

$$\frac{1}{z^n - 1} = \frac{1}{n} \sum_{k=1}^n \frac{\zeta_k}{z - \zeta_k}.$$

Let $z = \exp(i\theta)$. For $k = 1, 2, \dots, n$, we have

$$\frac{\sin \theta}{\cos(2\pi k/n) - \cos \theta} = \frac{\frac{z-z^{-1}}{2i}}{c_k - \frac{z+z^{-1}}{2}} = i \cdot \frac{z^2 - 1}{z^2 - 2c_k z + 1} = i \left(1 + \frac{\zeta_k}{z - \zeta_k} + \frac{\bar{\zeta}_k}{z - \bar{\zeta}_k} \right).$$

Since $\bar{\zeta}_n = 1 = \zeta_n$ and $\bar{\zeta}_k = \zeta_{n-k}$ for $1 \leq k < n$,

$$\begin{aligned} \sum_{k=1}^n \frac{\sin \theta}{\cos(2\pi k/n) - \cos \theta} &= ni + 2i \sum_{k=1}^n \frac{\zeta_k}{z - \zeta_k} = ni \left(1 + \frac{2}{z^n - 1} \right) \\ &= ni \cdot \frac{z^{n/2} + z^{-n/2}}{z^{n/2} - z^{-n/2}} = n \cot \frac{n\theta}{2}. \end{aligned}$$

Either side of the identity is undefined precisely when $\theta = 2\pi k/n$ for an arbitrary integer k .

PINEMI PUZZLE

	5		5			7			
8		9		9	8		9		5
		10			7	6		8	
	10		9			7	8		
3							8		8
6	9			10		10		7	
		10	11		9			8	5
6	10			10			8		
	8		9			10		10	
4		6			6		8		4

How to play. Place one jamb (|), two jambs (||), or three jambs (|||) in each empty cell. The numbers indicate how many jambs there are in the surrounding cells—including diagonally adjacent cells. Each row and each column has 10 jambs. Note that no jambs can be placed in any cell that contains a number.

The solution is on page 181.

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