

Sample *Mathematica* code for the paper "Modeling a Diving Board"

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■ Beam Data

■ Original Data

Remove headings from original data collected with World in Motion software.

NOTE: At $t = 1$ second, World in Motion recorded values of $(-2000, -2000)$ for the (x, y) data point at the 13th tape mark, at approximately $x = 1.46$ m. Looking at the nearby data, these clearly cannot be correct, so we interpolated the values from the corresponding points for $x = 1.46$ m at $t = 0.967$ sec, which was recorded as $(1.472478, 0)$, and at $t = 1.033$ sec, which was recorded as $(1.481429, 0.0582)$. These corrections are noted below in red.

```
In[1]:= Beam = Drop[{{"t (s)", "x1 (m)", "y1 (m)", "x2 (m)", "y2 (m)", "x3 (m)", "y3 (m)",
    "x4 (m)", "y4 (m)", "x5 (m)", "y5 (m)", "x6 (m)", "y6 (m)", "x7 (m)", "y7 (m)",
    "x8 (m)", "y8 (m)", "x9 (m)", "y9 (m)", "x10 (m)", "y10 (m)", "x11 (m)", "y11 (m)",
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```

```

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```

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```

Remove the first entry of each row, which corresponds to the time t at which the data was collected, for $t \geq 0$ seconds.

```
In[2]:= Beam = Drop[Beam, {}, {1}];
```

Define the positions for the tape pieces along the beam. Check the length of this list - there should be fifteen points.

```
In[3]:= xtape = Table[Beam[[1]][[i]], {i, 1, Length[Beam[[1]]], 2}
```

```
Out[3]= {0, 0.107415, 0.219305, 0.340147, 0.447562, 0.554977, 0.671343,
0.783233, 0.899599, 1.01597, 1.12338, 1.2308, 1.34716, 1.45905, 1.57094}
```

```
In[4]:= Length[xtape]
```

```
Out[4]= 15
```

■ Steady-State Solution

Solve the steady-state problem.

```
In[5]:= Clear[L, v, c, g]
```

```
In[6]:= DSolve[{-c^2 v''''[x] == g, v[0] == 0, v'[0] == 0, v''[L] == 0, v'''[L] == 0}, v[x], x]
```

```
Out[6]= {{V[x] ->  $\frac{-6 g L^2 x^2 + 4 g L x^3 - g x^4}{24 c^2}$ }}
```

```
In[7]:= v[x_] :=  $\frac{-6 g L^2 x^2 + 4 g L x^3 - g x^4}{24 c^2}$ 
```

```
In[8]:= v[x]
```

```
Out[8]=  $\frac{-6 g L^2 x^2 + 4 g L x^3 - g x^4}{24 c^2}$ 
```

■ Initial Conditions

Define initial conditions and model parameters. Length of beam is L meters. Parameters c , k , and g will be chosen dynamically below. The function $f(x)$ is initial displacement of beam. Initial velocity $g(x)$ is assumed to be zero. Define $h(x) = f(x) - v(x)$ to be the extra amount by which the beam is bent initially.

```
In[9]:= L = 1.6;
```

```
In[10]:= Fit[Partition[Beam[[1]], 2], {x, x^2}, x]
```

```
Out[10]= -0.0341084 x - 0.080823 x^2
```

```
In[11]:= f[x_] := a x - b x^2
```

```
In[12]:= a = -0.0341084;
```

```
          b = 0.0808230;
```

```
In[14]:= f[x]
```

```
Out[14]= -0.0341084 x - 0.080823 x^2
```

```
In[15]:= h[x_] := f[x] - v[x]
```

```
In[16]:= h[x]
```

```
Out[16]= -0.0341084 x - 0.080823 x^2 -  $\frac{-15.36 g x^2 + 6.4 g x^3 - g x^4}{24 c^2}$ 
```

■ Construct Model

■ Solve for the alpha values

```
In[17]:= Clear[alpha, α]
```

$$\alpha = \text{Table}\left[\alpha /. \text{FindRoot}\left[\text{Cos}[\alpha * L] + \frac{1}{\text{Cosh}[\alpha * L]}, \left\{\alpha, \frac{(2 n + 1) \pi}{2 L}\right\}\right], \{n, 0, 7\}\right];$$

```
α = alpha;
```

