FOCUS is published by the Mathematical Association of America in January, February, March, April, May/June, August/September, October, November, and December.

Letters to the editor should be addressed to Fernando Gouvêa, Colby College, Dept. of Mathematics, Waterville, ME 04901, or by email to fqgouvea@colby.edu.

Subscription and membership questions should be directed to the MAA Customer Service Center, 800-331-1622; e-mail: maahq@maa.org; (301) 617-7800 (outside U.S. and Canada); fax: (301) 206-9789.

Periodicals postage paid at Washington, DC and additional mailing offices. **Postmaster:** Send address changes to FOCUS, Mathematical Association of America, P.O. Box 90973, Washington, DC 20090-0973.

ISSN: 0731-2040; Printed in the United States of America.

FOCUS

November 2002

Volume 22, Number 8

On the cover: Nitin Saxena, Neeraj Kayal, and Manindra Agrawal of the Indian Institute of Technology in Kampur, who discovered a primality test that runs in polynomial time. See Carl Pomerance’s article on page 4.

FOCUS Deadlines

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Display Ads	October 29	November 26	December 20
Employment Ads	October 15	November 12	December 10

Judith Grabiner, Ranjan Roy, and Paul Zeitz Win Haimo Awards

The winners of this year's Deborah and Franklin Tepper Haimo Awards for Distinguished College and University Teaching of Mathematics are Judith Grabiner of Pitzer College, Ranjan Roy of Beloit College, and Paul Zeitz of the University of San Francisco. All three are honored as teachers who have been extraordinarily successful, both in their home institutions and also in a wider setting. As happens every year, the three winners will give special talks at the January Joint Meetings, where they will also receive the actual award. The three talks are scheduled for Friday, January 17, from 2:30 to 4:00.

Judith Grabiner

Judith Grabiner has an international reputation as a historian of mathematics, but she also excels as a teacher and lecturer. She is famous for giving talks that are knowledgeable, witty, beautifully organized, and that hold the attention of a wide range of audiences, from professionals to undergraduates. At Pitzer College, she has taught a wide range of courses, including two courses for non-mathematicians that have been particularly successful. She is also an exceptional expositor, as is witnessed by her two Lester R. Ford awards and her three Carl Allendoerfer awards, a record that few can match.

Grabiner's talk at the Joint Meetings is entitled *"You Can't Understand That Without Mathematics!" and You CAN Understand Mathematics*. She says "I regularly teach two liberal-arts courses for students who see themselves as non-mathematics students. One course, *Mathematics, Philosophy, and the 'Real World*, involves Euclidean geometry (we read Euclid himself), elementary probability and statistics, and problems arising from the individual students' own lives and interests. We also read a lot of philosophers whose work has been influenced by this mathematics. The other course, *Mathematics in Many Cultures*, focuses on the mathematics of non-literate peoples today and on non-western civilizations, including China, India, Rabbinic Judaism, and the Islamic world. Among the mathematical topics are combinatorics, graph theory, elementary group theory, and the relation between algebra and geometry. This course is linked with my institution's multicultural curriculum, and demonstrates by examples that every culture has mathematics, and that a culture's mathematics solves problems that the culture considers important." Grabiner's talk will discuss both courses.

Ranjan Roy

Ranjan Roy teaches mathematics as a body of deep and beautiful ideas, as a way of thinking that can improve the lives of all who study it. His interest in the history of mathematics and his systematic study of the original sources enrich his teaching, which displays an uncanny ability to connect mathematics to students' lives. Roy's student evaluations show how deeply he affects students. One had "Ranjan is God!" written in large letters across the standard form. Students say "Ranjan is the kind of teacher who changes your life," and this fact is recognized by Beloit College, which has named him "Teacher of the Year" twice. Roy is also an exceptional expositor; one of his articles for *Mathematics Magazine* won the Allendoerfer Award in 1991.

Roy's talk at the January meetings is entitled *Using the History of Mathematics in Teaching*. Roy says "I have found it helpful to introduce a little history of mathematics into mathematics courses at the undergraduate level in order to give students a perspective on mathematical ideas. As they learn about the development of mathematics and about the personalities of those involved, students may gain motivation and interest to pursue the subject further or to work harder in class. In my talk I will give a few examples which might be useful in undergraduate teaching."

Paul Zeitz

Paul Zeitz is a charismatic teacher whose passion for problem-solving permeates his teaching. He has been participating in mathematical competitions ever since he was captain of the Math Team at Stuyvestant High School. In 1974, he took first place in the USAMO and earned the right to be a member of the first American team to compete in the IMO. He remains involved in competitions, especially in the Bay Area, and this interest appears in his teaching too. Zeitz taught high school for six years before getting his Ph.D., and he still teaches younger students, coaching students for mathematics competitions and teaching Sunday afternoon classes at the Berkeley Math Circle. His humor and intensity keep class participation high, and he is successful in reaching students of all ability levels.

Zeitz's talk in Baltimore is entitled *The Chainsaw and the Giraffe*. Zeitz says "the talk will (probably!) be about how sometimes doing the 'wrong' or 'bad' thing in mathematics is a very good idea."

Grabiner, Roy, and Zeitz join a distinguished list of past Haimo winners that includes many of the most prominent American teachers of collegiate mathematics. The Deborah and Franklin Tepper Haimo Awards were created in 1991 to honor "college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions."

The Haimo Award winners are chosen from among the winners of the Teaching Awards given by each of the Sections of the MAA. More information, including a list of all past winners, can be found on MAA Online (click on "Awards" or go to http://www.maa.org/awards/haimo.html).

A New Primal Screen

By Carl Pomerance

Quick, is 91 prime or composite? OK, you have that one, but how about 12345678910111213? As with 91, the answer is still relatively easy to find, since this 17-digit number is divisible by the smallish prime 113, and so it is composite. But searching for factors often gets bogged down, and when a large number really is prime, a factor search is a *terrible* way to proceed.

In early August, Manindra Agrawal, Neeraj Kayal, and Nitin Saxena, three researchers from the Indian Institute of Technology in Kanpur, posted an algorithm that decides if a given number is prime or composite (see http://www.cse.iitk.ac.in/primality.pdf). Further, the number of elementary steps for this decision is bounded approximately by the 12th power of the number of digits of the number being tested. The new test is thus said to run in *polynomial time*, since the number of steps is bounded by a polynomial function of the length of the input data. It is the first deterministic polynomial-time primality test that has ever been put forward and rigorously analyzed.

Why should we care? Natural curiosity on such a fundamental question is a strong motivator. So strong, in fact, that none other than C. F. Gauss wrote in 1801: "The problem of distinguishing prime numbers from composite numbers ... is known to be one of the most important and useful in arithmetic. ... Further, the dignity of science itself seems to require that every possible means be explored for the solution of a problem so elegant and so celebrated.}"

In Sara Robinson's September 2002 *SIAM News* article, she quotes a 1956 letter of Gödel to von Neumann: "It would be interesting to know, for example, what the situation is in the case of determining whether a number is a prime number, and ... how strongly in general the number of steps of the exhaustive search can be reduced." And if this isn't good enough for you, there is an important application: Large prime numbers are the building blocks of modern public-key cryptographic systems in wide use, so any help in identifying them is appreciated.

Previously quite a lot has already been discovered about identifying whether a given number is prime. There are very practical *probabilistic* tests that are expected to rapidly identify composite numbers as such. So, if your number is not identified as composite, then it is highly likely that it is prime. Though you haven't proved it to be a prime, such a number might be used as a prime for practical applications; it is a so-called *industrial grade prime*. In addition we have (very difficult) probabilistic algorithms that are expected to find an actual proof of primality whenever the algorithm is given a prime, and to do so in polynomial time. These methods are based on the theory of elliptic curves and also Jacobian varieties of hyperelliptic curves of genus 2, and are due to Goldwasser, Kilian, Adleman and Huang. Impressed? You should be, the mathematics used by these four computer scientists is quite deep indeed.

What is all the more remarkable about the new test from India is that it is not so hard to understand, and most of it is accessible at the senior undergraduate level. In fact, two of the researchers from India, Kayal and Saxena, were undergraduates as recently as last spring! The idea behind the test stems from Fermat's little theorem (if p is prime and a is any integer, then $a^p - a$ is divisible by p) and the binomial theorem. A common mistake for students might be to expand the expression $(x + y)^p$ as $x^p + y^p$. Well, if p is prime and we are working modulo p, this is absolutely correct! All the other binomial coefficients are divisible by the prime p, as p appears as a factor in the numerator, but is not cancelled in the denominator.

$$\binom{p}{j} = \frac{p!}{j!(p-j)!}$$

So, it follows upon putting the two ideas together that the polynomial $(x + a)^p$ is congruent to the polynomial $x^p + a$ modulo p (meaning that corresponding coefficients are all congruent modulo p).

However, although true about primes p, it is not a property that is easily checked, since the expansion of $(x + a)^p$ has $p + 1$ terms. What Agrawal, Kayal and Saxena suggest is to check the identity modulo another polynomial $x^r - 1$, where r is a cleverly chosen prime number that is much smaller than p. It is not overly hard to find the remainder when $(x + a)^p$ is divided by $x^r - 1$ (especially since all the coefficient arithmetic is done modulo p), and the same for $x^p + a$. If you don't get the same remainder, you know that p cannot be prime.

As with many other proposals for primality testing, the question of the converse now appears. Just because

$$(x + a)^p - (x^p + a)$$

is divisible by $x^r - 1$ when working modulo p, does it mean that p must be prime? Well, the trio from Kanpur raise the ante. They propose to check this for every single value of a up to about $(\log p)^2$. They prove, with r chosen properly and with a few other minor things checked, then yes, if all the divisibilities work out, the converse is true and p must be prime.

