Who Was Miss Mullikin?

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1. INTRODUCTION. In 1946 R. L. Moore, responding to a prospective student's request for information about graduate study, closed his letter with a personal note: "Please remember me to Miss Mullikin" [**18**, p. 295]. Who was Miss Mullikin? And why did someone of Moore's stature wish to convey his best wishes to her?

Anna Margaret Mullikin (1893–1975) was Moore's third Ph.D. student, one of the first mathematicians in the world to write about connected sets, an inspiring high school mathematics teacher, a textbook author, a philanthropist, and a humanitarian. She is virtually unknown today, but for several reasons we believe she deserves greater recognition. For one, she was the first student to write a dissertation on topology under Moore, one of the towering figures of American mathematics who is well known for the teaching method he developed and for the impressive progeny of 50 Ph.D. students he produced by that method. Second, her published work, based on the dissertation, inspired a decade of intense investigations leading to applications and generalizations by two of the leading schools of topology at the time, one in Poland and the other in America. Indeed, it served as a catalyst for subsequent studies for another 50 years. Third, she devoted all her energy to high-school instruction in mathematics, inspiring students to pursue higher education through the doctorate and, in at least one case, to pursue a lifetime of research accomplishments.

In spite of this impressive record, Mullikin is cited in very few works on women mathematicians. She is mentioned neither in older references like [**13**] and [**21**] nor in recent ones like [**9**] and [**25**], even though her contributions were noted in two papers on the Moore school, [**28**] and [**58**]. We do not know why such an omission has occurred, but we hope the present work, in which Mullikin is cast as the central figure and not in a supporting role, reverses this regrettable situation. It has also come to our attention that a very recent book on female American mathematicians includes Mullikin and her achievements [**12**].

This paper begins with an account of Mullikin's only published mathematical research, her doctoral dissertation, which appeared in 1922. We detail those parts of the paper which involve fundamental concepts in $\mathfrak{R}^2$ and are appropriate for present-day students who are extending their conceptual understanding from $\mathfrak{R}$ to $\mathfrak{R}^n$. Next, we trace the catalytic effect her results had on the fledgling field of point-set topology. Called "the theorems of Miss Mullikin," they played an important role in one of the first truly international movements in mathematics in the United States. This analysis of Mullikin's research is followed by an account of her professional career as a high school teacher, citing particular educational influences on her students. The paper concludes with information about Mullikin's life and family.

2. DISSERTATION. Anna Mullikin received her A.B. degree in 1915 from Goucher College, where she displayed promise as a mathematician in her senior year by solving a problem on geometry in this MONTHLY [**11**, p. 166]. Figure 1 displays her photo from the 1915 yearbook. For the next three years she taught mathematics at the Science Hill School (KY) and at the Mary Baldwin Seminary (VA) [**A1**].[1] She arrived in Philadelphia in the fall of 1918 to begin graduate study in mathematics at

[1] This designation refers to the list of archival sources in Section 7 of the present paper.

the University of Pennsylvania (Penn), where she quickly came to Moore's attention and was enrolled in his graduate class. At that time Moore was only at the beginning of his career as a research mathematician and mentor of students. In the first year she discovered a counterexample presented below that became the starting point of her dissertation. In her second year, 1919–1920, she continued in Moore's class and advanced her research but also returned to secondary teaching. In 1920 Moore left the University of Pennsylvania for the University of Texas. There he arranged to have Mullikin appointed as an instructor so she could complete her thesis under his guidance, perhaps the first instance of a practice he continued for twenty-five years until the University insisted that he stop.

Figure 1. Anna Mullikin in the 1915 yearbook *Donnybrook Fair 1915–1916*, courtesy of the Goucher College Archives.

Mullikin's dissertation, "Certain theorems relating to plane connected point sets," appeared in the September 1922 issue of the *Transactions of the American Mathematical Society* three months after she officially received her doctorate. The major theme, as suggested by the title, is to characterize connected sets in the plane $\Re^2$. Sometimes, however, it is apparent that generalization to n dimensions is easily attainable. For instance, Mullikin's opening sentence defines a set M to be *connected* if M "cannot be expressed as the sum of two mutually exclusive point sets neither of which contains a limit point of the other." Today we might express this in the form: M is *connected* if M cannot be written as a union of *separated* sets A and B, meaning that $\overline{A} \cap B = \phi = A \cap \overline{B}$, where $\overline{X}$ denotes the closure of a set X. In general we state results in modern terminology.

Immediately following the definition of connectedness, Mullikin cited an important and recently proved result of the Polish topologist Wacław Sierpiński that served as the impetus for her investigation [**43**]:

Sierpiński's Theorem. *A closed, bounded, connected set M in $\Re^n$ cannot be expressed as a countable union of disjoint closed sets.*

Mullikin wrote: "It will be shown in the present paper that for the case where $n = 2$, this theorem does not remain true if the stipulation that M is closed be removed" [**38**, p. 144]. To accomplish this, she constructed an example of a bounded and connected set in $\Re^2$ that is the disjoint union of a countable collection of closed, bounded, and connected arcs in the following way. For each positive integer n, define an arc M_n to be composed of four line segments drawn from the x-intercept $\left(\frac{1}{2^{n-1}}, 0\right)$ to $\left(\frac{1}{2^{n-1}}, \frac{1}{2^{n-1}}\right)$, thence to $\left(\frac{-1}{2^{n-1}}, \frac{1}{2^{n-1}}\right)$ and $\left(\frac{-1}{2^{n-1}}, \frac{-1}{2^{n-1}}\right)$, and finally to $\left(1, \frac{-1}{2^{n-1}}\right)$. (The abscissa of the final point is always $x = 1$.)