■ Define position functions $X_n(x, t)$

```
In[20]:= X[x_, n_] := Cos[α[[n]] * x] - Cosh[α[[n]] * x] -
```

$$\frac{(\text{Cosh}[\alpha[[n]] * L] + \text{Cos}[\alpha[[n]] * L])}{(\text{Sinh}[\alpha[[n]] * L] + \text{Sin}[\alpha[[n]] * L])} (\text{Sin}[\alpha[[n]] * x] - \text{Sinh}[\alpha[[n]] * x])$$

- Define the A_n Coefficients - for the Manipulate command to work, the coefficients A_n have been found explicitly, which can be done with our choice of $f(x)$!

```
In[21]:= Clear[A, AN];
A[n_] := ((Sin[L α[[n]]] + Sinh[L α[[n]]) (-8 g Cos[L α[[n]]] + 8 c^2 L (a - b L) α[[n]]^4 + g (8 + L^4 α[[n]]^4) +
Cosh[L α[[n]]] (-8 g + Cos[L α[[n]]] (8 c^2 L (a - b L) α[[n]]^4 + g (8 + L^4 α[[n]]^4))) -
8 c^2 α[[n]]^2 (a α[[n]] Sin[L α[[n]]] + (a α[[n]] - 2 b Sin[L α[[n]]) Sinh[L α[[n]]])) /
(2 c^2 α[[n]]^4 (-L Cos[2 L α[[n]]] α[[n]] + L Cosh[2 L α[[n]]] α[[n]] - 6 Cosh[L α[[n]]] Sin[L α[[n]]] -
3 Cosh[L α[[n]]]^2 Sin[2 L α[[n]]] + (6 Cos[L α[[n]]] + 4 L α[[n]] Sin[L α[[n]]) Sinh[L α[[n]]] +
3 Cos[L α[[n]]]^2 Sinh[2 L α[[n]]]))
AN = Table[A[n], {n, 1, 7}];
```

- Define the B_n Coefficients

```
In[24]:= Clear[μ, B, BN];
μ[n_] := 1/2 Sqrt[4 c^2 α[[n]]^4 - k^2];
B[n_] := k AN[[n]] / (2 μ[n]);
BN = Table[B[n], {n, 1, 7}];
```

- Define time functions $T_n(x, t)$

```
In[28]:= T[t_, n_] := (AN[[n]] Cos[μ[n] t] + BN[[n]] Sin[μ[n] t]) e^(-1/2 k t);
```

- Define model $y(x, t) = v(x) + \sum X_n(x, t) T_n(x, t)$

```
In[29]:= Y[x_, t_] := v[x] + Sum[X[x, n] T[t, n], {n, 1, 7}]
```

Check to make sure our model is a function of c , k , and g :

```
In[30]:= Y[x, t]
```

```
Out[30]= (-15.36 g x^2 + 6.4 g x^3 - g x^4) / (24 c^2) +
e^(-k t / 2) ( (1/c^2) 0.01708 (3.26694 c^2 + 22.7594 g + 3.33742 (-8 g - 0.299633 (-3.94594 c^2 + 20.3624 g)))
Cos[1/2 Sqrt[7.54539 c^2 - k^2] t] + 1 / (c^2 Sqrt[7.54539 c^2 - k^2])
0.01708 (3.26694 c^2 + 22.7594 g + 3.33742 (-8 g - 0.299633 (-3.94594 c^2 + 20.3624 g)))
k Sin[1/2 Sqrt[7.54539 c^2 - k^2] t] )
(Cos[1.17194 x] - Cosh[1.17194 x] - 0.734096 (Sin[1.17194 x] - Sinh[1.17194 x])) + e^(-k t / 2)
( (1/c^2) 0.0000134008 (-393.464 c^2 + 493.665 g + 54.6543 (-8 g - 0.0182968 (-154.973 c^2 + 493.519 g)))
```