One aspect of the new primality test is not quite so elementary and it involves the selection of the prime r. One property that it must have is that $r-1$ is divisible by a prime q exceeding $r^{2/3}$. In fact a positive proportion of all primes r have this property, but this was not proved until 1985 by the French analytic number theorist Etienne Fouvry. His proof is very difficult, and is hardly suitable for a treatment at the undergraduate level. There is another unfortunate aspect of using Fouvry's theorem. Its proof ultimately relies on Siegel's theorem in analytic number theory, a beautiful and clever result, but one that is hopelessly *ineffective*. What this means is that short of proving some form of the Riemann Hypothesis, an actual concrete bound for the running time of the new primality test cannot be written down. Above I said that the time bound grows something like the 12th power of the number of digits. What would be nice is to have an actual explicit function, maybe $10m^{12}\log m$, where m is the number of digits of the number being tested, or something similar. But Fouvry's theorem does not allow the writing down of any such explicit function.

What is maddening is that almost surely the running time of the new test is far below the 12th power of the number of digits, more like the 6th power. But proving this seems beyond us right now.

However there is hope for at least some improvement. Hendrik Lenstra recently came up with an *effective* version of the new test with the 12th power running time. That is, Fouvry's theorem is no longer needed. Yes, the algebra may be a tad harder, but the analytic number theory used is *much* easier, almost nonexistent.

A question remains if the new test will actually be useful in proving numbers prime; currently it is not competitive with existing algorithms. Elliptic curve methods of Atkin and Morain have succeeded in finding primality proofs for numbers of no special form and with thousands of digits. Similar successes have been had with tests based on Jacobi sums, stemming from a 1983 paper of Adleman, Rumely and myself. A deterministic version of our test is "almost" polynomial in that the number of elementary steps for a given number p is bounded by an expression of the form $(\log p)^{c\log\log\log p}$. To be truly polynomial time, the $\log\log\log p$ needs to be missing from the exponent. On the other hand, there is a joke that though it has been *proved* that tends to infinity with p, it has never been observed doing so!

With the previous work nearly getting us there, many of us expected primality testing to end up as polynomial time. We just didn't expect the argument to be so relatively easy. One may wonder about other seemingly intractable problems. For example, take the notorious problem of factoring composite numbers. It is notorious because our best methods are painfully slow once the numbers grow beyond 150 digits or so, but nevertheless no one has ever proved that factoring *it must* be hard. Perhaps unbelievably, factoring has not even been proved harder than multiplication! However, most people *believe* factoring to be hard, and this belief is used quite concretely as the measure of security in many cryptographic systems. But who knows? If a bolt from the blue can make primality testing so much easier, perhaps another bolt could do the same for factoring. Thoughts such as this keep us going!

Carl Pomerance is in the Fundamental Mathematics Research Department at Bell Laboratories, Lucent Technologies in Murray Hill, NJ.

Seeking Nominations for MAA Officers

The Nominations Committee members for the 2003 elections are David Stone (Chair), Tom Banchoff, Aparna Higgins, Bob Megginson and Christine Stevens.

In the Spring the entire MAA membership will choose a President Elect and First and Second Vice Presidents. The committee solicits your suggestions for these three offices; please send recommendations (names and any supporting comments) to dstone@gasou.edu. In the Spring, we will present a slate of three persons for each office.

As a reminder, current officers (President-elect Ron Graham, 1st Vice-President Carl Cowen and 2nd Vice-President Joe Gallian) were elected in 2001 from the following slate:

PRESIDENT-ELECT: Ronald Graham, John Kenelly, Hugh Montgomery

1st VICE-PRESIDENT: Carl Cowen, Genevieve Knight, Bill Velez

2nd VICE-PRESIDENT: Susanna Epp, Joe Gallian, "Tino" Mendez.

John Kenelly was subsequently elected MAA Treasurer by the Board of Governors

The Nominations Committe thanks you in advance for your thoughtful responses.

1202–2002: Fibonacci's Liber Abbaci

By Heinz Lüneburg

Eight hundred years ago Leonardo Pisano, also called Fibonacci, finished writing the first version of the book that he called *liber abbaci* and also referred to as *liber de numero.* This first version of this book is not known to us: only the improved and enlarged second version of 1228 has survived. Fibonacci mentions explicitly in it that he composed the book in 1202. That's how we know. (The date 1202 refers to the Pisan calendar. It is the period from March 25, 1201 to March 24, 1202. The date 1228 is to be interpreted accordingly.

Abbacus, or rather *abacus*, is a word that has had several meanings in the past. It is still used to denote the Japanese and Chinese computing devices soroban and suan pan. Fibonacci uses it in a way peculiar to him. As one learns by studying his writings, he gives it the sense of mathematics with the exclusion of geometry. In certain contexts, he uses the word numerus, i. e., number, also in this sense. Thus *liber abbaci* or *liber de numero* simply means "book of mathematics." (In fact, the plural "numeri" meant mathematics already in antiquity.)

And what a book of mathematics! Way ahead of its time, its level was not reached again until three hundred years later by Luca Pacioli's *Summa de Arithmetica*, *Geometria, Proportioni & Proportionalita* (Venetia, 1494), in which Luca Pacioli writes explicitly that he resumes the work of Fibonacci. Then, of course, came Tartaglia, Cardano, Nunes, Bombelli, to mention just the major actors, and mathematical knowledge grew and grew and is still growing. However, during the period between Fibonacci and Luca Pacioli, the books of the former had their impact. I shall come back later to this topic.

Fibonacci starts his *liber abbaci* by introducing the nine Indian figures 9, 8, 7, 6, 5, 4, 3, 2, 1, as he calls them. These figures together with the zero 0 suffice to write all numbers, he claims, and he explains how to achieve this. He also teaches reading numbers written that way. In particular, one finds already there the hint to write large numbers arranging the digits in groups of three for easy reading. He shows that permuting the figures in a string changes its meaning and he explains the role of the zero.

It should be said already here that Fibonacci's computing is always in writing. One cannot overemphasize this fact. This was by no means the only way to do it: the counting board remained in use for centuries after Fibonacci. Paper was produced in Spain already in the middle of the 12th century and traded in Pisa in the 13th, but Fibonacci mentions only the wax tablet as his writing pad.

Before he explains how to perform addition, subtraction, multiplication, and division with remainder with numbers written in decimal, he surprises the modern reader—at least he surprised me—by introducing finger numbers. People in the Mediterranean had developed a system of gestures that enabled them to represent numbers from 1 to 99 by their left hand. The corresponding gesture made with the right hand meant the hundredfold. Thus, using both hands, they were able to represent all numbers between 1 and 9999. The explanation of what these finger numbers are used for comes a little later: The hand is used to hold carries, a very early dynamical storage device!

Addition and subtraction yield no surprise, but Fibonacci gives two procedures for multiplying two numbers. The second procedure is the same as ours. He recommends it for the multiplication of large numbers. The first one, the one he likes best, deals with numbers seen as polynomials where the indeterminate is replaced by ten. Here larger carries occur, so the left hand finger numbers come into their own. The left hand will suffice to hold carries as long as the numbers have less then 12 decimal places. Thus, when he says "large numbers" in this context, he really means *large* numbers.

Fibonacci's scheme for long division differs from ours, but that isn't really important. What *is* important is that the arithmetical procedures are based on the decimal representation of the numbers and that they are performed in writing.

A most important consequence of decimalization and the manipulation of numbers in writing is that 0 and 1 have to attain the status of numbers, since they play the same role as the other figures in the procedures of elementary arithmetic. Fibonacci uses the word number for them in his writings. This is by no means self-evident. Euclid defined numbers as being collections of (at least two) units. He does not say "at least two", but his formulations of propositions and their proofs shows that he means this. Accordingly, one still finds in texts of the 16th century the statement that 1 is not a number, but the origin of all numbers.

Fibonacci knows that all numbers less than or equal to n^2 can be factorized completely, if one knows all primes less than or equal to n. He gives the list of all primes below 100. Fibonacci also teaches how to perform the elementary operations for fractions. He introduces the symbol $\frac{1234}{5678}$ for fractions, with the bar between numerator and denominator.

Then the problems of the merchants are dealt with. One learns a lot about lengths, weights, currencies—even the recently (1180) coined *sterlingi* are mentioned—goods, centers of trade, and so on. It was this part, and, of course, the part on the elementary operations, that the *maestri d'abbaco*, the masters of reckoning, drew upon in the period between Fibonacci and Luca Pacioli.

Here one observes a phenomenon that occurs inevitably. The problems of the practitioner being solved, they become boring to the mathematician. Hence he leaves them aside and continues spinning his thread just for the sake of curiosity. Fibonacci studies beautiful linear problems that had no application whatsoever at the time. Many of them have a cyclic symmetry and some of them are solved using this symmetry. How can this be done in a situation where there are almost no notational facilities? Well, imagine that the four equations under consideration can be encoded, as we would say, by four fractions

$$\frac{5}{26} \quad \frac{4}{17} \quad \frac{3}{10} \quad \frac{2}{5}$$

and that the first solution is

$$5\cdot 17\cdot 10\cdot 5+5\cdot 4\cdot 10\cdot 5+\cdots+5\cdot 4\cdot 3\cdot 5+5\cdot 4\cdot 3\cdot 2$$

i. e., *numerator into denominator into denominator into denominator plus numerator into numerator into denominator into denominator plus etc.* This has to be shown, of course. But then, using the cyclic symmetry, one gets the second solution by applying this scheme to the fractions

$$\frac{4}{17} \quad \frac{3}{10} \quad \frac{2}{5} \quad \frac{5}{26}$$

giving the second solution

$$4\cdot 10\cdot 5\cdot 26+4\cdot 3\cdot 5\cdot 26+\cdots+4\cdot 3\cdot 2\cdot 26+4\cdot 3\cdot 2\cdot 5$$

Rearranging these fractions cyclically and applying the scheme again yields the third solution and so forth. This is the way Fibonacci proceeds. Most beautiful! It is the first use of symmetry in mathematics that I know.