This example can be used instructively today for students in a calculus or analysis course to extend the notion of a limit point from $\Re$ to $\Re^2$. Mullikin stated that each arc M_j contains a limit point of the union of every infinite subset of the collection $\{M_n : n \neq j\}$. Take, for example, M_3, which is composed of the line segments connecting $\left(\frac{1}{4}, 0\right)$ to $\left(\frac{1}{4}, \frac{1}{4}\right)$ to $\left(\frac{-1}{4}, \frac{1}{4}\right)$ to $\left(\frac{-1}{4}, \frac{-1}{4}\right)$ to $\left(1, \frac{-1}{4}\right)$. What point on M_3 is a limit point of the union of the entire collection $\{M_n : n \neq 3\}$? Mullikin's insightful idea was to form the countable disjoint union $M = \cup M_n$, which we will call "Mullikin's nautilus." Students should be encouraged to sketch M; we reproduce Mullikin's drawing in Figure 2, which shows the arcs M_n for $n = 1, 2, 3$. The website "Demos with Positive Impact" illustrates the figure by drawing one arc after another [http://mathdemos.gcsu.edu/mathdemos/mullikin/mullikin.html]. Students who can identify the limit point in question are probably poised to advance from limit points in a metric space to the concept of closure in a general topological space with little difficulty, so it seems appropriate to pose another question: What is the closure of Mullikin's nautilus, $\overline{M}$?

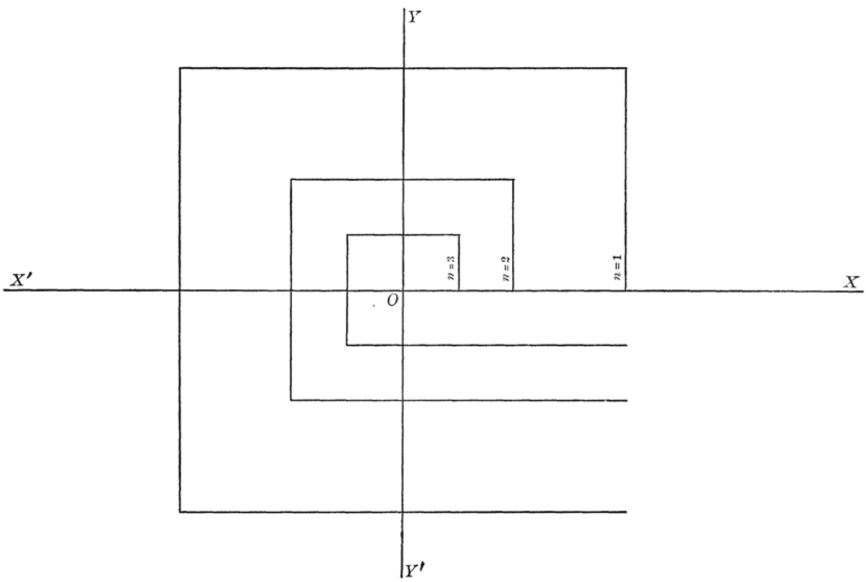

Figure 2. Mullikin's nautilus, Figure 1 in *Trans. Amer. Math. Soc.* **24** (1922), p. 145, printed with permission of the American Mathematical Society.

The set M is obviously bounded. Mullikin concluded that M is also connected by noting that each arc in the nautilus contains a limit point of every subset of M which consists of an infinite number of the remaining arcs. Her simple yet elegant nautilus thus provides the counterexample she sought of a bounded and connected set in $\Re^2$ that is the disjoint union of a countable number of closed and bounded sets. Moreover, it sparked several investigations afterwards. R. L. Moore modified it slightly to obtain additional results related to other theorems due to Sierpiński. (Footnote 2 in [**35**] cites

Mullikin's role.) And a few years later, Moore's student Gordon Whyburn constructed a similar counterexample, as shown in Figure 3.

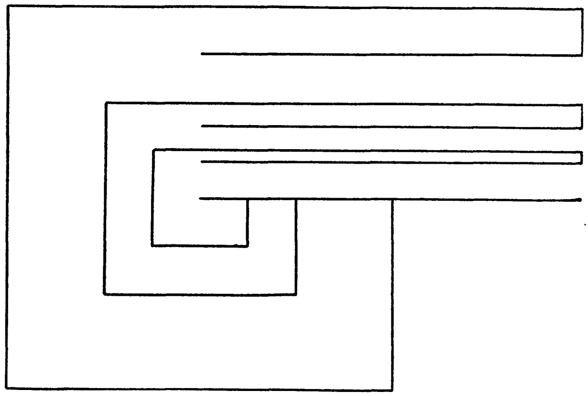

Figure 3. Whyburn's nautilus, Figure 3 in *Trans. Amer. Math. Soc.* **29** (1927), p. 399, printed with permission of the American Mathematical Society.

In typical R. L. Moore theorem-proof style, Mullikin moved directly from the nautilus to the first of five theorems in her dissertation.

Theorem 1. *If K and M are closed and disjoint sets, and H is a closed, bounded and connected set with $H \cap K \neq \phi$ and $H \cap M \neq \phi$, then there exists a connected subset L of H such that $L \cap K = \phi$ and $L \cap M = \phi$ but $\overline{L} \cap K \neq \phi$ and $\overline{L} \cap M \neq \phi$.*

Mullikin's proof relied heavily on two major results from the 1911 dissertation of Zygmunt Janiszewski that was published the next year [**23**].