$$\begin{aligned}
& \cos\left[\frac{1}{2}\sqrt{296.337c^2 - k^2}t\right] + \frac{1}{c^2\sqrt{296.337c^2 - k^2}} \\
& 0.0000134008\left(-393.464c^2 + 493.665g + 54.6543\left(-8g - 0.0182968\left(-154.973c^2 + 493.519g\right)\right)\right) \\
& \left. k \sin\left[\frac{1}{2}\sqrt{296.337c^2 - k^2}t\right]\right) \\
& (\cos[2.93381x] - \cosh[2.93381x] - 1.01847(\sin[2.93381x] - \sinh[2.93381x])) + e^{-\frac{kt}{2}} \\
& \left(\frac{1}{c^2}4.24787 \times 10^{-8}\left(80603.5c^2 + 3814.55g + 1288.99\left(-8g - 0.000775804\left(-1215.01c^2 + 3814.55g\right)\right)\right)\right) \\
& \cos\left[\frac{1}{2}\sqrt{2323.33c^2 - k^2}t\right] + \frac{1}{c^2\sqrt{2323.33c^2 - k^2}} \\
& 4.24787 \times 10^{-8}\left(80603.5c^2 + 3814.55g + 1288.99\left(-8g - 0.000775804\left(-1215.01c^2 + 3814.55g\right)\right)\right) \\
& \left. k \sin\left[\frac{1}{2}\sqrt{2323.33c^2 - k^2}t\right]\right) \\
& (\cos[4.90922x] - \cosh[4.90922x] - 0.999224(\sin[4.90922x] - \sinh[4.90922x])) + \\
& e^{-\frac{kt}{2}}\left(\frac{1}{c^2}3.42042 \times 10^{-10}\right. \\
& \left.(814491.c^2 + 14625.3g + 29803.9\left(-8g - 0.0000335527\left(-4665.69c^2 + 14625.3g\right)\right)\right) \\
& \cos\left[\frac{1}{2}\sqrt{8921.68c^2 - k^2}t\right] + \frac{1}{c^2\sqrt{8921.68c^2 - k^2}}3.42042 \times 10^{-10}\left(814491.c^2 + 14625.3g + \right. \\
& \left. 29803.9\left(-8g - 0.0000335527\left(-4665.69c^2 + 14625.3g\right)\right)\right) k \sin\left[\frac{1}{2}\sqrt{8921.68c^2 - k^2}t\right] \\
& (\cos[6.87221x] - \cosh[6.87221x] - 1.00003(\sin[6.87221x] - \sinh[6.87221x])) + \\
& e^{-\frac{kt}{2}}\left(\frac{1}{c^2}4.20671 \times 10^{-12}\right. \\
& \left.(1.99439 \times 10^8c^2 + 39951.8g + 689706.\left(-8g - 1.44989 \times 10^{-6}\left(-12749.7c^2 + 39951.8g\right)\right)\right) \\
& \cos\left[\frac{1}{2}\sqrt{24379.8c^2 - k^2}t\right] + \frac{1}{c^2\sqrt{24379.8c^2 - k^2}}4.20671 \times 10^{-12}\left(1.99439 \times 10^8c^2 + 39951.8g + \right. \\
& \left. 689706.\left(-8g - 1.44989 \times 10^{-6}\left(-12749.7c^2 + 39951.8g\right)\right)\right) k \sin\left[\frac{1}{2}\sqrt{24379.8c^2 - k^2}t\right] \\
& (\cos[8.83573x] - \cosh[8.83573x] - 0.999999(\sin[8.83573x] - \sinh[8.83573x])) + \\
& e^{-\frac{kt}{2}}\left(\frac{1}{c^2}6.66526 \times 10^{-14}\right. \\
& \left.(3.07785 \times 10^9c^2 + 89143.4g + 1.59603 \times 10^7\left(-8g - 6.26556 \times 10^{-8}\left(-28451.1c^2 + 89143.4g\right)\right)\right)
\end{aligned}$$

$$\begin{aligned}
& \cos\left[\frac{1}{2}\sqrt{54403.9c^2 - k^2}t\right] + \frac{1}{c^2\sqrt{54403.9c^2 - k^2}}6.66526 \times 10^{-14} \left(3.07785 \times 10^9 c^2 + 89143.4g + \right. \\
& \left. 1.59603 \times 10^7 (-8g - 6.26556 \times 10^{-8} (-28451.1c^2 + 89143.4g))\right) k \sin\left[\frac{1}{2}\sqrt{54403.9c^2 - k^2}t\right] \\
& (\cos[10.7992x] - \cosh[10.7992x] - 1. (\sin[10.7992x] - \sinh[10.7992x])) + \\
& e^{-\frac{kt}{2}} \left(\frac{1}{c^2} 1.24936 \times 10^{-15} (2.87302 \times 10^{11} c^2 + 173889. g + \right. \\
& \left. 3.69331 \times 10^8 (-8g - 2.7076 \times 10^{-9} (-55501.1c^2 + 173889. g))) \cos\left[\frac{1}{2}\sqrt{106129. c^2 - k^2}t\right] + \right. \\
& \left. \frac{1}{c^2\sqrt{106129. c^2 - k^2}} 1.24936 \times 10^{-15} (2.87302 \times 10^{11} c^2 + 173889. g + \right. \\
& \left. 3.69331 \times 10^8 (-8g - 2.7076 \times 10^{-9} (-55501.1c^2 + 173889. g))) k \sin\left[\frac{1}{2}\sqrt{106129. c^2 - k^2}t\right] \right) \\
& (\cos[12.7627x] - \cosh[12.7627x] - 1. (\sin[12.7627x] - \sinh[12.7627x]))
\end{aligned}$$

