Abstracts for the MAA Undergraduate Poster Session

Baltimore, MD
January 17, 2014

Organized by

Joyati Debnath
Winona State University

MAA

Organized by the MAA

Committee on Undergraduate Student Activities and Chapters

and

CUPM Subcommittee on Research by Undergraduates

MAA gratefully acknowledges
the generous support of the sponsor
of this year's Poster Session.

National Science Foundation

Dear Students, Advisors, Judges and Colleagues,

If you look around today you will see about 312 posters and 512 presenters, record numbers, once again. It is so rewarding to see this session, which offers such a great opportunity for interaction between students and professional mathematicians, continue to grow.

The judges you see here today are professional mathematicians from institutions around the world. They are advisors, colleagues, new PhD.s, and administrators. We have acknowledged many of them in this booklet; however, many judges here volunteered on site. Their support is vital to the success of the session and we thank them.

We are supported financially by the National Science Foundation.

Our online submission system and technical support is key to managing the ever-growing number of poster entries we receive. Thanks to MAA staff, especially Annie Baer, Maia Henley, and Jason Leibert for their work setting up and managing the system this year. Preparation of the abstract book is a time-consuming task. Thanks to Beverly Ruedi for doing the final production work on the abstract book.

There are many details of the poster session that begin with putting out the advertisement in FOCUS in February, ensuring students have travel money, and organizing tables in the room we are in today that are attributed to Gerard Venema (MAA Associate Secretary), Linda Braddy (MAA), and Donna Salter (AMS).

Zsuzsanna Szaniszlo (Valparaiso University) rallied volunteers to judge the session and coordinated the judge assignments. Angel R. Pineda (California State University, Fullerton), James P. Solazzo (Coastal Carolina University), and Dora Ahmadi (Moorehead State University) organized an orientation for the judges and authored the judging form. Thanks to all the students, judges, volunteers, and sponsors. I hope you have a wonderful experience at this year's poster session!

Joyati Debnath
Winona State University

Judges and Judge Affiliations

1. Edward Aboufadel, *Grand Valley State University*
2. William Abram, *Hillsdale College*
3. Alan Alewine, *McKendree University*
4. Sarah Anderson, *Clemson University*
5. Jeff Angel, *Department of Defense*
6. Kodwo Annan, *Minot State University*
7. Ted Ashton, *USG*
8. Abdellahi Aw, *Morgan State University*
9. Nicholas Baeth, *University of Central Missouri*
10. A. Bass Bagayogo, *Saint-Boniface University, Canada.*
11. Eric Bancroft, *Grove City College*
12. Padraic Bartlett, *UCSB*
13. Rachel Bayless, *Agnes Scott College*
14. Scott Beaver, *WesternOregon University*
15. Ghanshyam Bhatt, *Tennessee State University*
16. Eric Bone, *Department of Defense*
17. Matt Bowen, *U.S. Government*
18. Bernadette Boyle, *Sacred Heart University*
19. David Bressoud, *Macalester College*
20. Jim Brown, *Clemson University*
21. Kurt Bryan, *Rose-Hulman Institute of Technology*
22. John Burke, *Rhode Island College*
23. Jane Butterfield, *University of Minnesota*
24. Kristin Camenga, *Houghton College*
25. Naiomi Cameron, *Lewis & Clark College*
26. Alex Capaldi, *Valparaiso University*
27. Carmen Caprau, *California State University, Fresno*
28. Lori Carmack, *Salisbury University*
29. Zhixiong Chen, *New Jersey City University*
30. Tien Chih, *University of Montana*
31. Emily Cilli-Turner, *Salve Regina University*
32. Karen Clark, *The College of New Jersey*
33. David Clark, *University of Minnesota*
34. Steven Collazos, *University of Minnesota – Twin Cities*
35. Randall Cone, *VMI*
36. Daniel Cranston, *Virginia Commonwealth University*
37. Susan Crook, *Loras College*
38. Daniel Daly, *Southeast Missouri State University*

39. Mike Daven, *Mount Saint Mary College*
40. Courtney Davis, *Pepperdine University*
41. Louis Deaett, *Quinnipiac University*
42. Steven Deckelman, *UW-Stout*
43. John Diamantopoulos, *Northeastern State University*
44. Yi Ding, *New Jersey City University*
45. Elizabeth Donovan, *Bristol Community College*
46. Suzanne Doree, *Augsburg College*
47. Tim Dorn, *DOD*
48. Jill Dunham, *Hood College*
49. Justin Dunmyre, *Frostburg State University*
50. Joe Eichholz, *Rose-Hulman Institute of Technology*
51. Griff Elder, *University of Nebraska at Omaha*
52. Jennifer Ellis Royal, *Mercer University*
53. John Engbers, *Marquette University*
54. Frank Enstein, *The Tower*
55. Malena Espanol, *The University of Akron, Department of Mathematics*
56. Julius Esunge, *University of Mary Washington*
57. Eleanor Farrington, *Massachusetts Maritime Academy*
58. Janet Fierson, *La Salle University*
59. Brian Fisher, *Pepperdine University*
60. Breeann Flesch, *Western Oregon University*
61. Sharon Frechette, *College of the Holy Cross*
62. Andrew Gainer-Dewar, *Carleton College*
63. Joe Galante, *DOD*
64. Pete Geoghan, *DOD*
65. Whitney George, *West Chest University of PA*
66. Kris Green, *St. John Fisher College*
67. Lynn Greenleaf, *Stephen F. Austin State University*
68. Arthur Guetter, *Hamline University*
69. Thomas Höft, *University of St. Thomas*
70. Spencer Hamblen, *McDaniel College*
71. Muhammad Hameed, *University of South Carolina Upstate*
72. James Hartman, *The College of Wooster*
73. Nancy Heinschel, *DOD*
74. David Hendricks, *Abilene Christian University*
75. Edwin (Jed) Herman, *University of Wisconsin-Stevens Point*
76. William Higgins, *Wittenberg University*
77. Firas Hindeleh, *Grand Valley State University*
78. Joshua Holden, *Rose-Hulman Institute of Technology*

79. Matt Holzer, *George Mason University*
80. Larissa Horn, *DOD*
81. Debra Hydorn, *University of Mary Washington*
82. Lynne Ipina, *U of Wyoming*
83. Krishna Kaphle, *UMFK*
84. Mitchel Keller, *Washington & Lee University*
85. Annela Kelly, *Bridgewater State University*
86. Noureen Khan, *University of North Texas at Dallas*
87. Joel Kilty, *Centre College*
88. Klay Kruczek, *Southern CT State University*
89. Alicia Latten, *Morgan State University*
90. Joe Latulippe, *Norwich University*
91. Mike Lauzon, *DOD*
92. Shirley Law, *Washington College*
93. Glenn Ledder, *University of Nebraska-Lincoln*
94. Jessie Lenarz, *St. Catherine University*
95. Jessica Libertini, *University of Rhode Island*
96. Glenn Lilly, *DOD*
97. Sean Lynch, *University of Illinois at Chicago*
98. Yanping Ma, *Loyola Marymount University*
99. Marc Mace, *Department of Defense*
100. Crepin Mahop, *Howard University*
101. Audrey Malagon, *Virginia Wesleyan College*
102. Aldo Maldonado, *Park University*
103. Roummel Marcia, *University of California, Merced*
104. Marco V. Martinez, *North Central College*
105. Thomas Maxwell, *DOD*
106. Elizabeth McClellan, *Metropolitan State University of Denver*
107. Eileen McGraw, *Stevenson University*
108. Michael McLendon, *US Govt (DoD)*
109. May Mei, *Denison University*
110. Stephen Meskin, *UMBC*
111. David Milan, *University of Texas at Tyler*
112. Walter Morris, *George Mason University*
113. Erin Moss, *Millersville University of PA*
114. Bhamini Nayar, *Morgan State University, Baltimore*
115. Emma Norbrothen,
116. Hyunju Oh, *Bennett College*
117. Omayra Ortega, *Arizona State University*
118. Reyes M Ortiz-Albino, *University of Puerto Rico-Mayaguez*

119. Stephen Pankavich, *Colorado School of Mines*
120. Junkoo Park, *Houghton College*
121. Karoline Pershell, *American Association for the Advancement of Science Fellow*
122. M.Tip Phaovibul, *University of Illinois*
123. Alicia Prieto Langarica, *Youngstown State University*
124. Tariq Qazi, *Virginia State University*
125. Ashley Rand, *Bethany Lutheran College*
126. Tim Ray, *DOD*
127. Nathan Reff, *Alfred University*
128. Sandra Richardson, *Virginia State University*
129. John Rock, *California State Polytechnic University, Pomona*
130. Melvin Royer, *Indiana Wesleyan University*
131. Eric Ruggieri, *College of the Holy Cross*
132. Jan Rychtar, *University of North Carolina Greensboro*
133. Karin Saoub, *Roanoke College*
134. Timothy Sauer, *George Mason University*
135. Steven Schlicker, *Grand Valley State University*
136. Susan Schmoyer, *Worcester State University*
137. Therese Shelton, *Southwestern University*
138. Brendon Stanton, *DOD*
139. Colin Starr, *Willamette University*
140. Rebecca Swanson, *Colorado School of Mines*
141. Ahlam Tannouri, *Morgan State University*
142. Sandrine Tchatie-Leudeu, *Towson university*
143. Mohammed Tesemma, *Spelman College*
144. Narayan Thapa, *Minot State University*
145. Stephanie Thomas, *DOD*
146. Rob Thompson, *Macalester College*
147. Sunil Tiwari, *Sonoma State University*
148. Melissa Tolley, *Wingate University*
149. David Torain, *Hampton University*
150. Helene Tyler, *Manhattan College*
151. Violeta Vasilevska, *Utah Valley University*
152. Joseph Walsh, *Bethany College*
153. Kirsti Wash, *Clemson University*
154. Ursula Whitcher, *University of Wisconsin-Eau Claire*
155. Cassie Williams, *James Madison University*
156. Roger Wolbert, *University at Buffalo*
157. Elizabeth Wright, *DOD*
158. Justin Wright, *Plymouth State University*

159. Carolyn Yackel, *Mercer University*
160. Mohammed Yahdi, *Ursinus College*
161. Mohammed Yahdi, *Ursinus College*
162. George Yates, *Youngstown State University*
163. Rebekah Yates, *Houghton College*
164. Shenglan Yuan, *LaGuardia Community College, CUNY*
165. Yingxian Zhu, *Utah Valley University*
166. Jiehua Zhu, *Georgia Soouthern University*
167. Lori Ziegelmeier, *Macalester College*
168. Erica Zuhr, *High Point University*

Titles, Authors, Advisors and Abstracts

1. q-Symmetric Polynomials and nilHecke Algebras

Ritesh Ragavender South Brunswick High School
Advisor(s): Alexander Ellis, MIT PRIMES USA

Symmetric functions appear in many areas of mathematics and physics, including enumerative combinatorics and the representation theory of symmetric groups. A q-bialgebra of "q-symmetric functions" generalizing the symmetric functions was defined by Ellis and Khovanov as a quotient of the quantum noncommutative symmetric functions. In the $q = -1$ (or "odd") case of these q-symmetric functions, they and Lauda introduced odd divided difference operators and an odd nilHecke algebra, used in the categorification of quantum groups. In addition, Khongsap and Wang introduced an odd analogue of Dunkl operators, which in the $q = 1$ ("even") case have connections to Macdonald polynomials. Using diagrammatic techniques, we study relations for the q-symmetric functions when q is a root of unity other than 1 or -1. We then use q-analogues of divided difference operators to define a q-nilHecke algebra and describe its properties. We also show that odd divided difference operators can be used to construct the odd Dunkl operator of Khongsap and Wang, which we then use to give a representation of $\mathfrak{sl}_2$ on the algebra of skew polynomials.