In dealing with linear problems, negative numbers crop up. They are always interpreted as debts. Approximating the cube root of a certain integer, a negative number appears as an intermediate result. Fibonacci continues the computation correctly.

To approximate square roots and cube roots, one has to determine first the largest integer in the roots. For the square roots, one has to know by heart the squares $2^2 = 4$ up to $9^2 = 81$. Then Fibonacci remarks that, if n has $2k-1$ or $2k$ decimal places, then $\lfloor\sqrt{n}\rfloor$ has k decimal places. Then he shows how to determine $\lfloor\sqrt{n}\rfloor$ for $n = 743$ and $n = 8754$. The algorithm is the same one I learnt in school. Then he surprises the reader again. In order to compute $\lfloor\sqrt{12345}\rfloor$ and $\lfloor\sqrt{927435}\rfloor$, he refers to what has been learnt so far, giving simply that $123 = 11^2 + 2$ and $9274 = 96^2 + 58$, and then he repeats the algorithm that has worked already in the case of an integer with three or four digits. A neat recursion! He proceeds similarly for cube roots, recursion included.

Mathematics and geometry support each other and, hence, a book on mathematics cannot be without geometry, Fibonacci says in the prologue to his book. He also says that he tried to arithmetize the geometry he presents in his liber abbaci. He succeeded in doing so. Most remarkable in this respect is his presentation of the theory developed in Euclid's *Book X* on quadratic and biquadratic irrational quantities. I once taught *Book X* in class following Euclid first and switching to Fibonacci's method later. It really was a relief for the students.

Fibonacci's first acquaintance with the Indian way of writing numbers and manipulating them the Indian way took place at Bougie, a city on the North-African coast now belonging to Algeria. This city played an important role in trading goods and exchanging ideas between the Arabic east and the Arabic west, the Maghreb. Spain at that time was still occupied to a large extent by the Arabs. Fibonacci's father acted as notary public in the service of the city of Pisa for the Pisan merchants calling at Bougie harbor. He let come his son to Bougie to learn the Indian stuff, expecting it to lead to advantages for his son in the future. Later, Fibonacci traveled a lot in the Mediterranean to the centers of trade of Syria, Egypt, Greece, Sicily, and Provence to learn whatever he could learn about computing and mathematics. He also went to Constantinople. The rich fruit of these travels went into Fibonacci's *liber abbaci*. It became the basis of the reckoning of the Italian merchants on the one hand and on the other the basis Renaissance mathematics was built upon.

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Heinz Lüneburg can be reached at FB Mathematik der Universität, D-67653 Kaiserslautern, Germany, or by email at luene@mathematik.uni-kl.de.

Curriculum Foundations Workshop on Mathematics for Health and Life Sciences

By William J. Terrell and Thomas F. Huff

What mathematical experiences and knowledge should all Health and Life Sciences students gain during their first two years of study? And what can Mathematics Departments and colleagues in Life Sciences Departments do to interact productively to provide these experiences?

These issues were addressed by the *Curriculum Foundations* Conference on the Mathematics Curriculum for Health and Life Sciences students, held in May 2000 at Virginia Commonwealth University in Richmond, Virginia. Conference participants represented both mathematics and the life sciences. The summary report of the conference is a statement of the common ground discovered through the conference discussions, along with some general recommendations for the mathematics community from the life sciences participants. The summary report is available at the Bowdoin College web site at http://academic.bowdoin.edu/math/faculty/B/barker/index.shtml.

This article highlights two major issues that generated interesting discussion at this conference. On both issues there was a general consensus on basic requirements. These two issues are the need for a core mathematics curriculum for life sciences students, with an emphasis on mastery of mathematical concepts, and the need for flexibility in the curriculum of all life science disciplines, to allow all students access to this core mathematics curriculum.

A Core Curriculum: The consensus on the need for a core mathematics curriculum was reached very quickly. The essential requirement, on which the participants agreed, was that mastery of a mathematical concept means conceptual understanding combined with understanding of implementations/computations involving that concept. To achieve mastery, the conceptual discussion of mathematical concepts must be integrated with relevant computations in problem-solving. This requirement is currently being addressed by calculus courses using texts that give equal time to algebraic, geometric, and numerical aspects of mathematical concepts. However, conference participants from the life sciences were generally unaware of mathematics education reform efforts. The core curriculum should emphasize modeling discussions, and life scientists observed that modeling "requires a solid mathematical background in techniques and structures." Although mathematics in biology frequently uses discrete models, life scientists affirmed the importance of standard introductory courses in the continuous models of calculus and its extensions, as in differential equations. In addition, there were frequent calls for an emphasis on the use of statistics and data analysis.

Here are some additional statements that addressed the quest for mastery:

Graphing and visualization skills, including the use of log scales, and all aspects of the linear equation $y = mx + b$ should be mastered.

Some participants observed that "chemistry students tend to think more visually than physics students, who tend to think in terms of formulae." These habits, or inclinations, as the case may be, represent trade-offs in the overall mastery of concepts.

Students should master computer use and statistics for problem-solving, through use of a high level software package. The specific software package is less important than mastery of the concepts common to each class of package.

Mastery requires that theoretical understanding and computational skill be considered two sides of the same coin, the overall understanding of concepts. The desired balance between these aspects can be problem/question dependent. Some participants noted that biology students in the first two years "might not necessarily know how to solve a differential equation, but they should understand qualitatively what the various terms in the equation mean." This emphasizes basic conceptual understanding at some expense to the implementation aspect of understanding.

Flexibility: Life scientists observed that the value of mathematics is not restricted to the specific content of any particular course. Life sciences students should be permitted and indeed encouraged to take more mathematics courses even at the expense of some lesser amount of life sciences coursework. Thus, life sciences students might be offered a program structure that includes substantial mathematics courses. In the words of one participant, the flexibility needed for the benefit of life sciences students is that "the content of specific courses is not as important as the whole package of courses that the student is allowed to take." Some students might want and need to take additional courses beyond the recommended core material. One scientist commented on the needs of students with ambitions beyond the basic undergraduate level: "The current areas of biological interest will branch into further areas and mathematical issues will be crucial in forming links between these areas." The reader of this article can view the "most likely optional list" of additional mathematics topics as discussed by conference participants, by going to the web site given earlier in the article.

On the other hand, flexibility is needed on the part of Mathematics Departments, to offer program tracks with substantial life science courses for students with interests in those areas, even at the expense of taking some lesser amount of mathematics credits.

How do we all benefit? Successful interdisciplinary work often involves overcoming the difficulties of language or academic culture differences. Administrations must realize that instructors and researchers working across traditional academic boundaries need support and encouragement. The reward will be the realization of the creative potential that lies at these boundaries.

William J. Terrell is Associate Professor of Mathematics at Virginia Commonwealth University. Thomas F. Huff is Vice Provost for Life Sciences at Virginia Commonwealth University

Curriculum Foundations Workshops for Technical Programs in Two-Year Colleges

By John C. Peterson

The role and nature of mathematics needed for advanced technology programs was the subject of two CRAFTY workshops in October 2000 and a National Conference in May 2002. The workshops were conducted as part of the project "Technical Mathematics for Tomorrow: Recommendations and Exemplary Programs," awarded to the American Mathematical Association of Two-Year Colleges (AMATYC) by the National Science Foundation. Project directors, in addition to the author, are Mary Ann Hovis (James A. Rhodes State College, Lima, Ohio) and Robert Kimball (Wake Technical Community College, Raleigh, North Carolina).

Los Angeles Pierce College, Woodland Hills, California and J. Sargeant Reynolds Community College, Richmond, Virginia, served as host sites for the CRAFTY workshops. Participants were not mathematicians but people selected for their experiences in education or industry and their ability to provide suggestions on the mathematics needed by people preparing to work in the areas of Information Technology, Biotechnology and Environmental Science, Manufacturing and Mechanical Engineering Technology, and Electronics, Telecommunications, and Semiconductors. Participants addressed the mathematical content students must master during the first two years in order to complete their AAS program, enter the job market, advance up the career ladder, and continue their education.

The National Conference was held May 2002 in Las Vegas. The 83 participants included mathematics educators, technical personnel from business and industry, and technical faculty from two-year colleges. The non-mathematics educators were from the technical fields mentioned above and had attended one of the CRAFTY conferences sponsored by the project.

In most cases, with the exception of elementary statistics, the mathematical needs of these technicians did not extend beyond the content contained in most precalculus courses. In fact, the Biotechnology and Environmental Science report states:

"There is little need for mathematics past algebra when people enter the workplace. Some trigonometry is needed, but probably not advanced trigonometry....

"While everyone in the group agrees that a lot of advanced mathematics (such as calculus and trigonometry) is not needed in our fields, we also agree that everyone should know more mathematics than is required in their everyday job. However, this probably does not include such mathematics as calculus."

So, rather than advanced mathematics, it was felt that technicians must be able to think critically, solve problems, and function with linear and non-linear thought processes. The focus must be on reasoning skills and creative problem solving rather than specific content.

The Electronics, Telecommunications, and Semiconductors group reminded us that "Technicians are basically troubleshooters or repair persons. Some work in the field repairing items such as copy machines while others work on a test bench. Some technicians work for a manufacturing company and repair assembly equipment. The best job as a technician is working in an engineering lab. Some technicians are the highest trained technical people in the company... Technicians have had to locate and replace discreet parts in the past, but the troubleshooting of the future is at board level and requires more system troubleshooting."

In order to perform this troubleshooting function, technicians must be able to solve problems and communicate solutions. Mathematics faculty can help develop these skills by providing practical examples, using open-ended non-trivial problems that require the use of teamwork, and encourage the use of appropriate technology—including spreadsheets. One way that will help instructors relate the mathematics to the technical subject areas is for the material to be team taught with faculty from technical areas.