After proving Theorem 1 by constructing the desired set L, Mullikin moved directly—and without supplying motivation—to her second technical result. Its proof is much shorter than the first. We reproduce the proof with original wording and notation in order to convey a sense of the Mullikin, as opposed to, say, the Bourbaki, style of presentation. Using terms adopted by Sir Michael Atiyah, we might characterize the distinction as tactical (concerned with the minutiae of the argument) as opposed to strategic (involving the development of a theory, an architectural structure) [**4**, p. 87].

Theorem 2. *Let H be a closed and bounded set. If H contains disjoint closed subsets K and M but does not contain a closed, connected subset L with $L \cap K \neq \phi$ and $L \cap M \neq \phi$, then H is the disjoint union of two closed sets, of which one contains K and the other contains M.*

Proof. There exists a positive number ε such that no point of K can be joined to a point of M by a broken line made up of intervals of length less than ε such that the end points of these intervals are points of H. For otherwise there would be a closed, connected "limit set" as in [the proof of] Theorem 1. This limit set would belong to H, since H is closed, and it would contain a point of K and a point of M, since K and M are both closed. This is contrary to the hypothesis.

Now let H_1 denote the point set composed of K together with the set of all points $[P]$ of H such that P can be connected with some point of K by a broken line of intervals of length less than ε such that the end points of these intervals belong to H.

Let H_2 denote the point set composed of all other points of H. H_2 will contain M and it can easily be seen that neither H_1 nor H_2 contains a limit point of the other, since every point of H_2 is at a distance greater than or equal to ε from every point of H_1.

■

Next, Mullikin supplied a crucial definition, stating that a set A *disconnects* (or *separates*) $\Re^2$ if its complement $\Re^2 \setminus A$ is a union of separated sets. For example, the x-axis disconnects $\Re^2$ since its complement is the union of the upper half-plane and the lower half-plane, but none of the arcs M_n in Mullikin's nautilus does because the complement $\Re^2 \setminus M_n$ is connected. With this in mind, Mullikin stated and proved a powerful lemma that applies to these arcs M_n and was used in the proofs of the remaining three theorems in her dissertation.

Lemma. *If a closed set F does not disconnect $\Re^2$, then any two points in $\Re^2 \setminus F$ can be joined by a simple, continuous arc in $\Re^2 \setminus F$.*

Having established these preliminaries, Mullikin was ready to prove two principal results.

Theorem 3. *If a set M is a countable disjoint union of closed sets M_n that do not disconnect $\Re^2$, then M does not disconnect $\Re^2$.*

With this theorem, Miss Mullikin achieved one of the major goals stated in the introduction to her dissertation, to prove that "a plane point set, regardless of whether it be closed or bounded, which separates its plane cannot be expressed as the sum of a countable infinity of closed, mutually exclusive point sets, no one of which separates the plane" [**38**, p. 144]. The next major result in Mullikin's dissertation would live in the literature for another 50 years:

Theorem 4. *If M_1 and M_2 are closed, bounded, connected sets which do not separate the plane, then $M_1 \cup M_2$ will separate the plane if and only if $M_1 \cap M_2$ is not connected.*

The proofs of Theorems 3 and 4 required elaborate constructions and a deep analysis that constituted almost two-thirds of the *Transactions* paper. The power of the Moore Method is seen in the techniques that Mullikin discovered for these proofs. For instance, she generalized a result from Hausdorff's classic book on topology that had only appeared, and in German, a few years before [**20**] and she drew heavily upon three theorems from Moore's groundbreaking 1916 paper [**36**]. Theorem 4 would ultimately be called the Mullikin-Janiszewski Theorem, the Janiszewski-Mullikin Theorem, or simply J.M.T. after it was determined that Janiszewski had published it first, though, as we detail below, in a journal inaccessible to the Americans [**22**].

A footnote in the dissertation states, "Various parts of this paper were presented to the Society on October 25, 1919, December 28, 1920, and February 26, 1921" [**38**, p. 144]. All three meetings were held at Columbia University but Mullikin attended only the one in 1920. Moore read her paper at the 1919 meeting. AMS secretary F. N. Cole reported, "In one dimension no countably infinite collection of mutually exclusive closed point sets ever has a connected sum. One might rather naturally be inclined to believe that this proposition holds true also in two dimensions. Miss Mullikin shows by an example that this is, however, not the case" [**10**, p. 147]. Surely, the example presented at that meeting is Mullikin's nautilus. She must have returned home from

Austin during the holiday break in 1920 to deliver her first lecture at the Society's annual meeting. The title of the lecture, "Certain theorems concerning connected point sets," almost matches the title of the dissertation. The abstract in the secretary's report on the meeting indicates that Mullikin established Theorems 1 and 3 but it did not mention Theorem 2 or the Lemma [**41**, pp. 248–249]. Moreover, the presentation of Theorem 3 required that M be closed, a stipulation that Mullikin was able to eliminate shortly.

Theorems 4 and 5 were presented as one entity at the AMS meeting held two months later [**40**, p. 349]. This means that Miss Mullikin had completed her dissertation before leaving Austin. The final result in the dissertation reads:

Theorem 5. *Let M_1 and M_2 be closed, bounded, connected subsets of $\Re^2$, neither of which disconnects $\Re^2$. If $M_1 \cap M_2$ [is not connected but] is the disjoint union of two connected subsets, then $R^2 \setminus (M_1 \cup M_2)$ is the disjoint union of precisely two connected sets.*

Since Mullikin assumes here that $M_1 \cap M_2$ is not connected, it follows by Theorem 4 that $R^2 \setminus (M_1 \cup M_2)$ is the disjoint union of a collection of connected and separated sets. Therefore the essence of the proof of Theorem 5 was to demonstrate by contradiction that there cannot be more than two sets in the collection.