2. Finite Superideal Domain

Ashley Lawson Tennessee Tech University
Advisor(s): Andrew Hetzel, Tennessee Tech University

A *finite superideal domain* (FSD) is a new type of algebraic structure which was developed during a 2013 summer research project at Tennessee Tech University. We define a FSD to be an integral domain with the property that every nonzero ideal of the domain is contained in only finitely many ideals of the domain. Through counterexamples and proofs, we have shown the relationship between FSD's and other types of domain such as Dedekind domain, Noetherian domain, UFD, FFD (finite factorization domain), and CK-domain (Cohen-Kaplansky domain). We have also proved several results involving FSD including as the following characterization: Let R be an integral domain. Then R is a FSD if and only if for all nonzero ideals I in R, the factor ring R/I is a finite direct product of SPR's (special primary ring) and finite rings.

3. Counting generators in Temperley--Lieb diagrams

Sarah Salmon Northern Arizona
Michael Hastings Northern Arizona University
Advisor(s): Dana Ernst, Northern Arizona University

The Temperley–Lieb Algebra, invented by Temperley and Lieb in 1971, is a finite dimensional associative algebra that arose in the context of statistical mechanics. Later in 1971, R. Penrose showed that this algebra can be realized in terms of certain diagrams. Then in 1987, V. Jones showed that the Temperley–Lieb Algebra occurs naturally as a quotient of the Hecke algebra arising from a Coxeter group of type A (whose underlying group is the symmetric group). This realization of the Temperley–Lieb Algebra as a Hecke algebra quotient was generalized to the case of an arbitrary Coxeter group. In the cases when diagrammatic representations are known to exist, it turns out that every diagram can be written as a product of "simple diagrams." These factorizations correspond precisely to factorizations in the underlying group. Given a diagrammatic representation and a reduced factorization of a group element, it is easy to construct the corresponding diagram. However, given a diagram, it is generally difficult to reconstruct the factorization of the corresponding group element. In cases that include Temperley–Lieb algebras of types A and B, we have devised an algorithm for counting the number of times each simple diagram appears in a reduced factorization for a given diagram.

4. Cayley Graphs and the Cayley Isomorphism Property

Gregory Michel Carleton College
Hannah Turner Ball State University
Advisor(s): Sung-Yell Song, Iowa State University

For a finite group G and a subset S of G, the Cayley graph $\mathrm{Cay}(G, S)$ is the graph whose vertex set is G such that two vertices x and y are adjacent if $x^{-1}y$ is in S. A Cayley graph $\mathrm{Cay}(G, S)$ is called a Cayley Isomorphic (CI) graph if for any $\mathrm{Cay}(G, T)$ graphically isomorphic to $\mathrm{Cay}(G, S)$, there is a corresponding group automorphism σ of G with $\sigma(S) = T$. A finite group G is called a CI-group if every Cayley graph of G is a CI-graph. We show that G is not CI if it admits a non-CI subgroup or if it admits two non-isomorphic subgroups of the same order. We further show that this completely classifies the effect of subgroups on non-CI Abelian groups. That is, if an Abelian group is non-CI and its subgroups do not meet the conditions above, then any non-CI graph will be connected.

5. Generic Formal Fibers of Local Domains and their Polynomial Rings

Sander Mack-Crane Case Western Reserve University
Advisor(s): Susan Loepp, Williams College

We can define a metric on a local Noetherian ring R, and complete this metric space to obtain a new ring which we call the completion of R. One interesting relationship between an integral domain and its completion is the generic formal fiber. We extend previous results by finding necessary and sufficient conditions for the existence of a local integral domain A such that A has a completely specified completion and generic formal fiber (with countably many elements), and $A[x]$ has a specified completion and restrictions on its generic formal fiber. In the case of characteristic 0, we present necessary and sufficient conditions for the existence of an excellent local domain A with the above properties.

6. Local Generic Formal Fibers of Excellent Rings

Samuel Tripp Williams College
Peihong Jiang University of Rochester
Advisor(s): Susan Loepp, Williams College

Given a local Noetherian ring, we can define a new ring which we call its completion. One interesting relationship between a ring and its completion is the generic formal fiber. We will discuss the existence of excellent rings of characteristic 0 possessing a local generic formal fiber, and present original results and conjectures regarding the existence of excellent rings possessing a local generic formal fiber in characteristic greater than 0.

7. Exploring the Catenary Degrees of Arithmetical Congruence Monoids

Sherilyn Tamagawa Scripps College
Advisor(s): Scott T. Chapman, Roberto Pelayo, Sam Houston State University, University of Hawaii

Arithmetical congruence monoids (ACMs) play an important role in understanding the non-unique factorization theory of multiplicative submonoids of the natural numbers. The catenary degrees of these monoids, which measure differences in the factorizations of individual elements, have been previously analyzed for a subclass of these monoids known as regular ACMs. We present the catenary degrees for the remaining subclasses: local and global ACMs. We also include other interesting properties, including the classification of catenary degree 2 ACMs, the construction of ACMs with arbitrary catenary degree, cases when the catenary set is an interval, and periodicity.

8. The Survival Complex

Anna-Rose Wolff George Mason University
Advisor(s): Neil Epstein, George Mason University

Let $(S, \cdot, 0)$ be a commutative multiplicative semigroup with an absorbing element 0. Then one can form an abstract simplicial complex, called the *survival complex* of S, whose vertices are the nonzero zero-divisors of S and where one forms a simplex from $s_1, \ldots, s_n \in S$ whenever $s_1 \cdots s_n \neq 0$ — that is, whenever a subset "survives" multiplication. Let k be a field, let $X_1, \ldots, X_t$ be variables, and let I be a monomial ideal in $A := k[X_1, \ldots, X_t]$, and set $R := A/I$. Let S consist of the images of the monomials of A in R, under the inherited multiplication, along with

0. We investigate the survival complex of S, denoted by $\Sigma(S)$. This project's primary objective is to relate the algebraic properties of R to the combinatorial properties of $\Sigma(S)$. We compare $\Sigma(S)$ to the zero-divisor graph of S. In the case where I contains the squares of all the variables, we show that the associated Stanley-Reisner complex is a subcomplex of $\Sigma(S)$. We show fairly generally that $\Sigma(S)$ tends to contain isolated points; in particular, it is rarely pure. We also relate the dimension of R to the dimension of $\Sigma(S)$.

9. Factorizaton Properties of Leamer Monoids

Jason Haarmann Eastern Illinois University
Aleesha Moran McKendree University
Advisor(s): Robert, University of Hawaii at Hilo

The Huneke-Wiegand conjecture has prompted much recent research in Commutative Algebra. In studying this conjecture for certain classes of rings, Garc´a-Sánchez and Leamer construct a monoid S_Γ^s whose elements correspond to arithmetic sequences in a numerical monoid Γ of step size s. These monoids, which we call Leamer monoids, possess a very interesting factorization theory that is significantly different from the numerical monoids from which they are derived. We offer much of the foundational theory of Leamer monoids, including an analysis of their atomic structure, and investigate certain factorization invariants. Furthermore, when S_Γ^s is an arithmetical Leamer monoid, we give an exact description of its atoms and use this to provide explicit formulae for its Delta set and catenary degree.

10. An Invariant for Singular Links

Tsutomu Okano Carnegie Mellon University
Danny Orton California State University, Fullerton
Advisor(s): Carmen Caprau, California State University, Fresno

A knot is an embedding of a circle in three-dimensional space, and a link is an embedding of several disjoint circles in the three-dimensional space. Invariants for knots are efficient ways of distinguishing between different knots, which is the primary purpose of knot theory. In our work we consider a certain invariant for knots and links and extend it to an invariant for knot-like objects, called singular links. Specifically, we consider a state model of the one-variable specialization of the HOMFLY-PT polynomial via a solution to the Yang-Baxter equation, and extend it to singular links. A singular link with n-components is an immersion of disjoint n circles into three-dimensional space, which admits only finitely many singularities that are transverse double points. We explain what it means to have an invariant for singular links, and we show that our construction yields such an invariant.

11. Bent Functions and Difference Sets

Gavin McGrew University of Richmond
Erin Geoghan University of Richmond
Becca Funke University of Richmond
John Clikeman University of Richmond
Uthaipon Tantipongpipat University of Richmond
Tahseen Rabbani University of Virginia
Advisor(s): James A. Davis, University of Richmond

Bent functions are very important objects in coding theory and cryptography, where they can be used to encode information very effectively for error-correction or message obfuscation purposes. Difference sets are objects in group theory which are closely related to bent functions, in that every bent function is a difference set. We investigated groups of order 256 (of which there are 56092) in an effort to determine which of these contain difference sets and which do not. We will present constructions of difference sets in at least 11 previously unknown groups.

12. On the Schur Positivity of Differences of Products of Schur Functions

Jeremiah Emidih University of California, Riverside
Nadine Jansen North Carolina A&T State University
Jeremy Meza Carnegie Mellon University
Advisor(s): Rosa Orellana, Dartmouth College

The Schur functions are a basis for the ring of symmetric functions indexed by partitions of nonnegative integers. A symmetric function f is called Schur positive if when expressed as a linear combination of Schur functions $f = \sum_\lambda c_\lambda s_\lambda$ each coefficient c_λ is nonnegative. We wish to investigate the Schur positivity of expressions of the form $s_\lambda s_{\lambda^c} - s_\mu s_{\mu^c}$ where λ partitions n and μ partitions $n-1$ and the complements λ^c, μ^c are taken over a sufficiently large $m \times m$ square. We give a necessary condition that if $s_\lambda s_{\lambda^c} - s_\mu s_{\mu^c}$ is Schur positive, then μ is contained in λ. Furthermore, we show how both increasing the size of our box and conjugation preserves Schur positivity. We then conjecture a full characterization of when $s_\lambda s_{\lambda^c} - s_\mu s_{\mu^c}$ is Schur positive. Lastly, we use the Littlewood-Richardson rule to prove special cases of this conjecture.

13. Generic Polynomials in Characteristic p for $GL_n(\mathbb{F}_{p^r})$

Michelle Yuen Carleton College
Advisor(s): Jorge Morales, Louisiana State University

Let $\mathcal{A}$ be a finite-dimensional simple algebra over a finite field $\mathbb{F}_p$, where p is prime and let $\mathcal{A}^\times$ be the multiplicative group of $\mathcal{A}$. In this paper, we show the existence of a generic polynomial for the group $\mathcal{A}^\times \simeq GL_n(\mathbb{F}_{p^r})$ over $\mathbb{F}_p$ in nr parameters.

14. On a Class of Permutation Polynomials over Finite Fields

Christian Rodriguez-Encarnacion University of Puerto Rico, Rio Piedras
Alex Santos-Sosa University of Puerto Rico, Rio Piedras
Advisor(s): Ivelisse Rubio, Francis Castro, University of Puerto Rico, Rio Piedras

Permutation polynomials over finite fields are important in many applications, for example in coding theory and cryptography. Our goal is to provide families of polynomials that are rich in permutation polynomials, and study polynomials of the form $F_{a,b}(X) = X^m \left(X^{\frac{q-1}{2}} + aX^{\frac{q-1}{d}} + b\right)$, where $a, b \in F_q$, $q = p^r$, p prime, and $d \mid (q-1)$. We prove that the number of polynomials of the form $F_{a,b}(X)$ with value set of size $\left|V_{F_{a,b}}\right| = n$ is a multiple of d if d is even, or a multiple of $2d$ if d is odd and give a construction where, given a permutation polynomial $F_{a,b}(X)$ of F_q, we can construct a list of d or $2d$ coefficients a', b' such that $F_{a',b'}(X)$ is also a permutation polynomial of F_q.