As if to emphasize the above statement, the Information Technology (IT) group stated that "It can be difficult to identify specific math content required as job skills for individual IT positions; in fact, many IT technicians have jobs that require few quantifiable math skills. Therefore, academic mathematics preparation for students pursuing IT careers may not require advanced math but should include a solid foundation of fundamental content, with an accompanying strong emphasis on the analytical mental training that understanding mathematical concepts demands."

The Manufacturing and Mechanical Engineering Technology group emphasized this further by stating that "the student must be computer literate in a modern manufacturing setting and needs to be exposed to both mathematics software and simulation which would be used in both design and process planning and statistical process control applications in business. Again all technicians should be able to use standard business software as well as the Internet for communication and presentation skills."

However, it was the pedagogy that was the biggest concern to these groups. Perhaps the Manufacturing and Mechanical Engineering Technology group said it best when they stated, "Our technical community college faculty felt that major changes were required in the ways in which mathematics is taught in college today. Curriculum needs to be presented in a modular just-in-time format to suit the specific technical content area being taught. The mathematics problems need to be more connected to the real world and must be relevant to the technical field being studied. Experiment with different classroom approaches to provide real world experiences for the students.... Both the students and faculty should have the opportunity for internships and capstone projects with industry."

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While all four groups came up with extensive lists of the mathematical requirements for students in their technical areas, the recommendations boiled down to the following:

- Some topics should be emphasized: statistics, geometry, basic skills (arithmetic and algebra), estimation, and measurement conversions
- Topics should be de-emphasized if technology provides alternatives
- Modeling is vital

The mathematical content is not as important as the pedagogy. In response to the question "What instructional methods might mathematics instructors use to develop or reinforce non-mathematical skills or understandings in your discipline or company?" one group listed the following:

- Report writing and presentation skills (Communication)
- Just-In-Time Teaching—teach the mathematics concept and application as it is required in the content areas.
- Work together with technology colleagues to identify expectations, try different approaches, adjust the delivery, etc., to focus on the desired outcomes.
- Teach that there may be a range of answers versus a single answer.
- Use student teams on problem solving applications. Use of team problem solving techniques should begin as early as possible as the skill develops over time.
- Have students present how they solved a problem.
- Mathematics faculty should serve on technology advisory committees.

John C. Peterson teaches at Chattanooga State Technical Community College. *This article is based on work supported by the National Science Foundation under DUE grant no. 0003065. Any opinions, findings, and conclusions or recommendations expressed in this article are those of the author and do not necessarily reflect the views of the National Science Foundation.*

This issue includes two articles on Curriculum Foundations, a project of CRAFTY, the MAA Committee on Curriculum Renewal Across the First Two Years. Earlier articles have described the project as a whole (November 2000), the workshop on the mathematics courses needed by physics students (March 2001), computer science students (May/June 2001), chemistry students (September 2001), and students engaged in interdisciplinary programs (September 2001). Future articles will focus on other client disciplines. CRAFTY is a subcommittee of CUPM, the Committee on the Undergraduate Program in Mathematics, which is undertaking a review of the whole undergraduate curriculum.

USA Team Takes Third in 2002 International Mathematical Olympiad

At The 2002 International Mathematical Olympiad (IMO), held in Glasgow, Scotland, last July 29, 2002, the USA team placed third, behind the first place team from China and the second ranked Russian team.

Overall, the six members of the USA team won four gold medals, one silver medal and one honorable mention. Po-Ru Loh, of Madison, WI, distinguished himself by tying for second place in the total number of individual points by completely solving 5 of the 6 problems. Two Chinese students and a Russian tied for first place by correctly solving all six problems.Tiankai Liu of Sartoga, CA, Ricky Liu of Newton, MA, and Daniel Kane of Madison, WI, also received gold medals. Anders Kaseorg of Charlotte, NC, received a silver medal and Alex Xue of Chandler, AZ received an honorable mention. Adam MacBride, Academic Director of the IMO 2002, said, "This year's test was difficult, yet the students rose to the occasion. They met the challenge, and still managed to have a remarkable amount of fun in seeing Scotland."

US Olympiad team and coaches from left to right: Zuming Feng, Ricki Liu, Tiankai Liu, Alex Xue, Anders Kaseorg, Po-Ru Loh, Daniel Kane and Titu Andreescu.

The competition attracted 479 of the best young mathematicians from 84 countries. The IMO is the world's preeminent mathematical competition for high school-age students. The participating students won a total of 39 gold medals, 73 silver, and 120 bronze medals for solving, in a grueling nine-hour test administered over July 24 and July 25, six math questions that would challenge professional mathematicians.

The American Mathematics Competitions, a program of The Mathematical Association of America, is a series of challenging tests which culminate in the selection of the USA IMO team. Other major sponsors of the AMC and the USA-IMO team are the Akamai Foundation (http://www.akamai.com) and the University of Nebraska. The IMO team is chosen through a three-stage process of mathematics testing starting with almost half a million test-takers in middle school and high school. For more information, visit the AMC web site at http://www.unl.edu/amc.

NSF Beat

By Sharon Cutler Ross

The Course, Curriculum, and Laboratory Improvement (CCLI) program of the NSF has three tracks, Adaptation and Implementation (A&I), Educational Materials Development (EMD), and National Dissemination (ND). The last issue of this column reported the recent EMD awards; this time the focus is on A&I.

December 4, 2002, is the deadline for full proposals for the A&I track. Two types of projects are invited. The first, Type I, are ones that adapt and implement exemplary materials, laboratory experiences, or other educational practices developed and proven successful in order to achieve specific curricular goals. These goals may be for an individual course or laboratory, a cluster of courses, or an entire curriculum or program. The acquisition of instrumentation may be appropriate in a Type I project.

Type II projects are expected to support a group of faculty who will develop a plan to overcome challenges or barriers that prevent curriculum reform. Participants will explore available exemplary curricula, materials, and practices. Such projects might include intensive faculty enhancement activities, pilot use of externally developed materials or methods, or other investigations of ways to overcome the identified challenges.

The NSF anticipates $20 million to be available for the next round of A&I awards. Grants are typically for 2-3 years and can range up to $75,000 for a Type II project and up to $100,000 for a single course or $200,000 for a comprehensive Type I project. Full details are available on the Foundation's web site.

What sorts of activities are currently underway in A&I projects? Here are brief descriptions of the most recent A&I awards. Fifteen awards totaling roughly $1.3 million were made in the last round of funding. Some projects are focused on a single course while using a variety of pedagogical and technology-based approaches. These include:

- a project at East Tennessee State University (J. Knisley, PI) that follows a successful pilot of a technology-intensive laboratory in elementary statistics;
- an "investigative classroom" at Southern Polytechnic State University (M. Dillon) to support collaborative, active learning in Calculus I and II;
- a mathematics learning laboratory at Youngstown State University (R. Goldthwait) to support an active learning environment;
- an adaptation of a lecture-lab calculus course at North Carolina A&T State University (D. Clemence) to improve the success rates of undergraduate students, especially minority students.

Two consortia have projects. The project at San Diego Mesa College (M. Teegarden), American River College, and West Valley College will develop a collaborative, research-based approach to improve the teaching and learning of at-risk students in college algebra. Post-calculus probability and statistics is the target course for a collaboration between Athens State University (M. Lunsford) and Middle Tennessee State University (G. Rowell). The project will support active and cooperative learning, visualization, and simulations through the use of activity- and web-based materials. The University of Alabama Huntsville (T. Goodson-Espy) will provide evaluation for the project. Two courses at The College of New Jersey (T. Hagedorn), linear algebra and Principles of Mathematics will adapt the ATLAST materials.

The remaining projects on this list are intended to provide broad support for a variety of courses. Three propose the adaptation and implementation of a web-based homework delivery system, University of Tennessee Chattanooga (T. Walters), Radford University (C. Mett), and Arizona State University (J. Jones). All three focus on lower-division courses and have as a goal the training of faculty in the development of web-based problems. The project at East Central Oklahoma State University (L. Braddy) is designed to provide on-demand mathematics instruction for developmental mathematics students. A computer classroom at Birmingham Southern College (B. Spieler) will enable curricular change to be implemented in multivariable calculus, linear algebra, and differential equations courses. Finally, general support for distance learning through the development of communication tools for mathematics courses is the aim of the SUNY Stony Brook (G. Smith) project.

Call For Student Papers

Students who wish to present a paper at MathFest 2003 in Boulder, Colorado must be nominated by a faculty advisor familiar with the work to be presented. To propose a paper for presentation, the student must complete a form and obtain the signature of a faculty sponsor.

Nomination forms for the MAA student paper sessions are located on MAA Online at http://www.maa.org under students, or can be obtained from Dr. Thomas Kelley, Department of Mathematics, Henry Ford Community College, Dearborn, MI 49128, (313) 845-6492, mailto:tkelley@hfcc.net. Students who make presentations at the MathFest, and who are also members of an MAA Student Chapter, are eligible for partial travel reimbursement. The deadline for receipt of applications is July 1, 2003.

Pi Mu Epsilon student speakers must be nominated by their chapter advisors. Application forms for PME student speakers can be found at http://www.pme-math.org or can be obtained from the PME Secretary-Treasurer Dr. Leo Schneider, mailto:leo@jcu.edu. Students who make presentations at the Annual Meeting of Pi Mu Epsilon are eligible for partial travel reimbursement. The deadline for receipt of abstracts is June 1, 2003.

PREP Workshops—2002

By Victor J. Katz

Did you miss the opportunity to learn knot theory this summer? Or to gain new insights into teaching your preservice secondary teachers? Or to master new ways of teaching those courses for liberal arts students? Or to develop a new understanding of chairing a mathematics department? If you had signed up for one of the MAA's PREP Workshops, you could have spent a stimulating week in the company of faculty members from around the country studying these or other topics with some master teachers.