Neither Moore nor Mullikin was present at the 1921 meeting, so her paper was read by title: "A necessary and sufficient condition that the sum of two bounded, closed and connected point sets should disconnect the plane." Moore too submitted a paper that was read by title and dealt with issues very similar to those Mullikin presented at the annual meeting two months beforehand.

3. MATHEMATICAL LEGACY. When her instructorship at the University of Texas came to an end at the close of the academic year 1920–1921, Anna Mullikin returned to the University of Pennsylvania to complete requirements for her degree. The public defense of her dissertation was held in January 1922, so she officially received her degree from Penn that June, one of 19 Ph.D. recipients in mathematics in the U.S. that year [**8**]. It took some effort to get the dissertation published in the desired outlet because the *Transactions* frowned upon the submission of dissertations. Aware of this policy, J. R. Kline asked R. L. Moore to attest that it was "a piece of work of first grade" [**A2**]. Moore essentially defined the qualification for the Ph.D. as doing publishable work so he happily complied at once. Upon receipt, Kline replied, "I would be very glad to read Miss Mullikin's paper and would give it prompt attention" [**A3**]. The dissertation was accepted soon thereafter.

We have been unable to locate any reference to the paper the following year, but its impact on researchers became dramatically apparent in 1924 with a half-dozen references. Over the next several years, the dissertation would serve as a catalyst for international cooperation and competition between the schools of topology in the U.S. and Poland. We trace the impact of the dissertation in four distinct periods: 1924 (initiation), 1925–1929 (spread), 1929–1932 (crowning period), and 1952–1973 (modern generalizations). An account of the origin and early impact of the Moore Method briefly mentioned the influence that Miss Mullikin exerted on topology in the 1920s but it omitted many of the works discussed here [**58**]. Our development below fills these lacunae.

The initial period (1924) witnessed five papers by Moore and Kline, as well as one by the Polish mathematician Stefan Mazurkiewicz that included her name in the title, "Remarque sur un théorème de M. Mullikin" [**29**]. After stating her Theorem 3, Mazurkiewicz wrote, “I propose to give a new proof of this remarkable theorem based on several results on closed sets” [our translation]. His one-paragraph proof relied on a recent result of Kazimierz Kuratowski and Bronisław Knaster that had appeared in an earlier issue of the embryonic journal *Fundamenta Mathematicae* (*FM*), which had been established in 1919 to provide an outlet devoted exclusively to the theory of sets and its applications. Of course Moore and Kline were aware of Mullikin’s work earlier, but Mazurkiewicz’s citation suggests that the *Transactions* was becoming international.

All three Warsaw mathematicians primarily involved with founding *FM* played important roles in Mullikin’s legacy. We have already mentioned Zygmunt Janiszewski (1888–1920), who had recently died in the influenza pandemic of 1918–1919. The editors of the initial volume in 1920 were Stefan Mazurkiewicz (1888–1945) and Wacław Sierpiński (1882–1969).

The second issue of *FM* from 1924 contained not only the pivotal paper by Mazurkiewicz but an important contribution by R. L. Moore that drew upon Theorem 3 once, used Theorem 4 twice, and singled out Theorem 5 [**33**]. This work contains a telltale footnote referring to Theorem 4 as due to Janiszewski. Moore always insisted on giving credit to the original discoverer of a theorem, although in this case he vigorously defended Mullikin’s independent discovery by asserting, “A proposition which is a logical consequence of these theorems of Janiszewski’s has been recently established by Miss Anna M. Mullikin in her Doctor’s dissertation, which will appear soon in the *Transactions*... This paper had gone to the printers before either Miss Mullikin or I was aware that the proposition had already been proved. Apparently Janiszewski’s paper is printed in Polish” [**33**, p. 190, footnote 2]. Moore had still not seen Janiszewski’s paper by September 28, 1923, when he submitted his own paper to *FM*. (That date is revealed in footnote 4, p. 170, of [**30**]. His source for Janiszewski’s results was a recent paper by the Warsaw mathematician Stefan Straszewicz in *FM*.) But once Moore was aware of Janiszewski’s priority he insisted on referring to Theorem 4 as Janiszewski’s Theorem, as did his later students. These citations provide an early glimpse into the evolving, and ultimately symbiotic, relationship between the schools of topology in Warsaw and Austin. Moore also urged the reader to consult Mullikin’s Theorem 5, pointing out that it was presented at an AMS meeting in 1921. The essential result in Moore’s paper—Theorem 11, whose proof constituted almost half of it—was in the spirit of Mullikin’s Theorem 3, stating that the plane contains no unbounded continuum that is a disjoint union of a countable number of continua.

Besides these two papers in *FM*, three works by Moore and another by Kline made use of Mullikin’s dissertation in the *Proceedings of the National Academy of Sciences*, with Kline’s following two by Moore in the May 15, 1924, issue. Like Mullikin’s investigation, Moore’s two-page note [**30**] was inspired by the same Sierpiński work of 1918 [**43**]. In it, Moore proved one result, another in the spirit of Mullikin’s Theorem 3: if a closed and bounded set M is the disjoint union of a countable number of closed and connected sets M_n, then some M_j does not contain a limit point of the union of the set $\{M_n : n \neq j\}$. At the close of the proof he noted that his student’s nautilus example demonstrated that this result was false if M is not assumed to be closed; he then constructed an example to demonstrate that it was false if the M_n were not assumed to be connected. Moore concluded the paper by supporting his contention that his discovery and proof were carried out independently of Mazurkiewicz, who had stated and established it that year, declaring, “I submitted my proof for publication... Sept. 28. Sometime in November I received the reprint of the article by Professor Mazurkiewicz” [**30**, p. 170]. Moore’s second *Proceedings* article used Mullikin’s Theorem 4 twice, calling it “a theorem of Janiszewski’s” [**31**, p. 170]. In the very next paragraph, however, he relied on “a theorem of Miss Mullikin’s,” namely Theorem 5. In the next paper in that issue, J. R. Kline made crucial use of Theorem 4 in the proof of one of two major results he announced [**26**]. The remaining Moore paper from 1924 made a slight modification of Mullikin’s nautilus and relied on her Theorem 4 [**35**].