15. Applications of Invariant Theory on Spaces with Fixed Angular Momentum

Joshua Cape Rhodes College
Advisor(s): Christopher Seaton, Hans-Christian Herbig, Rhodes College, Charles University in Prague

Let there be k particles in $\mathbb{R}^n$ where each particle is characterized by position and momentum in each dimension. Under these considerations, the phase space describing all states of this system is $\mathbb{R}^{2kn}$. By imposing that the total angular momentum of the particles is zero, the new phase space is a subset of $\mathbb{R}^{2kn}$. If we further consider uniqueness up to rotations and reflections (i.e., quotient by the orthogonal group $O(n)$), the resulting space has a singularity; this yields an example of singular symplectic reduction. Experimentation suggests that for fixed k, all of these resulting spaces are equivalent for $n \geq 3$. We will demonstrate how the theory of invariant polynomials can be used to study the question of whether these spaces are equivalent.

16. Formal Fibers of Principal Prime Ideals with Countably Many Maximal Elements

Craig Corsi Williams College
Advisor(s): Susan Loepp, Williams College

Let T be a complete local (Noetherian) ring, C be a finite set of pairwise incomparable nonmaximal prime ideals of T, and $p \in T$ a nonzero element. In 2005, Chatlos, Simanek, Watson, and Wu gave necessary and sufficient

conditions for T to be the completion of an integral domain A containing the prime ideal pA whose formal fiber is semilocal with maximal ideals the elements of C. We show that the same conditions are necessary and sufficient when C is countably infinite. Furthermore, we give necessary and sufficient conditions for A to also be an excellent ring

17. Algebra associated with the Hasse graph of the n-dimensional semi-hypercube

Austin Riedl University of Wisconsin-Eau Claire
Mitchell Lemons University of Wisconsin - Eau Claire
Advisor(s): Colleen Duffy, University of Wisconsin - Eau Claire

The primary goal of our project is to determine the structure of a graded algebra, $A(\Gamma)$, that is associated to the Hasse Graph, Γ, of an n-dimensional semi-hypercube. We can consider a unit n-cube with one vertex at the origin, keep only those vertices with an even number of 1's, and form new simplex and semi-hypercube facets. Each symmetry of the n-dimensional semi-hypercube can be thought of as acting on the coordinates of the vertices. For each symmetry, we consider the Hasse graph consisting of fixed k-faces of the semi-hypercube under the action. From each Hasse sub-graph, we determine the graded dimension of subalgebras of $A(\Gamma)$ by counting the directed paths between each pair of levels in the graph. We have been able to determine a formula giving the number of fixed k-faces and how they are connected dependent only upon the symmetry acting on the semi-hypercube. Furthermore, we have determined part of the generating function that describes the algebra, that also is only dependent upon the symmetry.

18. Polygon Partitions and Reversions of Series

Alison Schuetz Hood College
Advisor(s): Gwyneth R Whieldon, Hood College

The Catalan numbers C_k were first studied by Euler, in the context of enumerating triangulations of polygons P_{k+2}. One generalization of the Catalan numbers, the Fuss-Catalan numbers $C_k^{(d)}$, counts enumerations of $(d+1)$-gon dissections of polygons $P_{k(d-1)+2}$. We present here a further generalization $C_k^{(d_1,d_2,\ldots,d_r)}$, arising in the enumerations of mixed polygonal dissections into $(d+1)$gons, where $d \in \{d_1, d_2, \ldots, d_r\}$. These mixed dissections arose in context of studying series arising from iterated polynomials, and we compute recursive (and closed) formulas for $C^{(d_1,d_2,\ldots,d_r)}$ by examining the reversions of certain power series.

19. Difference Set Transfers

Dylan Peifer Carleton College
Advisor(s): Vadim Ponomarenko, San Diego State University

Given a finite group G of order v, a subset $D \subseteq G$ is called a $\langle v, k, \lambda \rangle$-difference set if $|D| = k$ and the set $\{d_i d_j^{-1} | d_i, d_j \in D\}$ contains λ copies of each nonidentity element of G. A fundamental question in the study of difference sets is determining which groups contain difference sets and which do not, and a related question involves finding all difference sets in a group or set of groups. Exhaustive computer search can determine all difference sets in groups of order 16, but there are still many patterns to be found in what we term *difference set transfers* — where a difference set in one group of order 16 can be transferred to a difference set in a different group of order 16 using power-commutator presentations of these groups. In our research we discovered and proved many of the relations involving difference set transfers found in groups of order 16 and applied transfers to determine the existence of difference sets in some groups of order 64 and 144, where exhaustive search is infeasible and other standard methods for finding difference sets fail.

20. Extremal divisors on moduli spaces of rational curves with marked points

Morgan Opie University of Massachusetts Amherst
Advisor(s): Jenia Tevelev, University of Massachusetts Amherst

We study effective divisors on $\overline{M}_{0,n}$, focusing on hypertree divisors introduced by Castravet-Tevelev and the proper transforms of divisors on $\overline{M}_{1,n-2}$ introduced by Chen-Coskun. Results include include a database of hypertree divisor classes and closed formulas for Chen-Coskun divisor classes. We relate these two types of divisors, and from this construct extremal divisors on $\overline{M}_{0,n}$ for n greater than or equal to 7 that furnish counterexamples to the conjectural description of the effective cone of $\overline{M}_{0,n}$ given by Castravet-Tevelev.

21. Discrete Calculus on a Scaled Number Line

Avery St. Dizier Louisiana State University
Advisor(s): Allan Peterson, University of Nebraska - Lincoln

Calculus has been used to analyze many characteristics of continuous functions for centuries, and it has proven incredibly useful in many applications. Discrete Calculus, calculus implemented in non-continuous settings, has recently gained popularity for its provision of similar analysis on a variety of domains. As a result, discrete calculus has proven applicable in disciplines ranging from viscoelasticity to statistical methods and countless other applications. While the foundations of discrete calculus have been widely developed, techniques for solving difference equations are sometimes limited. Our research explores the use of discrete calculus and a corresponding Laplace transform on a domain called the "scaled number line," a generalization of the natural numbers. We develop an exponential function, and then proceed to define a Laplace transform. We further show that a unique Laplace transform exists under certain conditions, and we find it for common functions and examine its properties. We also demonstrate how Discrete Laplace Transforms can be utilized to solve difference equations on the scaled number line.

22. Classification of W-Measurably Sensitive Transformations

David Stevens Williams College
Advisor(s): Cesar Silva, Williams College

Sensitive dependence on initial conditions is one of the central ideas of chaos. In 2006, a notion of measurable sensitivity was introduced by the ergodic theory group of the SMALL Undergraduate Research Project as a measure-theoretic version of sensitivity. In the following year, a dichotomy for conservative ergodic Z-actions was given. Namely, they are either W-measurably sensitive, a notion that is in general strictly stronger than measurable sensitivity, or isomorphic to a uniformly rigid minimal isometry. Our work extends this classification theorem to more general abelian group actions G, leading to a result in measurable dynamics similar to the Auslander-Yorke dichotomy in topological dynamics.

23. Time Scale Calculus

Esmeralda Quintana State University of New York at Potsdam
Abigail Mosca Smith College
Juan Posso Lafayette College
Advisor(s): Victoria Klawitter, SUNY Potsdam

The Lebesgue Δ-measure is a measure defined on a time scale $\mathbb{T}$, where $\mathbb{T}$ is a nonempty closed subset of the real numbers. Finding the exact value of a Δ -integral of a Riemann Δ- integrable function on $\mathbb{T}$ is an open question. For the Cantor C as a time scale we develop the Lebesgue Δ-integration from basic notions. Using self-similarity properties of C and of the Δ-measure on the Cantor set we prove a recursive formula that allows calculations of Lebesgue Δ-integrals of polynomials. Our main theorem is a recursive formula for Taylor monomials defined through Riemann Δ-integration on arbitrary time scales. The formula provides a method for explicitly finding Taylor monomials of any order.

24. Properties of the infinite tower function $h(x) = x^{x^{x^{\cdots}}}$

Jimmy Scott University of Wisconsin-Stevens Point
Advisor(s): Edwin Herman, University of Wisconsin-Stevens Point

The tower function, $h(x) = x^{x^{x^{x^{\cdots}}}}$, is closely related to the Lambert W-function, which can be used to derive the power series of $h(x)$. Although $h(x)$ has some interesting properties on the real line, its behavior in the complex plane is far more interesting. Baker and Rippon showed that $h(z)$ converges on a region interior to the image of the map $\phi(z) = exp(z/e^z)$ of the unit circle. Beyond this region the tower function exhibits chaotic behavior. We will attempt to extend Baker and Rippon's results to characterize certain regions under which the function is periodic and examine the behavior of h(z) when non-principal branches are considered.

25. Stability Analysis of Solitary Wave Solutions of the KdV Equation

Alison Wilkman College of the Holy Cross
Advisor(s): Steven Levandosky, College of the Holy Cross

The KdV equation is a mathematical model for water waves in shallow surfaces. We consider the stability of solitary wave solutions of the generalized KdV equation, where our nonlinear term of interest is one of mixed powers. Through both numerical analysis and analytical approaches, we determine the range of speed c over which the solitary waves are stable for this equation. We apply the known theorem regarding stability, which depends on the function of wave speed $d(c)$ and its derivates over a range of speed c. Further, we consider a variation of this nonlinear term for the compound KdV equation and later investigate the original mixed power term applied to the fifth-order KdV model. We thank Dr. Dan Kennedy for financial support.

26. Realistic Agent Behavior in Repeated Economic Games

Edison Bailey George Mason University
Advisor(s): Harbir Lamba, George Mason University

Standard economic and financial models assume strongly rational/efficient behavior for all the participants. However many experiments have shown that actual human behavior is often far from optimal and may instead be based on simple heuristics. We develop such a model for how people might actually behave in two closely related simple economic games, the symmetric Minority Game and its asymmetric counterpart (the El Farol problem). In the Minority Game, at each round the participants select one of two choices and those in the minority win a symmetric payoff. In the asymmetric version, one of the choices has a higher payoff than the other and is thus preferred by the players but the minority still wins. We compare various measures of efficiency for our heuristic model against both the theoretical results for optimal behavior and experimental data for the symmetric game. In addition, our modeling assumptions predict that the aggregate behavior of the group will be significantly less efficient in the asymmetric case than the better-studied symmetric one.

27. The Weighted Laguerre Polynomials Form a Basis for $L^2(0, \infty)$

Lauren Hannah Butler University
Advisor(s): William Johnston, Butler University

The poster presents a new straightforward proof, following a method and Wheeler and Zygmund and a generalization by Johnston, and which undergraduates can understand, that the weighted Laguerre polynomials $L_n(x)\mathrm{e}^{-x/2}$, $n = 0, 1, 2, \ldots$, form an orthonormal basis for $L^2(0, \infty)$.