One of the functions of the MAA is to be a major source of professional development opportunities for its members. We are currently in a period of rapid and significant change in undergraduate mathematics education, and there is a strong need for faculty members at undergraduate institutions to keep ahead of the curve. The MAA is committed to making this possible by offering a variety of short courses and workshops at sites around the country. Many of these workshops are part of the Professional Enhancement Program, more commonly called PREP, funded by a three-year grant from the National Science Foundation. In 2002, there were eight PREP workshops, including one offered entirely online.

The workshop on *Knot Theory* was conducted by Colin Adams from June 24-28 at Wake Forest University, with a total of 24 participants. The five day program was an intense experience with knots—knots during morning lectures, knots during afternoon research groups, knots during dinner and on into the night. There was such enthusiasm that even during an evening excursion to a baseball game, some of the participants continued their discussions about knots. And although the schedule was intense, there was enough variety to keep everyone interested. As one participant wrote, "Just when we were starting to feel overloaded, we'd get a change of pace and make human knots."

Some participants were at the workshop to learn how to teach a course in knot theory. Others were there to learn how to direct undergraduate research in knot theory or to do research themselves. But everyone seemed to come away satisfied that they had attained their goals in attending. Colin Adams wrote that since this was a "dream class" of students all committed to spending a week of their time learning and thinking about knot theory, he would be happy to conduct the workshop again. (So if you don't know how to turn yourself into a "human knot," you can sign up next year and learn.)

Participants in the workshop *Presenting Mathematical Masterpieces and Powerful Techniques of Effective Thinking to Non-Science Students*, conducted by Ed Burger and Michael Starbird, were equally enthusiastic. Although there was much to do in the workshop outside the meetings themselves, the organizers designed these activities to be fun, interesting, and completable in a reasonable time. The discussions within the meetings were also very stimulating, particularly the ones on infinity and cardinality and on fractals. It is not easy to design an effective course for nonscience students at an elementary level, but most of the participants felt they had a much better idea how to do so after participating here.

Richard Scheaffer and Jeffrey Witmer presented *An Introduction to Statistical Methods Based on Regression* at Oberlin College. The workshop was designed for those teaching courses in statistics. After working with actual data sets and real applications and using the statistical software, virtually all of the participants felt that they were now better prepared to implement new ideas in future classes. They praised the organizers for putting together such an intensive, but well-organized, workshop.

Bill Haver organized *Assessment at the Departmental Level*. This was a three-part workshop for teams from various colleges and universities, with the first session held at the San Diego meeting, the second session at Virginia Commonwealth University in May, and the third session to be held at the Baltimore meeting. This workshop, although conducted under PREP, is a pilot for a series of similar workshops to be conducted under the *Strengthening Undergraduate Assessment in Mathematics* program, another grant funded by the NSF. (For more information on SAUM, visit http://www.maa.org/saum.) At the opening session, each college's team was asked to choose a particular component of its undergraduate program to assess. The team had to design the assessment process, using MAA guidelines, and report back to the second session. Further work will be done during fall, 2002, with final reports due in Baltimore. The general feeling of the participants was that this workshop helped to clarify their ideas on assessment. As one wrote, "I believe the workshop was useful in helping us to better define and focus the project that [one of us] is working on. Our initial presentation was pretty fuzzy in terms of what we were going to measure and how we were going to measure. It's sharper now, I think, because of the feedback we received at the workshop."

Leading the Academic Department: A Workshop for Chairs of Mathematical Science Departments was organized by Tina Straley, the Executive Director of the MAA, and took place at Towson University in Maryland in late June. Participants represented research universities, comprehensive state universities, liberal arts colleges, and two-year colleges. Plenary sessions featured Brit Kirwan, now Chancellor of the University System of Maryland; John Ewing, Executive Director of AMS and editor of *Towards Excellence*, and Michael Anselmi, attorney for Towson University. Most of the agenda was spent in small group sessions discussing a collection of case studies. These case studies were supplied by the discussion leaders, themselves experienced department chairs, and all were based upon real situations ranging from dealing with adjunct faculty to creating departmental strategic plans. Two of the case study sessions brought together chairs of same-type institutions and two brought together chairs from all types of institutions. The workshop concentrated on leadership skills rather than on the day to day operation of the department. However, the case studies approach led to discussions on all aspects of the job. The last session was a whole group discussion, led by the workshop leaders, on what the most important aspects of leadership are. The workshop leaders and case study writers were Jim Lewis, Catherine Murphy, Jon Scott, Ray Johnson, Arnold Ostebee, Jimmy Solomon, Michael Gealt, Dan Maki, Donna Beers, Bonnie Gold, Martha Siegel, and Tino Mendez. Virtually all of the participants, including some just becoming chairs and others who had been chairs more than five years, felt that the workshop met or exceeded their expectations. As one wrote, "I was a little afraid it might be aimed at research institutions, but the mix of people was great. As a brand new chair, I learned a lot and got many ideas. Very inspiring!" Since almost twice as many people applied to this workshop as could be accommodated, the workshop will be offered again in 2003. You, too, can come and be inspired.

The workshop on *Authoring Online Interactive Materials in Mathematics* was conducted entirely online from July 16-19 by Lang Moore, David Smith, and Frank Wattenberg. This had the advantage that participants, in local teams of two or three people, did not have to travel; all they needed was a comfortable office with the appropriate connections. Most of the participants were enthusiastic about the results, having increased their skill on various software packages, such as *Dreamweaver* or *Maple*, and about having learned how to develop online sessions for their own students. They generally felt ready to use this material in their fall classes. One participant commented that the president of the college insists on their students using technology, while the dean insists that faculty members provide opportunities for students to do self-assessment. He was, therefore, "real excited about the tools I now have to produce materials that will not only make the administrators happy... but I also will be able to produce worthwhile materials." After all, it is fine to have a personal webpage that has office hours on it, but students will find one with interactive materials much more useful in the long run.

Two of the workshops were designed for faculty preparing future secondary teachers. *Teaching Future Secondary Teachers: An Extended Workshop Experience* was conducted by Ed Dubinsky and Kathy Heid over a two week period this summer at the State University of New York, Potsdam. The program was quite intense, with discussions of both mathematical content for the secondary curriculum and also of more general pedagogical issues. Some of these were led by the organizers, and others by visitors, including Hyman Bass, Deborah Ball, and Dick Askey. According to the participants, among the highlights of the sessions were the model classes with undergraduates. "Watching the students working on mathematics and having the common experience to draw upon for examples was very helpful in our discussions," according to one participant. And although the trek to Potsdam was difficult for many, all the participants plan to return next summer for another session. During the intervening academic year, they will all be working on major projects, many of which will be related to ideas outlined in the CBMS report, *The Mathematical Education of Teachers*.

Mathematical Methods and Modeling for Secondary Mathematics Teacher Education was presented by John Dossey, Frank Giordano, and Sharon McCrone at Lewis and Clark College in Oregon. The workshop blended the coverage of mathematical content and secondary mathematics education methods throughout the week. The participants were divided into four groups, each of which was given a modeling problem. They worked outside of class on this as well as the homework and made substantial progress toward a solution in the limited time available. Furthermore, each of the participants will be creating a new modeling project to share with the group at their reunion at the Baltimore meeting. And the email list connected to the project already has had a lively discussion of the CBMS report.

As one of the participants commented, "This was a great workshop. I really appreciate the connections between and among the big ideas. The overview of the state of math education and the methods materials will definitely play in my methods courses." Most of the participants also noted that they would recommend this workshop to a colleague when it is offered again. "They would definitely get a sense for the pedagogical approach that can make mathematics more accessible and promote depth in knowledge." So if you would like to look at the models entitled "Car Financing or Terror Bird," and use these in your own methods classes, look for the reprise of this workshop in the coming summer.

As already noted, several of the 2002 workshops will be offered again in the summer of 2003. But there will also be some new workshops, including some interdisciplinary workshops offered in conjunction with other professional organizations, focused on modern applications of mathematics. Details of the workshops for 2003 will be in the December FOCUS, as well as on MAA Online.

Long range plans are to make PREP a continuing activity of the MAA. Although the MAA hopes to get further grant funding to help support participation in PREP, ultimately it must be self-funding. The MAA suggests, in its *Guidelines for Programs and Department in Undergraduate Mathematical Sciences*, that "departments should provide regular opportunities for and support the professional development of faculty members to learn of the most recent findings about teaching and learning in the mathematical sciences" and should "support professional development of faculty members to enable them to remain current with the most recent advances in the field." We now have regular programming to enable departments to fulfill these commitments. All of us should make the most of this opportunity for our own sakes and the sake of our students.

Victor J. Katz is Professor of Mathematics at The University of the District of Columbia, and is spending this year as a Visiting Mathematician at the MAA.

A Genuine Interdisciplinary Partnership: MAA Unveils *Mathematics for Business Decisions*

By Donald J. Albers

If your contribution to a problem can be replaced by one more line of computer code, then management will do that and buy a computer instead of hiring you. At the same time, it is unwise to ask a computer to display a number if you have absolutely no idea what it will be. Students at today's business schools often face these very problems. They find the mathematics education they receive is either insufficient or difficult to apply to the situations they face in professional settings. At the January Joint Mathematics Meetings 2003 in Baltimore, Maryland, the Mathematical Association of America will present an innovative, interdisciplinary course in mathematics for business students that not only solves these difficulties for students, but opens the door to new approaches in teaching mathematics. (See accompanying ad on page 17 for the details of this interactive presentation.) *Mathematics for Business Decisions* is a sparkling example of how interdisciplinary experiences for students can be shaped through the active collaboration of academic departments.

Dr. Richard Thompson of the mathematics department at the University of Arizona.

Dr. Chris Lamoureux of the business school at the University of Arizona.