During 1925–1929, Mullikin’s theorems were used as tools in proofs in at least eleven papers by Moore, his students Wilder and Whyburn, and W. A. Wilson, who was not associated with either the Austin or Warsaw schools. Moore relied heavily on her Theorem 2 twice in [**32**] and again in [**37**], while [**34**] cited Theorems 4 and 5 at critical junctures in proofs.

Moore’s first student at Texas, Raymond L. Wilder (1896–1982), came to Austin in 1921 after having earned a master’s degree at Brown. His 1923 dissertation developed some of Mullikin’s ideas and, like her, he presented his findings at more than one AMS meeting. Wilder’s third section provided a new characterization of continuous curves in any dimension, while he also drew upon a generalization of Mullikin’s Theorem 1 by finding a subcontinuum Q of a continuum M in place of Mullikin’s connected subset L of the closed, bounded and connected set H [**50**]. Mysteriously, he listed the year of her paper as 1923, a mistake repeated by several later writers. In connection with the continuing effort of Austin mathematicians to address priority issues with their Warsaw counterparts, Wilder acknowledged that, “In his article, Mazurkiewicz establishes numerous results and indicates that some of them were published in a journal to which I have not had access” [**50**, p. 344]. Theorem 1 became a prime component in Wilder’s proofs over the next few years ([**48**, pp. 334 and 339], [**49**, p. 620], and [**52**, p. 351]).

Two other young mathematicians drew upon Theorem 1 at about this time. One was Gordon Whyburn (1904–1969), R. L. Moore’s third doctoral student at Texas but first native Texan. His dissertation, which, like Mullikin’s, was published in the *Transactions*, made critical use of Theorem 1 on four separate occasions by asserting the existence of a connected set with certain properties [**47**]. We already referred to this paper for its nautilus-like figure. The first person outside the Austin-Warsaw axis to make use of Mullikin’s work was Wallace Alvin Wilson (1884–1948), who used Theorem 1 to prove a fundamental lemma that was central to the proof of his own major theorem in a paper published in the *Annals of Mathematics*, the only time her dissertation was cited in that journal [**53**]. Wilson went on to a long career at Yale.

By the time of the crowning period, 1929–1932, Mullikin’s results were so familiar that Wilder could introduce them without attribution, using the phrase “it is well known that” [**51**, p. 40]. But the biggest boost in this period came in the form of yet another thesis, this one by a member of the extended Moore school, Leo Zippin (1905–1995), who had won Penn’s Freshman Entrance Prize in 1923. Zippin was soon proving theorems in Kline’s Foundations of Mathematics course taught in Moore-Method fashion, culminating in the dissertation, “A study of continuous curves and their relation to the Janiszewski-Mullikin Theorem.” This was the first time Mullikin’s name earned equal billing with Janiszewski for Theorem 4; Moore, Wilder, and Whyburn always deferred to Janiszewski. The ongoing work of J. R. Kline’s students was delivered at AMS meetings, much like Moore’s, with Zippin presenting his on three occasions. The published version of his dissertation began, “In this paper... [the] principal theorems are devoted to the relation of such curves to the Janiszewski-Mullikin Theorem,” and a footnote added, “The theorem is readily seen to obtain on the surface of the sphere, from the manner of its proof in the plane” [**54**, p. 744].

Zippin is a prime example of the close cooperation among Moore disciples at Texas and Pennsylvania. In 1929, just before Zippin received his doctorate, Kline wrote to Moore, “Zippin has applied and asked to come to Texas to work with you” [**A4**]. So Zippin spent the ensuing year in Austin as a National Research Fellow. While there he began to refer to Theorem 4 as J.M.T. [**56**]. In two papers from 1932 Zippin continued to extend the J.M.T. to more general domains, with [**55**] extending to any $\Re^n$ a result due to Clark M. Cleveland, who had obtained his Ph.D. under Moore two years earlier. Zippin's other paper suggested that Theorem 4 had become commonplace in a considerably more abstract setting: "This is a simple theorem which we have had occasion to prove for locally compact continuous curves" [**57**, p. 709].

Also in 1932, another academic grandson of R. L. Moore, Edwin Wilkinson Miller (b. 1905), contributed to the Mullikin legacy by making essential use of her Theorem 2. Miller had received his doctorate in 1930 under R. L. Wilder, one of the 25 Wilder produced at the University of Michigan, and had joined W. A. Wilson at Yale. Then in 1935 the inaugural issue of the *Duke Mathematical Journal* featured a paper by E. R. van Kampen, who mentioned Mullikin tangentially by noting that characterizations of the 2-sphere lean heavily on some form of the Jordan curve theorem or the J.M.T. [**46**]. However, no direct mention of Mullikin would occur for another 17 years. And when it did, the centerpiece was Theorem 3, not the Janiszewski-Mullikin Theorem.