28. Information Reconciliation for a Multiple-Channel Communication Model

Rahul Dalal Harvard College
Phillip Jedlovec University of Notre Dame
Advisor(s): Susana Serna, Autonomous University of Barcelona

We establish a joint decoding strategy that can confidently reconcile information sent over multiple correlated channels and that requires less check information to be sent than in independent decoding. First, we look at sets of codes that can securely reconcile information sent over a single private binary symmetric channel and that work for a wide range of error rates. We choose from the existing literature on error-correcting codes used in *Forward Error Correction (FEC)* and adapt them to our information reconciliation context. In particular, we develop new methods for using BCH codes and concatenated codes in the information reconciliation context. Next, we present independent and joint decoding strategies using such a set of codes in a multiple-channel information reconciliation model and analyze the relative performance of these strategies through simulations and theoretical bounds. Our analysis shows that under the *Block-Fading Error Model*, joint decoding can perform significantly better than independent decoding and that if a good enough single channel set of codes were used for joint decoding, joint decoding should be able to perform better than the theoretical bound for independent decoding for some block error distributions.

29. Quantitative Portfolio Construction Using Various Optimization Methods

Yitzchak Solomon University of California, Los Angeles
Qinyu Wang Hong Kong University of Science and Technology
Oi Yee Liu Barnard College
Ho Yin Ma Hong Kong University of Science and Technology
Advisor(s): Ngai Wang-Kay, Hong Kong University of Science and Technology

To succeed in today's financial marketplace, it is essential that an investor maintains a well-constructed portfolio that accounts for their preferences and goals. In this project, we considered a variety of quantitative portfolio construction methods built loosely around the mean-variance theory of Harry Markowitz. The initial portion of the project is concerned with the performance of the HM model versus two Monte Carlo alternatives. We then go on to outline two additional algorithms: one based on clustering, the other on PCA analysis. We test the robustness and performance of these new algorithms as compared to the standard mean-variance method, and our results show that both methods exhibit interesting behavior, with the clustering-based algorithm often outperforming the traditional algorithm. Lastly, we look at strategy-switching methods that attempt to capitalize on the strengths of the different algorithms.

30. Linking Social Media to Crime and Disorder

Anna Ma University of California, Los Angeles
Yang Hu California State University, San Marcos
Advisor(s): Blake Hunter, University of California, Los Angeles

The increasing prevalence of social media in modern society means the existence of a new medium through which criminal activity can be operated. Easily accessible and wide-spread usage of social media technologies such as Facebook and Twitter facilitates the rapid diffusion of communication and therefore can potentially play a decisive role in social unrest. In recent times, police forces have begun to examine social media as a tool for monitoring social disorder. In particular, the Los Angeles Police Department (LAPD) has been utilizing Twitter, an online micro-blogging site where posts are limits to 140 characters. Due to the sheer volume of posts occurring in the LA county alone, it is difficult for the LAPD to monitor Twitter efficiently. The TSC method utilizies the time, location, and content of a Tweet to determine whether it is related to a 911 call for service. For location, we utilize the geography of our area of study, Hollenbeck, which is a division of Los Angeles County. Currently, the LAPD filters through Tweets using a keyword based search. Instead of keywords, we use topic modeling to take advantage of the meaning behind the content of a Tweet rather than just the words.

31. Nosé-Hoover Thermostats

Lora Weiss St. Olaf College
Karl Gross Texas A&M University
Brian Shi University of Texas-Austin
Advisor(s): Leo T. Butler, Central Michigan University

The equilibrium statistical properties of molecular systems is important to applied subjects such as biology, chemistry, computational physics and materials science. These equilibrium statistical properties are obtained as phase space integrals that depend on q as the position of the system, p as the momentum of the system, and have $H(q, p)$ as the total energy of the system. In 1984 S. Nosé introduced a thermostat to mimic the effect of a heat bath on a mechanical system. W. Hoover simplified this model and showed that even for the simple harmonic oscillator the system can exhibit complicated dynamics. In this presentation, we attempt to find exact solutions of the Nosé-Hoover thermostat; we look for periodic solutions to make a conjecture about the existence of invariant tori; we determine orbit averages along the solution curves; we analyze various numerical methods to solve the system; and we look at the existence of a first integral for the system, building on the work of Legoll, Luskin, and Moeckel.

32. Robust Low-rank Matrix Factorization

Lara Clemens St. Lawrence University
Advisor(s): Albert Ku, Hong Kong University of Science and Technology

Recent advances in signal processing have resulted in new methods for solving the low-rank matrix sensing problem and the low-rank matrix completion problem. Such problems arise in applications involving extremely large data sets that feature redundancy. These data sets can often be modeled by low-rank matrices. As the size of the data grows, solving these problems may become infeasible due to storage requirements. Algorithms which yield factorizations of the low-rank target matrix have been developed and have been proven to be optimal in certain scenarios. However, this method assumes that the matrix to be recovered is not corrupted and satisfies incoherency, or that the observations satisfy the RIP property. In this paper, we extend current algorithms to be robust to corruption by sparse noise. We perform numerical tests to support the validity of our methods and apply these algorithms to image colorization and disjoint clique inference.

33. Haarmony: Chord Recognition Using Wavelets and Other Methods

Katherine Weber State University of New York at Geneseo
Chance Rodriguez Mississippi State University
Advisor(s): Gregory Berkolaiko, Adam Larios, Texas A&M University

Our research examined whether a computer, using Matlab, can effectively distinguish between different musical chords. We employed the signal processing techniques of wavelets, Fourier analysis, and raw data analysis to determine which method best enabled the computer to correctly recognize guitar chords. We will discuss basic overviews of the theory behind each of these methods, explain how we collected our data and implemented our algorithm, and report on the accuracy of each method. In addition, we will provide evidence for why we think each method worked as it did.

34. Continuous Time Quantum Walks

Rebekah Herrman Christian Brothers University
Advisor(s): Travis Humble, Oak Ridge National Laboratory

Quantum computing offers a new approach to solving problems. Classical computers use bits in order to store information, but quantum computers use qubits. Qubits are the basis for quantum algorithms, and Continuous Time Quantum Random Walks (CQW) represent one model for implementing quantum computation. We formulate CQW in terms of walks on a graph and use these walks as computational primitives. The CQW dynamics depend on the Hamiltonian of the system, which is equivalent to the adjacency matrix of the graph. We have simulated CQW's along the K2 graph and the Binary Welded Tree. These simulations help explain how CQW's can be used for computation and how different graph structures effect computations. We constructed the CQW equivalent of quantum logic gates by using dynamics of the quantum state propagating from one vertex of a graph to another and on the way that the initial wave function evolved under the action of the graph Hamiltonian. The CQW simulations allowed us to determine how connections between certain vertices of a graph represent the behavior of the discrete logic gates found in other quantum computational models. Finally, we tested our ideas by converting circuits for quantum teleportation that use quantum logic gates into equivalent CQW's..

35. Lake Mead Temperature Modeling

Catherine Schepp Haverford College
Advisor(s): Lisa Driskell, Colorado Mesa University

The largest reservoir in the Colorado River system, Lake Mead, is a vital water source for many of the agricultural, industrial, and domestic needs of southwestern communities. As drought persists and demand continues to rise, there is much interest in protecting this resource and considering options to address scarcity. A model of temperature and energy flow within the lake has the potential to provide valuable information for studies of evaporation, water quality, biological communities, and hydrological systems. We discuss a parabolic PDE used to model temperature change with respect to depth and time based on numerous meteorological and physical processes and parameters. We compare the results of the PDE to empirical data from a site in Lake Mead and find that it reflects magnitude

and seasonal variation of the water temperatures. We investigate applications of the model and the effects of varying meteorological components representative of various climate and weather scenarios.

36. A Mathematical Model for Competition of Two Companies: E-Commerce vs. Traditional

Stephen Szulc New Jersey City University
Advisor(s): Zhixiong Chen, Yi Ding, New Jersey City University

We model the competition of two companies to help understand internet shopping's interaction with traditional shopping. Specifically, we use a two predator- one prey model to view competition, efficiency, and the markets effect on revenue of two companies. The two companies act as predators trying to scavenge the consumer's money to create revenue. The completion and use of numerical analysis of this model will allow us to predict the outlook for the companies.

37. Asymptotic Analysis of the Spectrum of the Discrete Hamiltonian with Period Doubling Potential

Maria Markovich Shippensburg University
Tara Hudson University at Buffalo
Meg Fields University of North Carolina at Asheville
Advisor(s): May Mei, Denison University

The discrete Hamiltonian operator with period doubling potential is a popular one-dimensional model of a quasi-crystal. The spectrum of this operator is a Cantor set for fixed coupling constants greater than zero. We numerically approximate this spectrum using both matrix truncation and a trace map method. We then computationally examine the asymptotic behavior (i.e. as the coupling constant goes to zero) of the box-counting dimension, thickness, and Hausdorff dimension for these approximations.

38. On the Spectrum of the Penrose Laplacian

Julie Pattyson University of Saint Joseph
Hailee Peck Millikin University
Christine Hoffman Smith College
Advisor(s): May Mei, Denison University

Since the early 1960's, aperiodic tilings have been a topic of particular interest to mathematicians. The Penrose tiling is one example of such a tiling. We present basic subdivision rules for generating the Penrose tiling using a set of four Robinson tiles. We use MatLab to create and store the Penrose tiling as a set of data structures and produce the Laplacian matrix of the tiling. In doing so, we provide insight into the spectrum of the Penrose tiling.

39. Oxygen mass transfer in a bubble-liquid system

Bernard Lipat New Jersey City University
Advisor(s): Zhixiong Chen, Yi Ding, New Jersey City University

Optimization of oxygen mass transfer is critically important in the field of bioreactor engineering, especially if biofuels are to become cost-competitive with fossil fuels. Herein is developed a mathematical model for oxygen mass transfer in a bubble column reactor, with a cost-optimized function for radius to which experimental data are compared. The model is derived from a generalization of a single-bubble system using equations for motion, mass transfer, and fluid stability, to a system of n independent bubbles, and finally to system of continuous bubble output

40. Mathematical Modeling of Dynamic Social Processes

Alexandra Zeller George Mason University
Advisor(s): Padmanabhan Seshaiyer, George Mason University

This project models the social interactions of human immigration and emigration within regions. Distinctive scenarios are proposed to impact the population movements between locations. Potential applications are discussed and include disease outbreaks, regulatory influences, and language differences. Mathematical modeling is combined with quantitative sociology in a system of differential equations that accurately projects a dynamic social process.

41. Text Analysis for Timely Discovery of Cyber-Security Concepts

Corinne Jones Penn State
Advisor(s): Robert Bridges, Oak Ridge National Laboratory

Entity extraction, an automated way to identify and classify key concepts in text, benefits many fields due to the existence of a high volume of textual data. In particular, our interest lies in using entity extraction for timely discovery of concepts that will benefit security analysts. To employ a supervised learning technique for entity extraction, a large annotated corpus is required for use as training data, and unfortunately, given a specific domain, usually no such data is available. We solve this problem in the cyber-security domain by creating a corpus via auto-labeling text tied to a database record. We then prototype a history-based maximum entropy model that processes this corpus to label new sentences and present results showing the Collins Perceptron outperforms maximum-likelihood estimation with L-BFGS and OWL-QN optimization for parameter fitting. While computationally more efficient due to the ability to use dynamic programming, history-based models constrain the feature set for a given word/label pair to the words in the sentence and the previous few labels, inhibiting performance. We therefore propose a method incorporating global features that escapes the computational difficulties associated with optimizing label sequences by applying simulated annealing.