In 1998, a group of deans and university administrators at The University of Arizona (UA) challenged Dr. Chris Lamoureux of the business school and Dr. Richard Thompson of the mathematics department to create a completely new two-course sequence of mathematics for all business and public administration majors. These new courses were to replace the traditional program of finite mathematics and brief calculus. With funding support from the National Science Foundation's Division of Undergraduate Education, Lamoureux and Thompson set themselves to the task. They first taught their creation in 1998-99. Says Thompson, "We learned that, if properly motivated and given suitable instructional material, business students can appreciate the value of mathematics in making sound business decisions." *Mathematics for Business Decisions* represents an entirely new publishing venture for the MAA, as the books for the courses are entirely electronic. The course itself is distinguished by the use of current technology and student teams rather than individual assignments. Students are allowed to embrace mathematics in a setting that more closely mirrors the professional environments they will encounter after college. Consequently their enthusiasm and retention increase.

Mathematics for Business Decisions has two parts. Part 1, *Probability and Simulation*, lays the groundwork for student understanding, while Part 2, *Calculus and Optimization*, works to finely tune student capability. The entire program consists of four substantial projects, each leading to a crucial business decision. These decisions are open ended, involving business insight as well as mathematical computation and understanding. In the capstone project, Bidding on an Oil Lease, students use computer simulation to discover a Nash equilibrium bidding strategy that is a research level result in auction theory, unknown until 2001. All mathematical and computer skills, including the use of PowerPoint, Excel, and Word, are introduced as tools for the solution of business problems and used in student presentations. Student understanding is enhanced with streaming video, computer simulations, and animations of mathematical topics.

Projects

Part 1, Probability and Simulation Loan Work Outs. Students use records from over 8,000 prior attempted work outs, along with specific characteristics of an individual borrower, to determine whether or not to enter into a work out plan. Tools: probability, Bayes' Theorem, and data searching.

Stock Option Pricing. Random variables and bootstrapping are used by students to establish a fair value for a European call option on a given stock. Tools: downloaded stock data, computer simulation, compound interest, and random sampling.

Part 2, Calculus and Optimization Marketing Computer Drives. Students generate demand functions by fitting trend lines to test market data. Marginal analysis is used to optimize profit, and the consumer surplus is explored. Tools: trend lines, differentiation, and integration.

Bidding on an Oil Lease. Normal distributions are used with Monte Carlo simulation to find ways of defeating the winner's curse in auction bidding. Students discover a Nash equilibrium strategy for adjusting their bids. Tools: variance, expected value, normal distributions and computer simulation.

"The new *Mathematics for Business Deci-*

sions curriculum has impacted the entire business school community," says Professor Chris Lamoureux, Head of the UA Finance Department and co-author of the new courses. "Students are more engaged because they feel that the curriculum is designed specifically for them as business students. This is also manifest through a real sense of accomplishment upon completing the curriculum. Recruiters sense that a business problem-centered curriculum prepares students better for employment. Recruiters are impressed by the realism of the business projects that the students work on in a math class as freshmen and sophomores. Faculty have a sense of continuity, finding it easier to hold students responsible for what they learned in earlier classes. Faculty also appreciate the fact that students mature as business students faster because of the business focus in the required math curriculum."

Mathematics for Business Decisions has now been taught at UA for five years. Six other institutions, including major research universities, smaller universities, and community colleges, have used the material as well. It has been taught by over 50 different instructors, including graduate teaching assistants, adjunct faculty, and research mathematicians. More than 1200 students are currently enrolled in the courses. Says UA Mathematics Professor William McCallum, "I think the *Mathematics for Business Decisions* courses are a remarkable example of collaboration between a mathematics department and a department whose students it teaches, and should serve as a model for the way mathematics departments handle their service teaching."

The success of the *Mathematics for Business Decisions* courses is obvious. Grades are higher and student retention rates are higher than those of the traditional courses in finite mathematics and brief calculus. Homework, which is completed using a word processor, even for equations, is of high quality and the level of the students' reports is outstanding. Student attitudes toward the use of mathematics have improved greatly. They recognize that what they are doing in *Mathematics for Business Decisions* will help them directly in the rest of their business program and in their later careers. The University's Instructional Assessment and Evaluation Services asked students in one section of *Mathematics for Business Decisions* if they saw what they were learning as of use to them in the real world. *100% agreed*! "Classes in the traditional sequence of finite mathematics and brief calculus were a chore for both students and instructors. Teaching the new material to the same students is now fun," explains Thompson, who has given course demonstrations to the Arizona State Board of Regents at the invitation of Dr. Peter Likins, President of UA.

Suppose that all individual parameters of a borrower indicate a loan work out attempt, when considered separately. Is it possible for them to predict against a work out attempt, taken collectively? Hint: Bayes' Theorem is a great tool for squeezing information out of a data set. Students of *Mathematics for Business Decisions* discover this quickly.

How many of us can clearly define a 20 week European call option on Walt Disney stock, with a strike price of $23? Students in *Part 1* of *Mathematics for Business Decisions* discuss this in their reports.

The Black Scholes method of determining a fair value for a stock option requires stochastic differential equations. Do you know how to bypass this, use historical stock data to estimate volatility, and bootstrap your way to a fair option value? Freshmen in *Part* 1 of *Mathematics for Business Decisions* do this.

Do your students think that demand functions are something that one finds in books (or maybe on the walls of caves)? Students in *Part 2* of *Mathematics for Business Decisions* get demand functions by fitting polynomial trend lines to data points found in test markets. How can they do something so sophisticated as this type of regression? Excel does it for them, thereby freeing them to consider the important business question of what type of trend line to use.

Do you know what happens if everyone bids his or her estimate of the value of an item in a first price sealed bid auction? Why is the result named the "winner's curse?" Students in *Part 2* of *Mathematics for Business Decisions* discover this for themselves during classroom auctions.

How can one defeat the "winner's curse?" What is it about the auction mechanism that makes it unlikely that colluding companies will stick together? These are important questions for bidders on oil leases, communications bands, or treasure bills. Students are expected to discuss these problems in their project reports.

Do you know how to find a Nash equilibrium strategy for bidding in a first price sealed bid auction? Despite a large scholarly literature in auction theory, nobody knew how to do this until 2001. In fact, the existence of such a strategy was discovered using only simulation with the tools of *Mathematics for Business Decisions*. Students in *Part 2* cover this topic in their project reports.

Can equally correct and valid mathematical computations lead to opposite conclusions in a business question? Of course they can. Students in *Mathematics for Business Decisions* encounter this in all of their projects. There is often no "correct" solution. Teams of students must back up their decisions with business considerations and an understanding of the human assumptions on which the mathematical computations are based.

Students express enthusiasm for *Mathematics for Business Decisions* too:

"I really enjoyed your class. I felt like I learned so much. It is such a different feeling coming out of *Mathematics for Business Decisions* than when I finished other math classes. I am really glad that I got the chance to take *Mathematics for Business Decisions*." *A student in Part 2*

"Finally a math class has been created that has useful applications towards the future." *A student in Part 1*

"I can't believe how much I have learned in this class. I work at A G Edwards & Sons and some of the brokers are amazed at

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this class and its usefulness. Others also say how great an idea it is to create this class and how lucky I am to be in it because it will benefit me in real life." (Note: This student was an undergraduate intern with the brokerage.) *A student in Part 1*

"I am working at my internship at Sony Studios. You have no idea how much that project [Marketing Computer Drives] helped me for what I am doing now for my job." *A student in Part 2*

Thompson and Lamoureux are pleased by the responses of students to the new courses and they emphasize that *Mathematics for Business Decisions* contains solid mathematics and revolves around important nontrivial problems. Adds Dr. George Davis, Provost of UA, "I am particularly delighted...by the way in which subsequent generations of students are now afforded a concrete and very immediate example of the symbiotic relationship between research and teaching."

Donald J. Albers is the Associate Executive Director for Publications and Electronic Services at the MAA.

2002 Fields Medals and Nevanlinna Prize Awarded

The 2002 Fields Medals and the 2002 Nevanlinna Prize were awarded last August 20 at the International Congress of Mathematicians in Beijing, China. The Fields Medals went to Laurent Lafforgue of the Institut des Hautes Études Scientifiques in Bures-sur-Yvette, France, and to Vladimir Voevodsky of the Institute for Advanced Study in Princeton, NJ. The Nevanlinna Prize went to Madhu Sudan of the Massachusetts Institute of Technology.

The Fields Medal is often described as "the Nobel Prize for mathematics." Presented by the International Mathematical Union, the prize is named for the Canadian mathematician John Charles Fields (1863-1932), who chaired the committee that established the prize in 1931. The Fields Medals have traditionally been presented to mathematicians no older than 40.


Laurent Lafforgue

Laurent Lafforgue was honored for making major advances in the "Langlands Program," a visionary set of conjectures by Robert Langlands that sketch out deep connections between number theory, analysis, and group representation theory. Lafforgue proved the global Langlands correspondence for function fields, building on the work of Vladimir Drinfeld, a l990 Fields medalist. Lafforgue's work confirms the fundamental insights of Langlands and is an important step towards the realization of the full program. It is said to be "characterized by formidable technical power, deep insight, and a tenacious, systematic approach."


Vladimir Voevodsky

Vladimir Voevodsky was honored for developing a new cohomology theory for algebraic varieties. Voevodsky's "motivic cohomology" (the name is related to Alexander Grothendieck's idea that there should exist objects, called "motives," that establish the connection between number theory and algebraic geomtry) builds on an idea first proposed by Andrei Suslin. Among other things, it creates a strong connection between algebraic varieties and algebraic K-theory and it provides a framework for studying many new cohomology theories. One consequence of Voevodsky's work is a proof of the Milnor Conjecture, for many years the main open question in algebraic K-theory. His work is said to be "characterized by an ability to handle highly abstract ideas with ease and flexibility and to deploy those ideas in solving quite concrete mathematical problems."

The Nevanlinna Prize has been awarded since 1982 by the International Mathematical Union. It recognizes exceptional work in the more mathematical aspects of computer science. Like the Fields Medal, it is traditionally given to young researchers, also no older than 40. The prize is named for Rolf Nevanlinna, a past president of the IMU.


