. 11, December 1973, pages 539-40.

Red, White, and Blue Weights

One way to solve the problem of six weights—two red, two white, and two blue—is first to balance a red and a white weight against a blue and a white weight.

If the scales balance, you know there are a heavy and a light weight on each pan. Remove both colored weights, leaving the white weights, one on each side. This establishes which white weight is the heavier. At the same time it tells you which of the other two weights used before (one red, one blue) is heavy and which is light. This in turn tells you which is heavy and which is light in the red-blue pair not yet used.

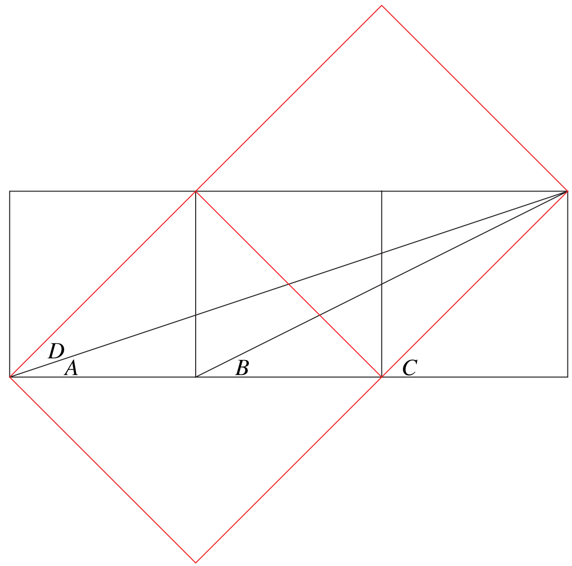

Figure 2. Construction for proof of the three-square theorem.

If the scales do not balance on the first weighing, you know that the white weight on the side that went down must be the heavier of the two whites, but you are still in the dark about the red and the blue. Weigh the original red against the mate of the original blue (or the original blue against the mate of the original red). As C. B. Chandler (who sent this simple solution) put it, the result of the second weighing, plus the memory of which side was heavier in the first weighing, is now sufficient to identify the six weights.

For readers who liked working on this problem, Ben Braude, a New York City dentist and amateur magician, devised the following variation. The six weights are alike in all respects (including color) except that three are heavy and three light. The heavy weights weigh the same and the light weights weigh the same. Identify each in three separate weighings on a balance scale.

DOI: 10.4169/194762110X524684