The final period of influence of Mullikin's 1922 dissertation begins in 1952 and ends in 1973 with two papers in *Fundamenta Mathematicae*, the journal that had played such a vital role in the early dissemination of her results. In 1952 W. T. van Est (of Utrecht) wrote, "Recently the simplification and modernization of Miss Mullikin's proof of [Theorem 3] was proposed as a problem by Wiskundig Genootschap at Amsterdam (apparently in ignorance of Mazurkiewicz's proof). The present author succeeded in giving such a proof and at the same time generalized Miss Mullikin's theorem for *n* dimensions (equally ignorant of Mazurkiewicz's article)" [**45**, p. 179]. The last statement in the paper, "The proof of Miss Mullikin's theorem utilizing this homology concept remains verbally the same," indicates the extent to which recently discovered methods had been brought to bear. Van Est's paper raised the question of finding a class of unicoherent Peano spaces in which Theorem 3 holds. It was answered in 1973 by E. D. Tymchatyn and J. H. V. Hunt (of Saskatoon) [**44**]. The authors of these two *FM* papers probably learned of Mullikin's work from Wilder's 1925 paper because they repeated his error of the year of publication. Two papers in the interim show Mullikin's continuing influence. In 1963, A. Glen Haddock stated a result with a one-line proof: "This theorem follows in a straightforward manner from the Janiszvoski-Mullikin [sic] theorems" [**17**, p. 636]. Two years later F. Burton Jones (1910–1999), one of R. L. Moore's most influential graduates, wrote that even though the foregoing 30-year period had seen much progress on characterizations of a 2-manifold of the Jordan-curve-theorem type, "no such progress has been made on results of the second [Janiszewski-Mullikin] type. The purpose of this paper is to initiate this progress" [**24**, p. 497].

Having elucidated Miss Mullikin's sole publication and the durable life of her Theorems 3 and 4, we provide a fuller picture of her character by describing in the next two sections her career and some aspects of her personal life.

4. TEACHING CAREER. When Mullikin finished her Ph.D. in 1922 she went to work for the School District of Philadelphia as a teacher of secondary mathematics. She already had several years of teaching experience. After her graduation from Goucher she had gone to the Science Hill School, a private prep school for girls in Shelbyville, KY, for two years. Then she moved for one year to the Mary Baldwin Seminary, then a women's junior college in Staunton, VA, where she was the only full-time teacher of mathematics. She took a year off from teaching during her first year at Penn but in 1919–1920 she joined the faculty at the Stevens School, a private girls' prep school in the Germantown section of Philadelphia. The following year she was in Texas teaching undergraduates, and in 1921–1922 she taught at the Oak Lane Country Day School, a private elementary school in the Philadelphia suburbs. Her first assignment in the Philadelphia School District was at the William Penn High School for Girls. After one year she was transferred to a coeducational academic high school, Germantown High, where she remained until she retired in 1959. During this 36-year tenure she earned a reputation as a demanding, sympathetic, and effective teacher of mathematics. Figures 4 and 5 picture Miss Mullikin in a high-school yearbook from 1945, after she had switched her first name to Anne.

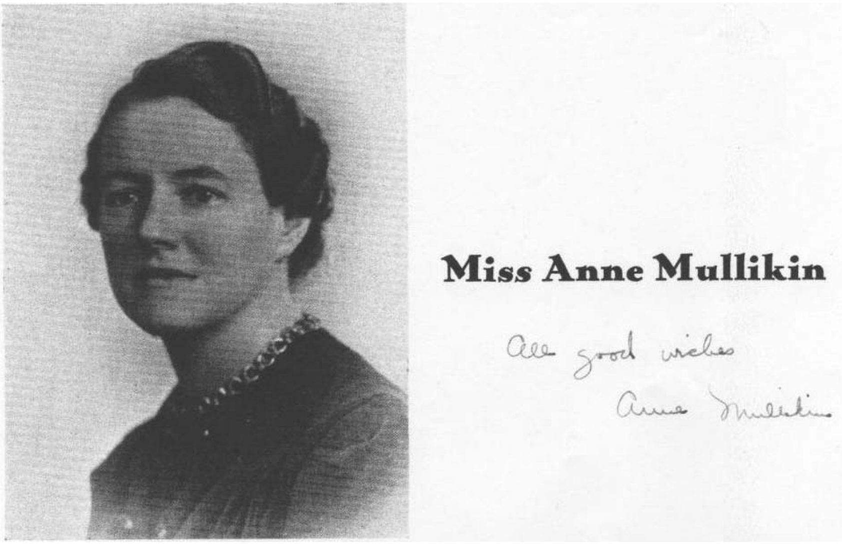

Figure 4. Anna M. Mullikin, Germantown High School Yearbook, January 1945.

Figure 5. Anna M. Mullikin, Germantown High School Yearbook, January 1945.

It also appears that after World War II she taught veterans who were filling colleges under the GI bill. Her report to the Goucher College Alumnae Association in 1949 mentions that she had been teaching part-time at "area colleges of the Commonwealth of Pa.," although we have not been able to determine which colleges these were or what she taught.

In 1952 Miss Mullikin was appointed Head of the Mathematics Department at Germantown High and a year later became "chairman of the Mathematics Advisory Committee—a committee which determines the curriculum with respect to mathematics in the Junior and Senior High Schools" [**A1**]. During this time she became a textbook author, publishing *Algebra and Its Use*, *Book 1* and *Book 2*, written with Ethel L. Grove, who had taught at Cuyahoga High School in Cleveland, and Ewart L. Grove of the University of Alabama [**14**], [**15**]. *Book I* moves very carefully through different types of equations from $x + a = b$ to quadratic equations with, according to the introduction, balance and equation as the "keynote." It is very much rule driven, with prescriptions followed by long exercise sets, the basic teaching strategy being practice in class followed by homework problems. There is no mention of commutativity, associativity, or distributivity. Each variant on a type of equation has its own prescription with little indication of common features. One innovation is student-written word problems, p. 131. After Mullikin retired in 1959 she completed another book with the Groves, *Basic Mathematics*, published in 1961 [**16**].