■ **Finding c, k, and g via the Manipulate command - Figures 3, 4, 5, and 6 in the paper are found this way!**

Since beam isn't moving until the fourth set of data points, drop the first four sets of data points.

```
In[31]:= Beam = Drop[Beam, {1, 4}];
```

```
In[32]:= data = Table[Partition[Beam[[i]], 2], {i, 1, Length[Beam]}];
```

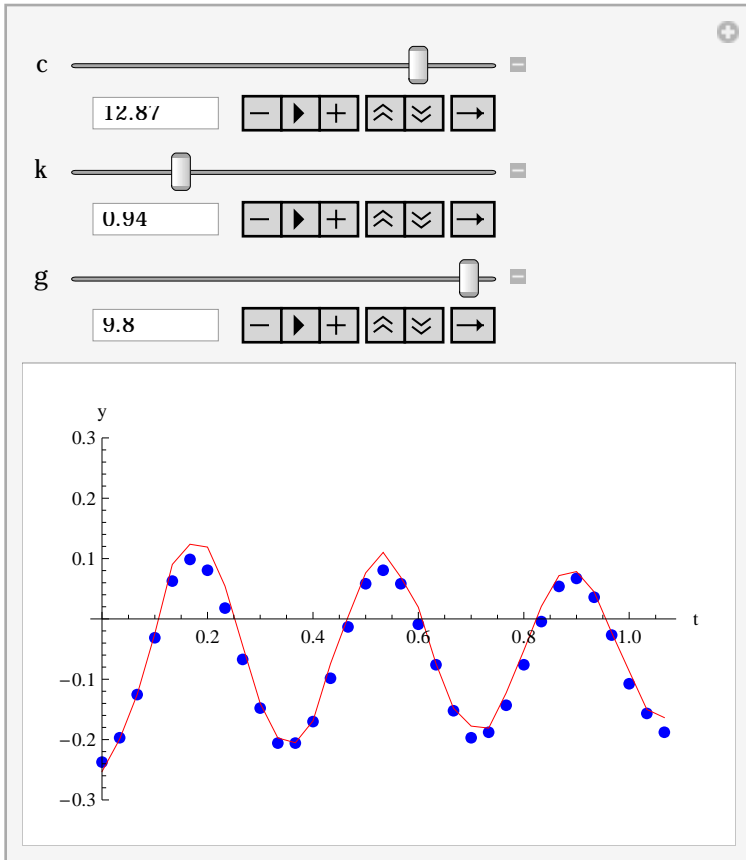
```
In[33]:= Clear[c, k, g]
```



```

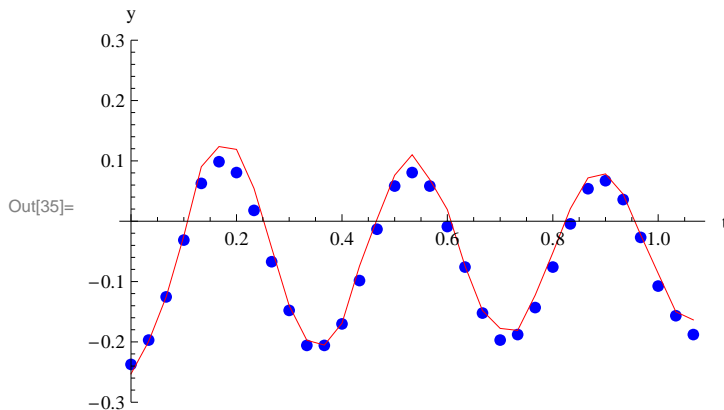
In[34]:= Manipulate[Show[ListPlot[Table[{ $\frac{j-1}{30}$ , data[[j, 15, 2]]}, {j, 1, Length[Beam]}], PlotStyle →
  {PointSize[0.02], RGBColor[0, 0, 1]}, PlotRange → {-0.3, 0.3}, AxesLabel → {"t", "y"}],
  ListPlot[Table[{ $\frac{j-1}{30}$ , Evaluate[y[data[[j, 15, 1]],  $\frac{j-1}{30}$ ]]}, {j, 1, Length[Beam]}] /. {c →  $\chi_1$ ,
  k →  $\chi_2$ , g →  $\chi_3$ }, PlotStyle → RGBColor[1, 0, 0], PlotRange → {-0.3, 0.3}, Joined → True]],
  {{ $\chi_1$ , 1, "c"}, 1, 15}, {{ $\chi_2$ , 0, "k"}, 0, 4}, {{ $\chi_3$ , 0, "g"}, 0, 10},
  TrackedSymbols → { $\chi_1$ ,  $\chi_2$ ,  $\chi_3$ }, SaveDefinitions → True]