42. Computational Flame Dynamics of Micro-Channels

Wendy Rummerfield University of Redlands
Advisor(s): Joanna Bieri, University of Redlands

For the last decade, the study of micro-combustion has increased tremendously due to its applications in aerospace and industrial technology as a more efficient form of power and heat source. Unfortunately, complications arise with these small channels creating much instability with the flame including repetitive extinction reignition patterns which are currently not understood. Our aim is to model these flames computationally and adjust specific variables such as the mixture ratio of fuel to oxidizer, heat loss through the channel walls, and channel width in the hopes of better understanding the parameters needed to create a stable flame. My research has involved expanding our understanding of non-symmetric flames and experimenting with new methods to obtain more accurate results from our computer code.

43. Long-term Behavior of Solutions to a Wave Equation with Degenerate Damping

Thomas Anderson New Jersey Institute of Technology
William Tritch Andrews University
Elizabeth Galvin Marist College
Jeffrey Kopsick James Madison University
Advisor(s): Daniel Toundykov, George Avalos, University of Nebraska-Lincoln

The model of interest is a semilinear wave equation with degenerate damping $u_{tt} - \partial_x(p(x)u_x) + |u|^r u_t = 0, x \in (0,1)$, $t > 0$ where $p(x) \geq c > 0$ is continuous, $r > 0$, and the sought after function u satisfies homogeneous Dirichlet conditions. The well-posedness of this problem has been studied; however, little is known to date about the energy dissipation rates of its solutions. The primary challenge is that it is not clear how to estimate the kinetic energy of a state in terms of the dissipation feedback that deteriorates at small amplitudes. We present an alternate proof of existence and uniqueness using a contraction mapping argument. In addition, we establish local existence of regular $(H^{1+k} \times H^k)$ solutions. Then, a numerical simulation of this model will be presented employing a combination of a finite-element scheme and the fixed point method. Some results concerning the asymptotic energy decay rates and their uniformity will be discussed. Examples of these energy decay rates will also be presented. This work was done as a part of an REU project at the University of Nebraska-Lincoln.

44. Modeling Heat Conduction in Micro-combustors

Freeman Levine University of Redlands
Advisor(s): Joanna Bieri, University of Redlands

Micro-combustion is an area of engineering and applied physics that is not well understood. It is difficult to produce flames in such enclosed spaces without high heat loss and inefficiency, and the parameters required to produce a flame in the first place are not always known. However, the devices store a huge amount of energy per unit mass

compared to batteries, so there are many applications for devices where total mass is a constraint. We seek to better understand the dynamics of these devices using a computational approach. This approach allows us to search through all possible parameters and learn about what kind of flames can be created and stabilized within small channels. We use a system of partial differential equations to model the flames and to explain how the flames move and change as we alter the input parameters. Currently we seek to understand heat conduction inside the channel walls and how it affects the flame's position, strength, and stability. Understanding the dynamics of these flame systems will allow experimentalists to more easily create stable flames, as we can create testable hypotheses about their behavior.

45. A Linear Analysis of a Straight Rod under Tension both with and without Drag

Victoria Kelley James Madison University
Advisor(s): Eva Strawbridge, James Madison University

Starting with a straight rod under tension, we are studying the perturbations in twist and bend using the Kirchhoff Rod Model. Additionally, we can include the effects of drag approximated by resisted force theory. Finally, this model can inform us about the response of internal forces compared to external ones. This work has applications to the study of worm locomotion, bacterial flagella, and DNA.

46. 2-D Smoothing of Images by an Edge Map

Ileana Treyes California State University, Stanislaus
Leonard Marchese CSU Stanislaus
Jesus Navarro CSU Stanislaus
Advisor(s): Jung-Ha An, Michael Bice, CSU Stanislaus

The main purpose of image processing is to reduce unnecessary noise but to preserve the boundary of region of interest. The constant smoothing term over the domain removes the noise from the image but causes Gaussian blurriness on the boundary. Therefore, adding an edge detector function to the smoothing term allows smoothing over the region while still preserving the boundary. We propose an efficient 2-D smoothing model which incorporates an edge map. The presented model reduces noise in a region of interest while maintaining the strength of an image's boundary by using an edge map. Optimal solutions of the proposed model are derived by finding Euler-Lagrange equations of the presented algorithm. In addition, we approximate solutions to our model numerically in MATLAB through implementation of the method of steepest descent and finite differences. The effectiveness of our new model is demonstrated by applying it to a brain image (http://brainweb.bic.mni.mcgill.ca/brainweb) and a synthetic image containing a simple region with Gaussian noise artificially added. The numerical results show the effectiveness of the proposed model.

47. Line-of-sight Pursuit in Montone Polygons

Alana Shine Pomona College
Lindsay Berry University of Texas, Austin
Junyi Wang Macalester College
Zachary Keller University of Minnesota
Advisor(s): Andrew Beveridge, Macalester College

We examine a pursuit-evasion game in a simply connected polygon. In this turn-based game, a pursuer P chases an evader E. Both players can move to any point within distance 1 from his current location. P wins if he is within unit distance of E; the evader wins by eluding capture forever. Applications of pursuit-evasion include robotic surveillance, search and rescue operations, and ecological management of invasive species. In our version of the game, E always knows the location of P, but P only has line-of-sight visibility: P observes the evader's position only when the line segment connecting them does not intersect the boundary of the polygon. Therefore P must search for E when the evader is hidden. We provide a winning strategy for P in a *monotone polygon*. A polygon Q is monotone with respect to a straight line L when every line orthogonal to L intersects Q at most twice. Our algorithm uses a capture strategy that is new to the pursuit-evasion field. This *rook strategy* complements the well-known *lion strategy*. The capture time is linear in the area of Q, and independent of the number of vertices of the polygon. Finally, we also extend our algorithm to *sweepable polygons*, a generalization of monotone polygons.

48. Numerical Methods for the Solution of the Hilbert-Schmidt Integral Eigenvalue Problem

Haocheng Bian Illinois Institute of Technology
Advisor(s): Greg Fasshauer, Illinois Institute of Technology

The analytical solution of the Hilbert-Schmidt eigenvalue problem $\int_\Omega K(x,z)\varphi(z)\rho(z)\mathrm{d}z = \lambda\varphi(x)$ for general K, Ω and ρ is very difficult. We study the accuracy and efficiency of numerical solution techniques based on various levels of discretization by testing them against known analytical solutions. We first turn the infinite-dimensional integration problem into a manageable finite-dimensional one by representing eigenfunctions as linear combinations of basis functions. The resulting equation is then enforced on a set of collocation points to yield a generalized matrix eigenvalue problem. This gives a semi-discrete method provided we can evaluate the integrals of the kernels times the basis functions analytically. More generally, we also need to discretize the integrals. We do this by employing our collocation points also as quadrature nodes and obtain a standard matrix eigenvalue problem. Knowing the eigenvalues and eigenfunctions of a positive definite kernel is important in numerical analysis, machine learning and statistics. This work was performed under the supervision of Greg Fasshauer and Mike McCourt and supported by a summer research stipend from IIT's College of Science.

49. Using simulated data to test the effectiveness of regularization techniques as it applies to fuel cells

Grant Sander Arizona State University
Advisor(s): Rosemary Renaut, Arizona State University

The optimization of performance in fuel cells calls for an approximation of the distribution of relaxation time (DRT). The DRT cannot be measured directly, requiring the solution to an inverse problem where the DRT and impedance are related by means of a Fredholm integral equation. A standard quadrature approach is taken, making the problem ill-posed; regularization methods are used to improve the solution by reducing quadrature error and noise. Regularization is performed on simulated data to assess the effectiveness of certain techniques that are used in finding an approximate solution to the DRT.

50. Bar Code Localization Using Machine Learning

Keenan Hawekotte Nebraska Wesleyan University
Mikaela Cashman Coe College
Elizabeth Newman Haverford College
Dung Nguyen Bard College
Advisor(s): Thomas Hoft, University of St. Thomas

We develop an algorithm for the automatic localization of 1-D bar codes in images using machine learning techniques. Despite the ubiquity of operational bar code scanners, we focus on low resolution camera-based scanners and challenging environments where traditional methods fail. We develop attributes that help distinguish bar codes from other objects in the image such as text and logos. These attributes are based on the discrete wavelet transform (DWT), the discrete Fourier transform (DFT), and gradient analysis. To create a bar code detection process robust to image distortions such as rotation, glare, noise, oblique viewing angle, uneven and dim illumination, and an abundance of surrounding text, we use the information from our individual detection methods in neural network, linear discriminant analysis (LDA) frameworks, and simple boosting. We analyze the effectiveness of these attributes and report on performance for a range of degraded images.

51. Mathematical Modeling, Sensitivity Analysis, and Optimal Control of Agroecosystems

Cara Sulyok Ursinus College
Advisor(s): Mohammed Yahdi, Ursinus College

This project develops mathematical models and computer simulations for cost-effective and environmentally-safe strategies to minimize alfalfa damage from pests with optimal biodiversity levels. Predator and plant diversity can control potato leafhopper (PLH) damage to the host-plant alfalfa, the most cultivated forage legume in the world. A mathematical model including eleven size- and time-dependent parameters was created using a system of non-linear differential equations. The model was shown to accurately fit results from open-field experiments and thus predict outcomes for scenarios not covered by the experiments. Steady state solutions were determined and a sensitivity analysis established the relative importance of each parameter to reduce the plant damage. Optimal control theory led to designing practical controls on the diversity levels to minimize the plant damage while preserving the plant production in a polyculture setting. In conclusion, the project provides a framework for designing cost-effective and environmentally-safe strategies to minimize alfalfa damage, determine critical parameters, and utilize the enemies hypothesis and polyculture diversity.

52. The Effect of Structure on Physiologically Based Pharmacokinetic Modeling

Sarah Laper College of Wooster
Archana Patel The College of New Jersey
Jerrell Mure Morehouse College
Amy Kern University of Southern Mississippi
Advisor(s): Marina Evans, Environmental Protection Agency

In physiologically based pharmacokinetic (PBPK) modeling, the structure of the model has impacts the accuracy of the output. Modelers frequently use pre-existing model structures and do not consider alternate structures once one has already been developed. The values of parameters used in the model also impact the model output. Modelers often optimize for parameters without considering whether or not the model is actually sensitive to the parameters. In this study, four different models were developed to identify how the complexity of a model affects the accuracy of the model. The four different model structures are: (1) a standard, 5 compartment model; (2) the standard model with isolation of the brain; (3) the standard model with the cardiovascular system isolate; and (4) the standard model with the brain and cardiovascular system isolated. Methyl tert-butyl ether and bromochloromethane served as the test chemicals for this study. Sensitivity analysis was performed on each model to determine which parameters were estimable and could therefore be optimized for. The error between the inhalation data and model output was used to determine which model gave the most accurate fit after optimization. It was found that, in general, increasing model complexity improves model accuracy.

53. The Moduli Space of $\mathbb{Z}_2$-graded $4|1$-Dimensional Non Nilpotent Complex Associative Algebras and their Deformations

Michael Loper University of Wisconsin-Eau Claire
Bryan Picchiottino University of Wisconsin-Eau Claire
Advisor(s): Michael Penkava, University of Wisconsin-Eau Claire

Algebras are familiar objects. Most people know about polynomial algebras, the integers, and the rational, real, and complex numbers, which are all examples of algebras. Shifting the point of view from considering algebras as sets of rules to thinking of them as objects in a certain space changes the type of questions that we can ask about them. The new point of view is that an associative algebra is a codifferential, which is a special type of coderivation. Two algebras are isomorphic if their codifferentials are equivalent. The set of isomorphism classes of codifferentials on a fixed vector space V is the *moduli space* of algebra structures on V. The moduli space of $4|1$-dimensional non nilpotent complex associative algebras consists of 506 types of algebras, including 11 one parameter families and 495 isolated algebras. We computed versal deformations of each of these algebras. Using the versal deformations we were able to determine all deformations of these algebras, which tells us how the moduli space is glued together. We will explain the methods of our construction using fundamental theorems from deformation theory. We will also illustrate how certain algebras deform to others and indicate how the deformations of these algebras are computed.