In 1954 Goucher College held a science conference in conjunction with the dedication of the new Hoffburger Science Building. Several Goucher alumnae who had achieved distinction in the sciences were honored. The citation for Mullikin reads [**3**]:

> Originator of the Mullikin theorem in point-set topology in the plane, inspiring and beloved teacher in the Germantown High School of Philadelphia, by your ability, patience, good humor, and enthusiasm you have imparted to hundreds of students a sure knowledge of elementary mathematics and its significance in modern life and thought. From the richness of your own mind you have developed in many of these young people the beginning of an appreciation for mathematical ideas.

Some of R. L. Moore's later students came to understand that he was "heartbroken" by Mullikin's decision to "waste her talents" as a high-school teacher [**39**, p. 242]. Jeanne LaDuke has written that some American women who earned a Ph.D. in mathematics in the early part of the twentieth century would "emphasize teaching—because they want to or because they are pushed in that direction" [**13**, p. xix]. Was Mullikin pushed or did she gladly choose to teach secondary mathematics?

Mullikin's own education at a women's college and her first three years teaching in women's institutions may have led her to envision herself as an educator of young women. She might have decided to pursue the Ph.D. to qualify for positions at women's colleges, which provided almost the only employment opportunities for scientific women in the early part of the twentieth century [**42**]. If so, she had changed her mind by the time she finished her degree in 1922, and was exploring positions in China. J. R. Kline wrote to R. L. Moore in September 1922, "Miss Mullikin is teaching in some girls high school in Philadelphia... Her China job did not materialize" [**A5**].

It is beyond the scope of this paper to attempt a conceptualization of the careers of American women Ph.D. mathematicians, but it is germane to compare Mullikin's career path with those of the other three women who received their degrees in 1921–1922. Margaret Buchanan (1885–1965) graduated from West Virginia University before enrolling at Bryn Mawr College, where she wrote her dissertation under Anna Pell Wheeler. She then returned to her *alma mater* and taught there for the rest of her career. A West Virginia alumni award is named in her honor. Claribel Kendall (1889–1965) received her bachelor's degree from the University of Colorado in 1912 and joined the faculty the next year while pursuing a master's degree, which she received in 1914. She attended the University of Chicago during summer sessions and ended up writing her dissertation there under Ernest Wilczynski. She remained at Colorado for the rest of her career. Eleanor Pairman (1896–1973) was a human computer in Karl Pearson's Department of Applied Statistics at the University of London before enrolling at Radcliffe College, where she received her Ph.D. under G. D. Birkhoff. She and fellow graduate student Bancroft Huntington Brown were married in the same year their degrees were conferred. Mr. Brown was appointed to the faculty at Dartmouth College where Mrs. Brown found that she shared research interests with Rudolph E. Langer, with whom she published a joint paper [**27**]. In these two respects Pairman seems to be an exception among our four women; she is the only one who married and the only one who published anything other than her dissertation. The others, like Mullikin, remained single and embarked on teaching careers, albeit at their home universities.

We believe Miss Mullikin was drawn, not pushed, into secondary teaching. Her decision to teach in 1919–1920 and 1921–1922, when she had fellowship support from Penn, suggests that she enjoyed teaching. The reports of her students confirm that she took pleasure from their successes. She also had the model of her two older sisters who were both teachers in the Baltimore schools [**1**], [**2**]. Confirmation that Mullikin made a free choice comes from her student and friend Mary-Elizabeth Hamstrom, who attended Germantown High School from 1941 to 1944. Two years later, as an undergraduate at Penn, she wrote to Moore for information about his program at Texas. His reply closes with the request to extend greetings to Miss Mullikin quoted at the beginning of this article. Hamstrom went on to take a Ph.D. from Moore at Texas and to become a distinguished research topologist at the University of Illinois. She has written to us that Mullikin found that "doing research was much too hard work for her tastes" and that Mullikin chose to teach in public schools because the pay was better than in private schools and colleges [**19**].

The citation quoted above from the 1954 Goucher Science Conference, probably written by Hamstrom, who was then on the Goucher faculty, praises Mullikin for her "patience, good humor, and enthusiasm." In a solid geometry class at Germantown H. S. in 1942 Hamstrom was challenged and encouraged by Mullikin. She "stimulated good students and was patient and pleasant... She was really pleased when a student solved a hard problem and showed it with a broad smile. She had a good supply of problems—many hard—for good students" [**19**].

Other students agree. Three from the class of January 1945 have written to us. Earle W. Barber, Jr. recalls that students and faculty alike regarded her as one of the best teachers in the school, certainly the best in the mathematics department. She "treated all her students with respect" but "did not forgive those who did not do their homework." At Germantown each class had a dual slate of officers, a young man and a young woman for each office, and two faculty advisors, one male and one female. For his class Barber was the male class president and Mullikin was the female advisor. He says that all the class officers found Mullikin more approachable and supportive than her male colleague but "she never let me get away with any shenanigans" [**5**]. Wallis D. Bolton recalls Mullikin as "gentle but firm... warm and friendly" [**6**]. He reports that she worked hard to develop mathematical competence in all her students but "took special pleasure in challenging those members [of her class] who were math-bright." He supports Barber's remark that she did not forgive slackers. Although he was one of the bright students and generally rose to Mullikin's challenge to earn an A, when he took a part-time job just before graduation and neglected his homework, he received a grade of C. James Buck discloses that Mullikin had some of the talent-spotting ability for which her advisor Moore was famous. In tenth grade he was assigned to an algebra class for students of low ability. But Mullikin recognized that he could do better. "Each day she gave me separate, more complex homework assignments.... Miss Mullikin challenged me everyday...and brought me to being a straight A student in my math classes.... How she ever picked me out as having ability, I don't know" [**7**]. But it changed his life; he went on to a career in the new computer industry, working at IBM.