Madhu Sudan

Madhu Sudan of the Massachusetts Institute of Technology won the prize for his work on "probabilistically checkable proofs, to non-approximability of optimization problems, and to error-correcting codes."

Sudan's work is wide-ranging and brilliant. He was a main player in the development of methods for a "verifyer" (typically a computer) to establish with a high degree of probability whether a proof is correct. His work on non-approximability of solutions to certain problems relates to the "P versus NP" problem in theoretical computer science, showing that for many NP-hard problems approximating a solution is just as difficult as finding the actual answer. In the theory of error-correcting codes, Sudan's work showed that certain coding methods could correct many more errors than was previously thought possible.

For more about the Fields Medal, visit http://www.mathunion.org/medals/index.html. See also http://www.maa.org/news/fields02.html on MAA Online.

The Undergraduate Student Poster Session: History, Present and Future

By Mario Martelli

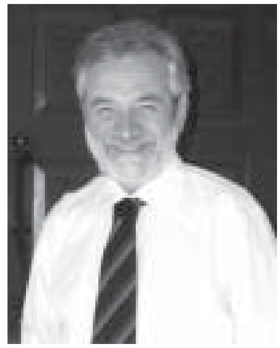
The Undergraduate Student Poster Session will be one of the highlights of the Friday afternoon, January 17, activities at the Joint Meeting of the AMS, MAA and AWM in Baltimore, Maryland. If the participation of the last two years in New Orleans (2001) and San Diego (2002) and the posters already received are any indication of what the Baltimore session will be like, then we will once again see at least 80 posters presented by an enthusiastic crowd of at least 200 undergraduates from all over the country. Each poster will be carefully evaluated by three professional judges. The fifteen best posters will receive a prize of $100 each (the prizes are provided by the MAA, AMS, AWM, CUR and private donors). Refreshments will be served for the students, the judges and the visitors. If you'd like to participate, send your poster as soon as possible to Mario Martelli, Mathematics Department, Claremont McKenna College, Claremont, CA, 91711 (909-607-8979) mmartelli@mckenna.edu). Space is limited and will be allocated on first come, first serve basis.

The Undergraduate Student Poster Session started very modestly at the beginning of 1990. I still remember the sessions in Cincinnati (1994) and San Francisco (1995), organized by John Greever of Harvey Mudd College. There were about 15 participants in each session. John provided the prize money out of his own pocket. Judith Palagallo organized the poster sessions in Orlando (1996) and Baltimore (1998). Some prize money was offered by the MAA and Judith was able to add a few more prizes with private donations. She also printed nice booklets with titles and abstracts of all posters. Both sessions were a resounding success.

Aparna Higgins and Joe Gallian handing out awards during the poster session.

I organized the poster session in San Diego (1997) and despite my efforts, the number of participants was modest. Aparna Higgins took over the organization of the poster session in San Antonio (1999). I was supposed to co-organize the session with her, but Aparna did most of the work. She devoted an enormous amount of time and energy to it. Her efforts paid off. The number of participating posters rose to about 40. In 2000 (Washington DC), 2001 (New Orleans), 2002 (San Diego) I organized the session and streamlined the evaluation of the posters. AMS, MAA, AWM and CUR contributed prize money, thanks to the efforts of Joe Gallian and Colin Adams. There were over 60 posters in Washington, and over 80 both in New Orleans and San Diego.

REU programs have consistently contributed to the success of the poster session. Judging is now in the hands of about 100 professional mathematicians who donate their time and expertise. Many of them get in touch with me way ahead of the session to ask if they are going to be in the group again. Organizing the session is time-consuming and absorbs my winter break entirely, but doing it is a rewarding experience. The organizers of the joint meeting have made significant changes to acknowledge the growth, both in numbers and in importance, of the Poster Session. Refreshments are now provided, and the Undergraduate Student Lecture has been moved to early Friday afternoon.



Mario Martelli

We are still working on securing financial support to help those students who are not funded by their institutions. Thanks to the efforts of Dan Schaal we should receive a three-year grant from NSF to pay at least 50% of the traveling and accommodation expenses for 30 students. I consider this just the first step in the right direction. We obviously need a more stable source of income, like an endowment, to ensure that no student will be kept away for financial reasons only. Next spring I will write an article describing in detail how the session is organized, how judging takes place, and how prizes are awarded. Hopefully, some mathematician who really believes in undergraduate research will step forward and offer to replace me in this task. I am not tired of it and I can continue, but I feel that somebody with bold ideas and more energy can bring the session to a new level of importance and recognition.

Mario Martelli, who teaches at Claremont McKenna College, has been involved with the student poster sessions at the winter Joint Meetings for several years.

MathFest Memories

The 2002 MathFest was held in Burlington, VT, and as usual it was a large and exciting meeting with lots of stuff going on. With a total of 1271 participants (1003 mathematicians and 268 guests), this was the largest MAA MathFest yet. (The summer meeting has been the MAA MathFest since 1997; previous meetings were joint summer meetings with the AMS.) The scientific program included lots of large lectures (most notably the Hedrick Lectures by László Lovász and the Leitzel Lecture by Jim Lewis), but there were also many special sessions on topics ranging from Discrete Methods in Geometry to Recreational Mathematics in the Classroom. Several sessions were dedicated to student presentations (see more on this on page 21).

Several publishers were present, and as usual their displays attracted a lot of attention and became the natural place to go to meet people. And meeting people is a big part of MathFest: the relaxed atmosphere made it particularly nice to catch up with friends or get to know colleagues. Many social events made this easier to do. The standout social event was probably the late afternoon cruise on Lake Champlain.

Project NExT was a big presence at the meeting, with both new fellows and veterans taking part in many events. One former NExT fellow is now a member of the MAA Board of Governors!

For the MAA, the summer meetings are an opportunity to conduct business and to honor those who have served the association in various ways. The Board of Governors met the day before MathFest proper began, and many MAA committees held meetings. At a special awards session, the MAA honored the authors of the best expository papers of the last year. (The full list of award winners was in the September issue of FOCUS and is still on MAA Online.) The President of the MAA hosted a special reception to thank the MAA's most generous donors; it was held on the boat, immediately before the cruise on Lake Champlain.

This section of FOCUS collects several short reports on MathFest events. The next MathFest will be on July 31–August 2, 2003, in Boulder, Colorado. Mark your calendar!

Board of Governors Actions at MathFest

Among the many items discussed by the Board of Governors at their meeting in Burlington, several elections stand out. The Board elected Nancy Hagelgans of Ursinus College in Collegeville, PA as chair of the Committee on Sections. David Stone of Georgia Southern University in Statesboro, GA, who had just left the position of chair of the Committee on Sections, was chosen to chair the Nominations Committee. Also on the Nominations Committee are Chris Stevens, Tom Banchoff, Aparna Higgins, and Bob Megginson.

Richard Anderson, MAA President Ann Watkins and Frank Morgan in Burlington at this summer's MathFest.

The Board also elected the Leitzel, Polya, and Hedrick Lecturers for next year. The Leitzel Lecturer, who will speak at the 2003 MathFest in Boulder, CO, is Joan Leitzel, former President of the University of New Hampshire. The Hedrick Lecturer, who will give a series of three lectures at the 2003 MathFest, is Henri Darmon of McGill University. Finally, the Pölya Lecturer, who will be available as a lecturer for several MAA Section meetings over the next two years, will be David Bressoud of Macalester College.

Reception for GLBT Mathematicians

The first annual on-site reception for Gay, Lesbian, Bisexual and Transgendered mathematicians and friends was held at Mathfest. We had an excellent turnout of GLBT mathematicians, friends, and MAA officers. The food, wine, and good conversation were plentiful. Our receptions are open to all attendees of the meetings. Please come to our Baltimore reception. If you would like to be added to our confidential email list, please contact George at drgeopgh@aol.com.

A Visit to Ben and Jerry's

The SIGMAA on Statistics Education sponsored a statistics tour of Ben & Jerry's in Waterbury, VT for members of the SIGMAA. (Space permitting, non-members were allowed on the tour, but had to pay $15 for transportation. No charge for members.) Our chair, Mary Sullivan, arranged the tour and arranged for transportation from the convention center with Mountain Transit, Inc.

Eleven of us went on the tour and all enjoyed it. We left the convention center shortly after Robin Lock's talk on *Fun and Games for Teaching Statistics*. We took the regular tour, accompanied by Daron Byerly (Ben & Jerry's statistician). During the tour we got to taste some Ben & Jerry's ice cream (Cherry Garcia and Chocolate Chip Cookie Dough) and then to shop in their gift store. We met with Daron for an interesting hour afterwards. He told us how Ben & Jerry's uses statistics for quality control and to design and analyze experiments in order to improve their production processes. We had many questions for Daron and he had a few for us. Then we returned to MathFest, getting there just in time for the afternoon sessions.

Students With Varied Math Interests Continue to Add to MathFest

As always, a major feature of the MAA MathFest was the student paper sessions. The MAA Committee on Undergraduate Student Activities and Chapters (MAA-CUSAC) organized eight student sessions, four on August 1st and four on August 2nd. A record total of 64 students participated in 58 presentations. A complete list of the MAA student speakers, titles, and abstracts can be found at http://adm.hfcc.net/~tkelley/ (scroll down to the *MathFest 2002* section of the page).

Barbara Beechler, Mario Martelli, and Carolyn Staples on the Lake Champlain Dinner Cruise.

More than half of the students were members of various REU (Research Experiences for Undergraduates) programs being held in the summer at Cornell, Grand Valley State University, Mt. Holyoke College, SUNY Potsdam, and Williams College. Most students were from the Northeast, but some traveled from as far away as California and two students listed their home institution as being in China. Sixteen of the students took advantage of their membership in their home institution's MAA Student Chapter and received travel grants (up to $600) to help defray the cost of attending the meetings.