54. The Algebra of Block Permutations

Isabel Corona Metropolitan State University of Denver
Matthew Sarmiento Columbia University
Ryan Contreras Columbia University
Advisor(s): Rosa Orellana, Dartmouth College

A *block permutation* of $[n] = \{1, 2, \ldots, n\}$ consists of two set partitions $\mathcal{A}, \mathcal{B} \vdash [n]$ having the same number of blocks and a bijection $f : \mathcal{A} \to \mathcal{B}$. We show that the set of block permutations of $[n]$, BP_n, is closed under multiplication and hence is a monoid. We define a Hopf algebra of block permutations, which is a generalization of the Hopf algebra of uniform block permutations. The algebra BP_n is a subalgebra of the partition algebra and is therefore a diagram algebra. Furthermore, we study the subalgebra of planar diagrams, P_n, and give a presentation for both P_n and BP_n.

We consider the planar rook algebra RP_n and show that it is isomorphic to P_{n+1}. Using this isomorphism we can give all the irreducible representations for P_{n+1}. In addition, we are in the process of constructing the irreducible representations for BP_n, which will be indexed by Young diagrams with $1, 2, \ldots, n$ boxes. We conjecture as to the degree of the irreducible representations and investigate the induction of representations of BP_n to BP_{n+1}.

55. Orbits of a fixed-point subgroup of the symplectic group on partial flag varieties of type A

Jefffrey Cai Ridge High School
Advisor(s): Daniel Thompson, Massachusetts Institute of Technology

In this paper we compute the orbits of the symplectic group Sp_{2n} on partial flag varieties GL_{2n}/P and on partial flag varieties enhanced by a vector space, $\mathbb{C}^{2n} \times \mathrm{GL}_{2n}/P$. This extends analogous results proved by Matsuki on full flags. The general technique used in this paper is to take the orbits in the full flag case and determine which orbits remain distinct when the full flag variety GL_{2n}/B is projected down to the partial flag variety GL_{2n}/P. The recent discovery of a connection between abstract algebra and the classical combinatorial Robinson-Schensted (RS) correspondence has sparked research on related algebraic structures and relationships to new combinatorial bijections, such as the Robinson-Schensted-Knuth (RSK) correspondence, the "mirabolic" RSK correspondence, and the "exotic" RS correspondence. We conjecture an exotic RSK correspondence between the orbits described in this paper and semistandard bi-tableaux, which would yield an extension to the exotic RS correspondence found in a paper of Henderson and Trapa.

56. Fusion of Modular Invariants

William Clark Ohio University
Advisor(s): Alexei Davydov, Ohio University

Traditionally modular invariants are discrete data controlling the way two chiral 2-dimensional conformal field theories are glued together into a full 2d CFT. In this work we look at modular invariants as data describing surface defects between 3 dimensional topological field theories. Fusion of surface defects results in an operation on modular invariants which is explicitly computed for a series of examples.

57. Symplectic Transformations as R-Lagrangian Subspaces

Brennan Langenbach Grinnell College
Chris Hellmann Grinnell College
Advisor(s): Michael VanValkenburgh, Grinnell College

The graph of a real symplectic linear transformation is an R-Lagrangian subspace of a complex symplectic vector space. The restriction of the complex symplectic form is thus purely imaginary and may be expressed in terms of the generating function of the transformation. We provide explicit formulas; moreover, as an application, we give an explicit general formula for the metaplectic representation of the real symplectic group.

58. Betti Numbers of Splittable Graphs

Brittany Burns Shawnee State University
Haley Mansfield Kansas State University
Ola Sobieska Kent State University
Zerotti Woods Morehouse College
Advisor(s): Reza Akhtar, Miami University

Let G be a graph with n vertices. One way of studying the combinatorial information encoded in G is by examining its edge ideal, which is a monomial ideal in the polynomial ring $k[x_1, \ldots, x_n]$, where k is some base field. While the ideal itself can be rather complicated, a lot of useful information can be obtained by computing its Betti numbers. This approach has been particularly fruitful in studying geometric properties of graph varieties. Even more information is encoded in the higher secant ideals of G; these are defined in terms of the edge ideal, but have not received nearly as much attention. In this project, we use edge-splitting techniques to compute the Betti numbers of the edge ideal and of the higher secant ideals of two classes of graphs: wheels and complete tripartite graphs. Our calculations were assisted by the use of the software Macaulay2.

59. How Far is a Chicken McNugget® From Being Prime? And Almost Everything Else a Mathematician Could Possibly Want to Know about Chicken McNuggets®

Emelie Curl Bryn Mawr College
Staci Gleen Langston University
Advisor(s): Roberto Pelayo, University of Hawaii at Hilo

Numerical monoids have long been studied for their interesting (i.e., non-unique) factorization properties. While numerical monoids of embedding dimension 2 are relatively well-understood, the presence of a third minimal generator makes these monoids more difficult and interesting to study. We provide a complete analysis of $\text{M}^c\text{N} = \langle 6, 9, 20 \rangle$, an embedding dimension 3 numerical monoid whose elements correspond to the amounts of Chicken McNuggets® one can purchase using the traditional order sizes of 6, 9, and 20. Our analysis includes a closed formula for $\omega(x)$, the omega-primality of an element, which measures how far that element is from being prime in the monoid. Furthermore, we also develop formulae for the elasticity $\rho(x)$ and delta sets $\Delta(\text{M}^c\text{N})$, quantities which measure non-uniqueness of factorizations in M^cN. After presenting computational data, we will also provide conjectures for more general classes of embedding dimension 3 numerical monoids.

60. Catenary Degree of Numerical Monoids

Christopher Miller University of Wisconsin-Madison
Dhir Patel Rutgers University
Marly Corrales University of Southern California
Advisor(s): Scott Chapman, Sam Houston State University

Let M be a commutative cancellative monoid. For m a nonunit in M, the catenary degree of m, denoted $c(m)$, and the tame degree of m, denoted $t(m)$ are combinatorial constants that describe the relationships between differing irreducible factorizations of m. These constants have been studied carefully in the literature for various kinds of monoids, including Krull monoids and numerical monoids. In this talk, we show for a given numerical monoid S that the catenary degree and the tame are both eventually periodic. In addition, we will also talk about the numerical monoids generated by arithmetic sequences and the catenary degree of all the elements in the monoid. We show similar behavior for several functions related to the catenary degree which have recently appeared in the literature. These results nicely complement the known result that the delta sets of S also satisfy a similar periodicity condition.

61. Weak Rational Ergodicity of Rank-One Trasformations in Infinite Ergodic Theory

Francisc Bozgan University of California, Los Angeles
Jane Wang Princeton University
Advisor(s): Cesar E. Silva, Wiliams College

In 1979, Aaronson found an analogue of the Weak Ergodic Theorem in the infinite measure ergodic theory, namely if T is measure-preserving on X, with $\mu(X) = \infty$, and if there exists a special set $F \subset X$ of finite positive measure such that for all $A, B \subset F$,

$$\frac{1}{\sum_{k=0}^{n-1} \mu(F \cap T^k F)} \sum_{k=0}^{n-1} \mu(A \cap T^k B) \to \frac{\mu(A)\mu(B)}{\mu(F)^2}$$

then T is called *weakly rationally ergodic*. We show that all rank-one infinite measure-preserving transformations are subsequence weakly rationally ergodic and rank-one transformations with a bounded number of cuts are weakly rationally ergodic, extending work of Dai, Garcia, Padurariu and Silva. We further prove that there exist rank-one transformations that are not weakly rationally ergodic. We also study the properties of zero type and multiple recurrence in this context.

62. Generalized Mandelbrot Sets

Aaron Schlenker Butler University
Advisor(s): William Johnston, Butler University

A complex point z_0 is in the famous Mandelbrot Set fractal when an iterative process applied to z_0 and using the function z^2 stays bounded. We investigate what happens if we change the function so that z^2 is now composed

with a Mobius transformation, indexed on a parameter a. The Mandelbrot set corresponds to $a = 0$. What happens when we change $a = 0$ to other values, repeating the same iterative process and then drawing the sets? Do these "Generalized Mandelbrot Sets} have similar properties as the original? This presentation describes some surprising results, illustrates the sets in computer-generated movies, and uses transcendental functions to produce further set generalizations.

63. A Climate Change Analysis

Roshil Paudyal Howard University

Advisor(s): Robert Vanderbei, Princeton University

While the community of people acknowledging climate change seems to be expanding, there are still doubts over the extent to which temperatures have risen over the years. In a summer research project at DIMACS, NJ, with Dr. Robert Vanderbei from Princeton University, I created a world climate change map in which each location (weather station) was analyzed by solving a daily model with sinusoidal seasonal fluctuations. Daily temperature data for last 55 years was retrieved from the NOAA database and was used to create a regression model, which was then solved using LOQO, an optimization software. Initial results showed an average temperature rise of 2.26°F per century over the last 55 years. The data was then divided into three different time frames which yielded several interesting results including a period of "global cooling} in the 1950's and a sudden slowdown in rate of temperature rise in last 15 years. We also analyzed the effect of solar cycle in climate change by including it in the overall regression model, which induced minor changes in the initial results.

64. Mutual estimates for the dyadic Reverse Holder and Muckenhoupt constants for the dyadically doubling weights.

Temitope Ode Baylor University

Advisor(s): Oleksandra Beznosova, Baylor University

Muckenhoupt and Reverse Hölder classes of weights play an important role in harmonic analysis, PDE's and quasiconformal mappings. In 1974 Coifman and Fefferman showed that a weight belongs to a Muckenhoupt class A_p for some p if and only if it belongs to a Reverse Hölder class RH_q for some q. In 2009 Vasyunin found the exact dependence between p, q and the corresponding characteristic of the weight. The result of Coifman and Fefferman works for the dyadic classes of weights under an additional assumption that the weights are dyadically doubling. We extend the Vasyunin's result to the dyadic Reverse Hölder and Muckenhoupt classes and obtain the dependence between p, q, the doubling constant and the corresponding characteristic of the weight. We obtain our results using the method of Bellman functions.

65. Shuffle Showdown: Patterns of Perfect Shuffles of Even and Odd Sized Decks

Allysa Starkweather Nevada state College

Advisor(s): Aaron Wong, Nevada State College

Perfect shuffles are typically only an interest of magicians and card counters in Las Vegas casinos, but the art of perfect shuffling is actually a complex mathematical process. A perfect shuffle occurs when two equal piles of cards from a deck are interlaced. There are two types of perfect shuffles, in-shuffles and out-shuffles. The difference between the two comes from the decision of which pile was used to start the interlacing process. In an in-shuffle, the original top card becomes the second card in the shuffled deck. In an out-shuffle, the original top card remains the top card in the shuffled deck. Typical research on the patterns that arise between the periods of the various deck sizes usually only extends to even-sized decks. This research will explore the patterns among the periods of odd-sized decks that are perfectly shuffled, as well as the similarities and differences among the patterns for even and odd perfect shuffles.