5. LIFE AND FAMILY. In this section we indicate something of Anna Mullikin's personal life and character. Miss Mullikin was born in Baltimore, MD, on March 7, 1893, the youngest of four children of William Lawrence Mullikin (1846–1915) and Sophia Ridgely Battee Mullikin (1854–1921), who were descended from seventeenth-century Scottish and English immigrants to the Chesapeake Bay area. She had two sisters, Mary Hester Mullikin (1884–1947) and Caroline (Carrie) Battee Mullikin (1890–1969), and a brother, Richard Nicholas Mullikin (1888–1945). The sisters graduated from Goucher College in 1907 and 1913, respectively, and became mathematics and science teachers in the public schools of Baltimore. Her brother earned an A.B. and a Ph.D. in chemistry from Johns Hopkins University and worked as a chemist for the Dupont Corporation. He was the only one of the siblings to marry and have children.

When Mullikin was assigned to Germantown High School in 1923, she became a resident of the Germantown section of Philadelphia. She lived in rented rooms and apartments for many years before purchasing a home in 1944 just a block from the school [**A6**]. In 1962 she was diagnosed with arteriosclerosis and had to curtail her activities. She lived the remaining thirteen years of her life in nursing homes.

Mullikin had a variety of interests outside mathematics. In college she managed her class basketball team and was on the swimming team and the student governing board. As an adult she enjoyed walking, gardening, and raising dogs. Beyond that we know little about her friendships and activities but there are two stories that seem to reveal something of Anna Mullikin's character.

One concerns volunteer work she did during World War II. The Alumnae Records Collection in the Penn Archives contains only one item for Anna Mullikin, a postcard dated 1943 on which she reported her address, her place of employment, and the cryptic remark that she was doing "work at a control center under civilian defense" [**A7**]. In May 1941, with war raging in Europe and U.S. involvement anticipated, Philadelphia set up a Council of Defense to establish and coordinate many public programs to support the war effort. A secret program, disclosed only late in 1944 when victory in Europe seemed assured, consisted of six control centers in various locations around the city. Each center was able to call on a technical staff trained for police, fire, and emergency work and each was staffed around the clock by volunteer "telephonists," working in four-hour shifts [**A8**]. Harry S. McDevitt, Chairman of the Council of Defense, wrote at the end of 1944, "Too much cannot be said of the faithful but never seen volunteers who served in obscurity, mostly in basement Control Centers" [**A9**]. It seems that Anna Mullikin was one of these faithful but obscure telephonists but we do not know where she did this work or how much time she devoted to it.

The other story is one of philanthropy. Early in 1956 Miss Mullikin transmitted to the senior minister of the First Methodist Church of Germantown funds to establish the Julia Morgan Memorial Fund. Julia Morgan (1891–1948) was already a medical student at Penn when Mullikin arrived in 1918 to study mathematics. They met and became friends shortly thereafter. At the same time that Mullikin finished her Ph.D., Morgan completed her residency at the University of Pennsylvania hospital. She enrolled with the Women's Foreign Missionary Society of the Methodist Episcopal Church and went to China to serve in Christian hospitals as a physician and professor of medicine [**A10**]. She returned to the United States in 1942 and became a professor of medicine at Penn until her death in 1948. Eight years later Mullikin established the Julia Morgan Memorial Fund to support overseas medical missionaries. Shortly before her death she broadened its purposes so that it is now used to support a variety of mission activities of the Church.

Miss Mullikin suffered a cerebral thrombosis in June 1975 and succumbed on August 24 at age 82.

6. CONCLUSION. Anna Margaret Mullikin may be remembered as a mathematician, a teacher, a philanthropist, and a humanitarian. In mathematics, she was one of the earliest American researchers in point-set topology, advancing knowledge of connectedness in the plane and characterizations of 2-manifolds in terms of connected sets, and fostering an international collaboration among topologists in Poland and the United States. Although her only research was the doctoral dissertation, resulting in one published paper, it contained such deep results that all five theorems found application in subsequent investigations lasting another 50 years.

As a high-school teacher Mullikin served as an excellent mentor and role model for all her students during a 40-year teaching career. With a combination of firmness, patience, and kindness, she guided them both in mathematics and in the art of living. She identified and encouraged students of strong mathematical ability, taught a meticulous and orderly approach to mathematics to all her students, and tailored her lessons to the abilities of individual students. Although her pupils were unaware of her earlier exploits and some of them did not even know that she held a Ph.D., they benefited by experiencing firsthand a brilliant and serious mathematical mind at work. Perhaps this was especially true for her female students, but all her students remember her with admiration and respect.

As a person Miss Mullikin was generous of spirit and purse, leaving a financial legacy which continues to benefit humanity. Although all of her immediate family died before she did, some of them prematurely, and her own health was poor for several years, she maintained an optimistic outlook, welcoming visitors graciously. Firm in her sense of how to do mathematics, how to teach, and how to live, she was patient, modest, and considerate in her relations with others.

7. ARCHIVAL SOURCES. In addition to published items, the authors have drawn material from a number of important archival sources:

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