```



■ Here's how to plot Figure 6 without using the Manipulate command

```
In[35]:= Show[ListPlot[Table[{ $\frac{j-1}{30}$ , data[[j, 15, 2]]}, {j, 1, Length[Beam]}],
  PlotStyle -> {PointSize[0.02], RGBColor[0, 0, 1]},
  PlotRange -> {-0.3, 0.3}, AxesLabel -> {"t", "y"}],
ListPlot[Table[{ $\frac{j-1}{30}$ , Evaluate[y[data[[j, 15, 1]],  $\frac{j-1}{30}$ ]]}, {j, 1, Length[Beam]}] /.
  {c -> 12.87, k -> 0.94, g -> 9.8},
  PlotStyle -> RGBColor[1, 0, 0], PlotRange -> {-0.3, 0.3}, Joined -> True]]
```



■ Find error with a choice of c, k, and g via a Mean Sum of the Squares for Error Calculation

Find the error with the choices of c, k, and g (entered as c1, k1, and g1 so that the Manipulate command will work correctly) that give a reasonable fit graphically. The function $\sqrt{H(c, k, g)}$ computes the mean of the sum of the squares for error (MSSE) over all data points.

```
In[36]:= H[c_, k_, g_] :=
  Evaluate[Sum[(data[[j, i, 2]] - y[data[[j, i, 1]],  $\frac{j-1}{30}$ ])^2, {i, 1, 15}, {j, 1, Length[Beam]}] /
  (Length[Beam] * 15)]
```

```
In[37]:=  $\sqrt{H[12.87, 0.94, 9.8]}$ 
```

Out[37]= 0.0103875

■ Find values of c, k, and g that minimize MSSE via the FindMinimum command, starting with c = c1, k = k1, and fixing g = 9.8 m/sec². Then compute error with the resulting c and k!

```
In[38]:= c1 = 12.87;
k1 = 0.94;
```

```
In[40]:= FindMinimum[H[c, k, 9.8], {c, c1}, {k, k1}]
```

Out[40]= {0.000105073, {c -> 12.842, k -> 1.00698}}

```
In[41]:=  $\sqrt{H[12.84, 1.01, 9.8]}$ 
```

```
Out[41]:= 0.0102511
```

- **Plot model vs. measured data at all points with our new choices for c and k with g fixed at 9.8 m/sec^2 - this is Figure 7 in the paper:**

```
In[42]:= g1a = Show[Table[Graphics3D[{RGBColor[0, 0, 1], PointSize[0.011],
    Point[Table[{xtape[[j],  $\frac{i-1}{30}$ , Beam[All, 2 j][[i]]}, {i, 1, Length[Beam[All, 1]]}]]],
    {j, 1, 15}], Axes -> {True, True, True}, AxesLabel -> {"x", "t", "y"},
    BoxRatios -> {3, 2, 1}, ImageSize -> 600];
```

```
In[43]:= g2a = Show[
    Table[ParametricPlot3D[{xtape[[i],  $\frac{j}{30}$ , y[xtape[[i],  $\frac{j}{30}$ ]} /. {c -> 12.84, k -> 1.01, g -> 9.8}],
    {j, 0, 32}, Axes -> {True, True, True}, AxesLabel -> {"x", "t", "y"},
    Ticks -> {xtape, Automatic, Automatic}, BoxRatios -> {3, 2, 1},
    PlotRange -> {{0, L}, {0, Length[Beam] / 30}, {-0.25, 0.25}},
    ImageSize -> 600, PlotStyle -> {Thick, Red}], {i, 1, 15}];
```

```
In[44]:= Show[g2a, gla, Boxed -> False, ViewPoint -> {0.8, -1.5, 1.5}]
```