66. Combinatorial Structure of Finite Frames

Allison Theobold Colorado Mesa University
Alice Chan Pomona College
Logan Stokoles University of California at Berkley
Advisor(s): Sivaram K. Narayan, Central Michigan University

A *frame* in an n-dimensional Hilbert space H_n is a possibly redundant collection of vectors $\{f_i\}_{i \in I}$ that span the space. A *tight* frame is a generalization of an orthonormal basis. A frame $\{f_i\}_{i \in I}$ is said to be *scalable* if there exist nonnegative scalars $\{c_i\}_{i \in I}$ such that $\{c_i f_i\}_{i \in I}$ is a tight frame. Here we study the combinatorial structure of frames and their decomposition into tight or scalable subsets. We prove conditions which these decompositions must satisfy and use these to fully determine when a frame in H_2 exists with a given tight subframe decomposition. We also study when a frame can be scaled to have a given tight subframe decomposition. This research was done as part of the Central Michigan University REU program.

67. Construction and Evaluation of a Ranked Retrieval Model Using Implicit Feedback from User Activity Data

Feng Zhu Princeton University
Alice Dilanchian University of California, Berkeley
Advisor(s): Zicong Zhou, University of California, Los Angeles

In this project we describe user-activity-based evaluation measures for the current Quick Search system on the Shoah Foundation's Visual History Archive (VHA) database, as well as a proposed ranked retrieval system for the database. The VHA is a database of over 50,000 video testimonies from Holocaust survivors which have been manually indexed with keywords. The evaluation measures based on user search and view data—click entropy, precision, and recall—demonstrate how user clickthrough data can provide useful implicit feedback about the performance of information retrieval systems. The proposed ranked retrieval model incorporates a vector space model and a PageRank-like algorithm with a rich weighting scheme incorporating keyword metadata and user activity data, and combines these features using a ranking support vector machine with training data provided by user clickthrough data. Human evaluation of our proposed model showed that its results were preferred to those of the current Quick Search system by 75% of human evaluators.

68. Nucleation in a Two Component Metal Alloy and the Corresponding Boundary Conditions

Kalea Sebesta University of Nevada, Reno
Advisor(s): Evelyn Sander, George Mason University

This is a numerical study that explores the phase separation phenomenon, known as nucleation, specifically in a two component metal alloy. The aim of this study is to understand the change in the number of components both as a function of time and a function of parameters. In order to accomplish this, numerical topology code to find the number of components was developed and analysis was used to develop some heuristic arguments. For the purpose of this research, a stochastic equation was used implying, that there are necessarily large deviations in behavior in an individual run. Therefore, it was necessary to perform and average a large number of simulations to see the full scope of the behavior. These arguments paved way for predictions of the expected behavior both in time and in parameter variation. Furthermore, the stochastic behavior is what gave rise to the predictions for how the behavior should change in time. The results are based on the theory of large deviations which say that the time to nucleation should depend on the largest eigenvalue.

69. Modeling and Analysis of Strategies in the Symmetric and Asymmtric El Farol Problem

Weicheng Ye Stony Brook University
Advisor(s): Harbir Lamba, George Mason University

The El Farol Problem is related to minority games where agents use mixed strategies to make attendance decisions. Cross et al. established a strategy of asymmetric game based on agents' dominant psychological characteristic. In this research, we establish both symmetric and asymmetric El Farol games. Due to agents' psychological behavior and the unpredictable occurrence, we introduce noise and strategies for periodicity recognition. We compute the hamming distance to see agents' decision based on the same strategy space. We then analyze the model to test

the periodicity, and establish the comparison of strategies between symmetric and asymmetric games. We study the agents' attendance's relationship to the past aggregate or individual attendance/minority side history. In the last section, we compare our program result of attendance behavior to the experimental result. Our results suggest that the simple model of human decision making in the El Farol game suggested by Cross et al, when modified with noise and periodicity recognition, is broadly consistent with experimental data in the symmetric game scenario. When the model is extended to deal with the asymmetric game we postulate, based upon the above observations, that the aggregate behavior is far less efficient.

70. Modeling Population Growth of the Invasive Bighead and Silver Asian Carp in the Illinois River

Matthew Delbert New Jersey City University
Advisor(s): Zhixiong Chen, Yi Ding, New Jersey City University

Asian carp, specifically the Bighead and Silver variety, are currently the subject of intense focus in the Mississippi River basin. They pose a biological threat to the native species, and the potential to establish breeding populations in the Great Lakes. This could negatively impact the multi-billion dollar fishing and recreation industry of the area. Asian carp were originally imported to the United States in the latter half of the 20th century to help reduce the amount of algae present in a given waterway. Flooding events allowed them to escape into the Mississippi River system, where they eventually migrated to the Illinois River, and have since dominated native competition. We modeled the population of Bighead and Silver Asian carp in the Illinois River, considering different aspects affecting carp populations, such as egg viability and commercial fishing. We used this to make predictions regarding future populations. The potential to profit from the cultivation and sale of Asian carp for domestic consumption and exportation to other countries was also examined. The results will allow us to determine whether or not sufficient large-scale fishing is being done to successfully reduce Asian carp populations in the Illinois River, and the benefits of taking such action.

71. The Dynamics of Offensive Messages in the World of Social Media: the Control of Cyberbullying on Twitter

Krystal Blanco Boston University
Aida Briceno Columbia College
Javier Tapia St. Mary's University
Andrea Steele Medgar Evers College (CUNY)
Advisor(s): Kamuela Yong, Sherry Towers, Arizona State University

The 21st century has redefined the way we communicate, our concept of individual and group privacy, and the dynamics of acceptable behavioral norms. The messaging dynamics on Twitter, an internet social network, has opened new ways/modes of spreading information. As a result cyberbullying or in general, the spread of offensive messages, is a prevalent problem. The aim of this report is to identify and evaluate conditions that would dampen the role of cyberbullying dynamics on Twitter. We present a discrete-time non-linear compartmental model to explore how the introduction of a Quarantine class may help to hinder the spread of offensive messages. We based the parameters of this model on recent Twitter data related to a topic that communities would deem most offensive, and found that for Twitter a level of quarantine can always be achieved that will immediately suppress the spread of offensive messages, and that this level of quarantine is independent of the number of offenders spreading the message. We hope that the analysis of this dynamic model will shed some insights into the viability of new models of methods for reducing cyberbullying in public social networks.

72. Atomistic Simulations Using an Improved Kinetic Monte Carlo Method

Zephyr Penoyre University of Cambridge (Trinity Hall)
Advisor(s): Ted Yu, UCLA/ Cal State Long Beach

When dealing with large atomistic simulations there can be huge numbers of particles moving simultaneously. Simulating all of these movements at once is incredibly computationally expensive, so instead we often use a powerful approximation, the kinetic Monte Carlo method, which picks and moves one particle at a time randomly, weighted by their rate of movement. Efficient solutions exist for these simulations when there are only a few discrete rates in the system, but we have developed a variation on the method that allows not only for a continuos distribution of rates, but also accommodates rates varying over huge ranges, with an efficient and versatile algorithm.

73. A Contagion Model of Emergency Airplane Evacuations

Junyuan Lin Pepperdine University
Advisor(s): Timothy Lucas, Pepperdine University

Inspired by the Asiana Flight 214 crash in San Francisco this summer, this project focuses on modeling an emergency airplane evacuation. Our models are based on the Particle Swarm Optimization (PSO) algorithm, where each agent's position is compared to a fitness function that describes the current environment. Each agent moves according to its knowledge of its own previous best position and the group's current best position. The static environment is modeled by a potential function that describes the layout of the airplane that includes the exits and physical barriers such as the seats. We model the interactions within the swarm by an attraction-repulsion force. Finally, we chose to incorporate the spread of an emotion such as fear or panic that influences the behavior of agents within the swarm. Our project includes an analysis of how the parameters and scaling of different parts of the model affect the swarm behavior. We also compared simulations with and without fear to study the impact of emotion on individual behavior as well as the ability of the entire group to safely exit the aircraft. We hope that this will lead to increased understanding of how panicked crowds behave in evacuation situations and that this will lead to better, safer evacuation designs.

74. Modeling the Asian Carp Invasion Using Mathematical Evolutionary Game Theory

Jasmine Everett Bennett College
Hakimah Smith Bennett College
Advisor(s): Hyunju Oh, Bennett College

Asian Carp were imported from China in the 1970s to improve water quality of aquaculture and to control aquatic vegetation ponds. However, they subsequently migrated from ponds into the Mississippi river, where they quickly reached high population density. They are considered invasive species, highly detrimental to the ecological balance, as they threaten the native fish population, eating up the algae and other microscopic organisms essential for the survival of the native fish. The Asian Carp now threatens the Upper Mississippi River and the Great Lakes. The goal of our project was to gain a better understanding of the interaction between native species and Asian Carp using mathematical model of evolutionary game theory. Our model consists of three players, one Asian Carp, Silver Carp and two native fishes Gizzard Shad and Largemouth Bass and various parameters of the game. We found best strategy to slow down or even stop the invasion depends on parameters through the simulations using MAPLE.

75. An Asymptotic and Numerical Study of Two Euler-Bernoulli Beams Connected By Standard Types of Dissipative Joints

Chris Abriola Marist College
Advisor(s): Matthew P. Coleman, Fairfield University

In the study of collinearly linked beams, there are four standard types of linear dissipative joints. We consider the case of two identical Euler-Bernoulli beams, coupled via any of these dissipative joints, and subject to the standard energy-conserving boundary conditions at the outer ends. Specifically, we would like to estimate the vibration spectrum of all 40 possible cases (4 joints, 10 boundary condition combinations). Our approach is two-pronged, involving both an asymptotic and a numerical method. While the latter is more accurate and, indeed, is the closest to an exact solution that we can get, the former allows us to see similarities between various types of joints, and makes it much easier to see if two joints might be equivalent, at least in a limiting/asymptotic sense. Indeed, we do find that two of the four joints are asymptotically equivalent, for each possible choice of boundary conditions. We also find a number of other interesting relationships. We conclude with a comparison between the asymptotic and numerical results, as well as a comparison of the numerical results for different values of the damping parameters.

76. Spectral Theory for Expanding Maps with Countable Markov Partitions and Holes

Lisa Naples Fairfield University
Paul Frigge Northwestern University
Allison Grossman Fordham University
Advisor(s): Mark Demers, Fairfield University

We study expanding maps of the unit interval that admit countable Markov partitions after the introduction of a hole. We prove that the transfer operator associated with such open systems is quasi-compact and admits a spectral

gap on an appropriate function space. In this context, the eigenfunction corresponding to the maximal eigenvalue represents a physically relevant quasi-invariant distribution for the open system. As a by-product, we prove that the spectrum of the transfer operator scales precisely after the introduction of the hole so that the spectral gap does not deteriorate as the hole gets larger as long as a mixing condition is satisfied. This work was completed as part of the Fairfield University REU during Summer 2013.

77. Bouncing Bifurcations: A comparison of the bifurcation and chaotic behaviors between two dynamical systems

Matthew Cessna University of La Verne
Advisor(s): Michael Frantz and David Chappell, University of La Verne

A bouncing ball on a sinusoidally vibrating plate is a classic physics model that has been thoroughly studied over the last few decades with the results being utilized for various applications. Similarly, a droplet bouncing on a bath of the same viscous fluid that oscillates vertically has been a popular subject of study in recent years to gain further insight into applications of fluid dynamics. In this presentation, we create a mathematical model of the bouncing ball by constructing a system of iterated equations with dimensionless parameters while employing the high-bounce approximation to simplify the analysis of the ball's behavior. We determine the fixed points of the system and mathematically analyze where the threshold of bifurcation occurs and when chaotic regimes materialize. We then vary the values of the dimensionless parameters of the bouncing ball model and create bifurcation diagrams in Matlab. We compare our findings with results obtained from our laboratory experiments with bouncing droplets of silicon oil to investigate whether the two systems exhibit similar bifurcation and chaotic behavior.