Cash awards of $150 were distributed by the MAA for 9 presentations judged as outstanding and one award was given by the Council on Undergraduate Research Award. Their names, institutions, and presentation titles appear in the table on this page.

The titles give a hint as to the depth and breadth of the topics covered by student presenters at the 2002 MathFest. Congratulations are due, not only to these award winners, but also to every student who presented at the conference. The MAA Committee on Undergraduate Student Activities and Chapters looks forward to continuing this fine tradition at the 2003 MathFest in Boulder, Colorado.

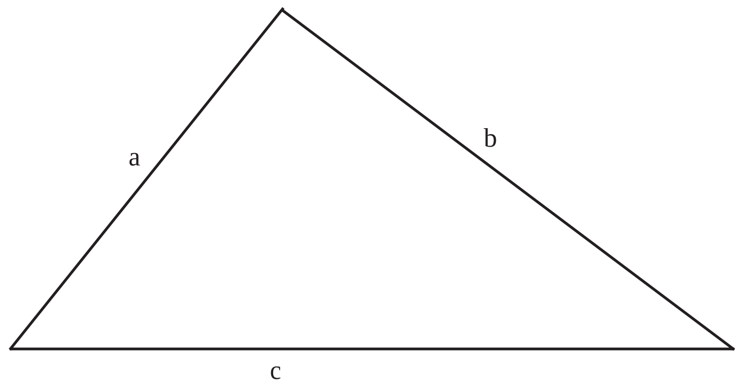
MAA MathFest 2002 – Student Presentation Awards

Council on Undergraduate Research Award		
Daniel G. Treat	University of Missouri-Rolla	PDEs on the Octagasket Fractal
MAA Outstanding Presentations		
Katherine A. Bold	University of Texas Austin	Examining Bifurcations of the Forced Van Der Pol Equation
Lisa DeKeukelaere	Colby College	Square Functions and Weights on Various Intervals
Elisa Golfinopoulos Ryan McCarthy	Hamilton College	Thinning Out Divergent Series
Sarah Iams	Williams College	The Dynamics of R^2 Actions
Robert Lopez	Williams College	Symmetries and Comparisons of Minimizing Double Bubbles
Scott Nickleach	Slippery Rock University	Unordered Mastermind
Michael Piatek	Duquesne University	High Performance Shoelace Tightening
Irma Servatius	Worcester Polytechnic Inst.	How Few Radii?
Amy C. Ulinski	Duquesne University	Give me a Boost: Ups & Downs Learning Monotonic Boolean Functions.

A Nifty Derivation of Heron's Area Formula by 11th Grade Algebra

By Reuben Hersh

The area of any triangle equals the square root of the product of four linear combinations of the side lengths. Heron's formula (sometimes credited to Archimedes) is stated in terms of the "semi-perimeter". Such a simple, useful formula ought to have a simple, direct derivation. The classical one, by Euclidean geometry, is ingenious but too long and tricky for standard courses. Heron can be easily derived from the law of cosines, but here is a simpler way, which gives the law of cosines as a bonus.



$$s = \frac{a+b+c}{2}$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

We're looking for a formula A(a,b,c) for area in terms of side lengths. It has to be symmetric, even, and homogeneous of degree 2. The first try would be a quadratic function, which could only be a non-zero constant times $a^2 + b^2 + c^2$. But the degenerate triangle with sides 1,1,2 has area zero,so that won't work.

The next simplest possibility is the square root of a quartic,

$$\text{area} = \text{const} \times \sqrt{P(a,b,c)},$$

P a symmetric homogeneous quartic. There are two easy ways to identify P, either by the Factor Theorem, or by simple facts about symmetric functions of 3 variables.

To use the Factor Theorem, just remember that a "degenerate" triangle has a + b = c or a + c = b or b + c = a, and area = zero. So as a polynomial in a, P has roots at b + c, b - c, and c - b. So by the factor theorem, it has factors a - b + c, a + b - c, and -a + b + c. Since it is a quartic, it must equal the product of these three times a linear factor, and since it is symmetric, that has to be a constant times a + b + c. All that's left is to evaluate the constant, using any triangle of known area, for instance, one with a = b = c = 1. The constant is 1/16 under the square root sign. To recognize this as Heron, replace s by (a + b + c)/2 in Heron.

The other way is to notice that an even homogeneous quartic has to be a linear combo of the sum of 4th powers and the sum of products of squares. This leaves two constants to be determined. Using, say, one degenerate triangle along with the equilateral, you get -1 and 2 respectively as the coefficients. Of course this expression is just the same as the one you get by multiplying out in Heron. As before, this has to be normalized, with a factor of 1/16.

This is a derivation, not a proof. To verify the derived formula, put in x-y coordinates, with A at (0,0), B at (1,0), and C at (x,y) (y > 0). Then the area is y/2, and the multiplied-out form of the quartic uses only squared lengths, no square roots, so it's not that hard to check.

To go from Heron to the law of cosines, use the elementary formula for area, A = ha/2, and express the altitude h as b sin C. Square both sides, and set this area squared equal to the squared Heron formula, multiplied out. Replace $\sin^2$ by $1 - \cos^2$, and solve for ab cos C.

A student research project could be to find the corresponding formulas on the unit sphere and in the hyperbolic non-Euclidean plane. A final joke is to write a very messy-looking algebraic identity and ask for a one-line proof. (It would just be Heron's formula for a triangle as the sum of Heron's formulas for the two sub-triangles formed by an altitude.)

It is now tempting to look for a three-dimensional analogue of Heron's theorem. Why not a formula fqor the volume of a tetrahedron as a function of the lengths of the six edges? But the lengths of the edges determine neither the shape nor the volume of a tetrahedron.

Counterexample: two right tetrahedra, with equilateral bases. One has base 1,1,1 and slant edges $\sqrt{2}$. Its volume is ($\sqrt{5}$)/6. The other has base edges all $\sqrt{2}$, and slant edges 1. It's rectangular, with volume = 1/6.

Peter Lax has shown me how to transform the determinant formula for the volume to an expression in the lengths of the six sides. This expression depends on which of the four vertices you take as the origin of coordinates.

Reuben Hersh lives in Santa Fe, N.M., and has recently taught at St. John's College, Santa Fe Preparatory School, and the University of New Mexico.

In Memoriam

Cliff Long, 1931–2002

Cliff Long passed away peacefully at home Tuesday August 6, 2002 after a 7 year fight with multiple myeloma. He had been very active in the MAA and the Ohio Section, serving in many leadership capacities. He was on the Board of Governors 1988-91, received the MAA Meritorious Service Award in 1994, was Chair of the MAA Ohio Section 1980-81, and Program Committee Chair 1978-79. He had a near perfect record of attendance at MAA meetings, even when he was well into his illness.

He was born Clifford Allan Long on the south side of Chicago on April 10, 1931. His parents were Canadian. Because of the depression Cliff worked in the stockyards and began attending Wilson Junior College and then the University of Illinois (Navy Pier). After a few years, Cliff moved to the Champaign-Urbana campus. There he obtained his bachelor's degree, master's degree, and finally a doctorate in mathematics, under Pierce Ketchum. Cliff began teaching mathematics at Bowling Green State University (Ohio) in 1959, and he taught there for the next 35 years.

He witnessed the infancy of the computer at the University of Illinois, and followed it through its adolescence while at BGSU, guiding the integration of computers into everyday university life. He was one of many who helped computing reach a certain level of maturity in our time, when the computer seems but a simple and useful appliance in so many homes. He had a special interest in computer graphics, and the visualization of mathematical ideas. He started doing computer graphics when it was really difficult to do. It now seems so easy, but he started doing 3-D computer graphics on a line printer in the 60s. He even had a small computer-controlled milling machine in his office to generate models of 3-D surfaces. It was built by one of his students.

Cliff had an early interest in computer aided design. He was involved with a colleague in early work at Ford Motor Company on Bezier curves and surfaces. That seemed to drive most of his work of the next 30 years. He even taught his milling machine to sign his name to his works with Bezier curves.

Bezier curves are closely related to his interest in linear algebra and its teaching. He was in the linear algebra reform movement before there was such a thing. He and colleagues (but mostly him) started teaching a linear algebra course at BGSU in the 70s which had as its goal the singular value decomposition (SVD) and an emphasis on applications previously found only in numerical analysis courses. This followed the work of Gilbert Strang which has changed the course of linear algebra instruction. He was joined in this work by his son Andy. His *Mathematics Magazine* article on using a digitized model of a bust of Abe Lincoln (which he digitized by hand) to demonstrate the SVD is continually cited.

Cliff Long

Cliff was also a very early pioneer in the calculus reform movement and the introduction of technology in teaching mathematics. He had to push the technology, which meant building a portable cart himself with monitor and Apple II computer to wheel into a classroom, producing super-8 movies, slide sets, milled models, and even a View-master reel of quadric surfaces. All this to try to teach mathematics through visualization. One aspect of the visualization of mathematical ideas that many people appreciated was his sculpting: Cliff expressed his mathematical interests in art works which he carved from wood and stone. Even those petrified of mathematics enjoyed seeing and touching his mathematical sculptures.

Cliff was most committed to serving students through his teaching. Over the years many students paid their respects to him in various ways, and many became mathematicians and teachers.

An expanded version of this article may be found on the Ohio Section page, at http://www.maa.org/Ohio.

Call For Minicourse Organizers

Have you attended a minicourse at one of our national meetings, and found it both informative and enjoyable? Then why not give a minicourse yourself? Even if you have not attended one, perhaps you have an area of interest that you would like to share with persons attending the meetings. Minicourses are four hours in length, and are divided into two parts. Topics vary considerably (just look at the schedule of courses for Baltimore) but should be of interest to those teaching college level mathematics. Courses should be interactive and not designed to promote a specific text or piece of software. For further information, go to the MAA website or contact George Bradley at bradley@duq.edu.