78. Modeling Coalition Formation Patterns and Pareto Optimization

Carrie Giesen Loyola Marymount University
Ben Whitesell Loyola Marymount University
Advisor(s): Ben Fitzpatrick, Loyola Marymount University

This paper examines algorithms for coalition formation in an agent-based population model. Agents are assigned a political position and resource value. Coalitions are formed through a utility function that balances a desire for resources with similarity of political positions. Agents move in a grid and test if they want to join with a neighbor based on their utility functions, aiming to find Pareto-optimal coalitions. Computing all possible coalitions to seek Pareto optimality is difficult for even small populations. Thus, we developed simulation-based approaches. Six different methods of coalition formation were modeled. The first were "backroom pair"—two agents made decisions based on their own personal needs and not the entire coalition, "democracy"—over half of the agents in each coalition had to benefit from the joining, and "communism"—all of the agents had to benefit from the merge. The other models contained the same stipulations as the first three—backroom pair, democracy, and communism—with an additional option. Agents were allowed to "jump} from their coalition and join another if it was beneficial. Through multiple simulations, we found patterns in the output of these algorithms and parameters that lead to different distributions of coalition structures.

79. Modeling Civil Violence Patterns Among Coalitions

Carrie Giesen Loyola Marymount University
Ben Whitesell Loyola Marymount University
Advisor(s): Ben Fitzpatrick, Loyola Marymount University

In this paper, we present an agent-based model of civil uprisings. Starting with Epstein's well-known computational model of civil violence, we extended the population structure to include individual resources and political positions. The model is a cellular automata structure, with agents walking randomly on a two-dimensional grid. Epstein's model involves civilian and government (policing) individuals, with civilians characterized by individual hardship, risk tolerance, and government legitimacy, parameters with which agents make decisions about whether or not to revolt. Our model connects hardship with resources and legitimacy with political positions. The Epstein based model was then combined with a coalition forming model to observe how groups of agents go active. We also considered a model of charisma as a means for leaders to emerge to direct and motivate coalition activity. Lastly, the model was extended to permit directed, rather than random walking, movement, making agents more likely toward areas more heavily populated with members of their coalitions. We present a variety of simulation results demonstrating the effects of the newly introduced models of agents on the population-wide behaviors.

80. Linearly Stable Relative Equilibria Utilizing a Dominant Mass

Gopal Yalla College of the Holy Cross
Margaret Hauser College of the Holy Cross
Advisor(s): Gareth Roberts, College of the Holy Cross

We consider the linear stability of relative equilibria (rigid rotations) in the $(1+n)$-body problem where one mass is 'dominant' in relation to the others. For various n values, and for stable solutions, we numerically search for the smallest possible ratio between the dominant mass and the sum of the remaining masses. Based on a conjecture of Moeckel's, we apply the technique of gradient flow to locate minima of the Newtonian potential function constrained to a level surface of the moment of inertia. These minima are possible candidates for linearly stable configurations. Then, by computing the Jacobian of the equations of motion at the relative equilibrium, linear stability was determined by finding whether or not all the eigenvalues were purely imaginary. Using Matlab, various programs were written to follow different families of central configurations and calculate the dominant mass ratio. This program provided several revealing pictures and captivating animations.

81. Approximating the Geometry of Eye Blinks

Dylan Chapp University of Delaware
Advisor(s): Richard Braun, University of Delaware

The tear film is a dynamic system that regulates the health of the eye and quality of vision. Models of time-evolution of tear film fluid have been numerically solved on a variety of domains, and in order for these models to be useful to ophthalmologists, the model domain should accurately reflect the physiology of the eye. We present a method for generating a smoothly time-varying 2D domain on which numerical solutions to model equations for tear film evolution can be computed. This time dependent domain originates with digital video of a blink and employs image segmentation techniques and parameterized least squares fitting to generate a the boundary of a 2D eye-shaped domain for each frame of the input. These domains are then fit again by least squares to produce a smoothly time-varying domain.

82. Substance Abuse via Legally Prescribed Drugs: The Case of Vicodin in the United States

Wendy Caldwell University of Tennessee, Knoxville
Benjamin Freedman Bucknell University
Luke Settles Southern Illinois University Edwardsville
Michael Thomas Kennesaw State University
Advisor(s): Erika Camacho, Stephen Wirkus, Arizona State University, West Campus

Vicodin is the most commonly prescribed pain reliever in the United States. An estimated two million people currently abuse Vicodin, and the majority of those were initially exposed to it via prescription. Our goal is to determine the most effective strategies for reducing the overall population of Vicodin abusers: prevention methods aimed at educating doctors and patients on the potential for drug abuse or treatment methods implemented after a person abuses Vicodin. We consider compartmental models in which medical users of Vicodin can transition into an abuser compartment or leave the population by no longer taking the drug. Once abusers, people can transition into a treatment compartment, with the possibility of leaving the population or of relapsing and re-entering the abuser compartment. A linear model assumes no social interaction, while a non-linear model considers interaction with abusers affecting both the relapse rate and the number of new prescriptions. Sensitivity analyses are conducted to determine which strategy has the greatest impact on the population of abusers. From these models, we determine that prevention has a greater impact treatment. We also note that increasing the rate at which abusers seek treatment has a greater impact than the treatment success rate.

83. Finite Volume Model for Analyzing Dam-Break Caused By Mud Expansion

Raden Hadi Indonesia University of Education
Indra Hardiyana Indonesia University of Education
Marjan Nurjanah Indonesia University of Education
Advisor(s): Sumanang Muchtar Gozali, Indonesia University of Education

Failure by oil drilling company has impact in the plains region, one of which may result hot mud flood from below earth. As happened in eastern Java, Sidoarjo, dam used to block hot mud flood. The dam-break can cause great

destruction in that region around the dam. Therefore, to understanding the nature of the mud expansion that cause dam-break is an important problem. In this poster, dam-break problems solved using the Finite Volume Method for one-dimensional case. The mathematical equation we use is the continuity equation and the momentum balance $\frac{\partial A}{\partial x} + \frac{\partial Q}{\partial x} = 0 \frac{\partial Q}{\partial x} + \frac{\partial}{\partial x}(\frac{Q^2}{A} + gI_1) = g\frac{\partial I_1}{\partial x} - gAS_f$ which could explain the mud expansion in dam and its analytical solution to the dam-break problem. Central-upwind method which is one kind of finite volume method to be tested method to solve the dam-break problem.

84. Exact and Modeled Solutions of Total Variation Minimization

Erik Bates Michigan State University
Advisor(s): Yang Wang, Michigan State University

Total variation (TV) minimization has been widely used as an image denoising model, allowing for the smoothing of a known but noisy signal. This model is particularly useful in application when the desired, but unknown function is well approximated by piecewise constant functions with a small number of pieces, as solutions to TV optimization are obtained by penalizing variation. Moreover, application has been made possible by numerical algorithms converging to unique minimizers of a given TV model. Relatively less, however, is known about the actual form of these solutions. We present an exact description of TV minimizers when the given signal is monotonic or piecewise linear. From these fundamental cases, we develop a more complete understanding of the behavior and performance of TV minimization.

85. Putting Humpty Dumpty Together Again

Yiwen Hu Macalester College
Sophors Khut Macalester College
Advisor(s): Rob Thompson, Macalester College

Humpty Dumpty sat on a wall, Humpty Dumpty had a great fall, how do we put Humpty Dumpty together again? We show how the method of invariant signatures can be used to match three dimensional curves based only on shape, without regard to position or orientation. This curve matching technique is applied to the problem of reassembling a broken surface such as an eggshell. Our work is based on new geometric algorithms for automated assembly of jigsaw puzzles. All the king's horses and all the king's men would love to see this.

86. Effects of a Contact Lens and the Blinking Cycle on Tear Film Deposition and Drainage

Jonathan Horton George Mason University
Advisor(s): Daniel Anderson, George Mason University

This study analyzes the dynamics of the eye's pre-lens tear film when a contact lens is in place. The blinking cycle replenishes the tear film and causes motion of the contact lens. With assumptions based on lubrication theory, equations governing the fluid dynamics of the tear film are simplified to provide an evolution PDE describing the rate of change in the tear film's thickness. Further research will explore numerical solutions, which may provide necessary insight for treating dry eye in contact lens wearers.

87. Metaheuristics Using Agent-Based Models for Swarms and Contagion

Daniel Moyer University of California, Los Angeles
Douglas de Jesus University of California, Los Angeles
Advisor(s): Jesus Rosado, UCLA

Swarming is a natural phenomenon observed in numerous animal species, such birds, fish, and humans. Groups of individuals move cohesively and exhibit complex group behavior despite the lack of obvious leadership or communication. Previous mathematical literature has explored particle based models for reproducing such swarms, with more recent work introducing an emotional contagion along with the dependence on space and velocity. In the present work we explore a contagion model that allows parameters to respond to selection pressure. In particular, we present numerical results of simulated evolutionary selection on a variety of parameters. Two separate genetic algorithms were implemented and run for a large number of generations on the same numerical framework. The framework itself implemented a multi-threaded linked cell scheme. Both algorithms show apparent convergence to the same biologically interpretable values.

88. Applications of Mathematical Ecology to Rotational Grazing

Mayee Chen Jamestown High School
Advisor(s): Junping Shi, The College of William and Mary

It is a common understanding that rotational cattle grazing provides a better yield than continuous grazing, but a quantitative analysis is lacking in agricultural literature. In this paper, we mathematically show how production yields and stockpiled forage are greater in rotational grazing. We also reveal ideal configurations to maximize the number of cattle sustained per acre. In rotational grazing, cattle periodically move among paddocks in contrast to continuous grazing, in which the cattle graze on a single plot for the entire grazing season. Using the logistic equation for grass growth and a modified Michaelis-Menten equation for the cattle consumption rate, we construct a vegetation grazing model on a fixed area. With parameters from agricultural publications and keeping the minimum value of remaining forage constant, the differential equation model is integrated over a one-year time span with various grazing patterns. Our results show that both the number of cattle per acre and stockpiled forage increase for all tested rotational configurations. More specifically, the former increases more than 50

89. Automatic Detection of Rare Bird Species Using Neural Networks

Nicole Bender Marist College
Advisor(s): Arik Kershenbaum, NIMBioS

Wildlife Conservationists are constantly developing new methods for endangered species management. Since many species use vocalizations to communicate, researchers have begun monitoring population dynamics of endangered species by taking advantage of this natural sound source. For example, the prairie warbler, native to the Eastern United States, has recently been placed on the Audubon Conservation WatchList as a result of a decline in their natural habitats due to anthropogenic causes. The concern for prairie warbler populations made them an ideal species to concentrate on. The goal of this project was to design an algorithm to detect prairie warblers using recorded audio signals. This process was broken up into four separate components: 1) data collection, 2) data preprocessing for input into a neural network, 3) training the chosen neural network using large data sets, and 4) performance testing of the algorithm on two data sets obtained both by ground-based and balloon recordings.

90. The Dynamics of Immune-Mediated Laser Therapy of Cancer

Bryan Dawkins University of Central Oklahoma
Advisor(s): Sean Michael Laverty, Professor/University of Central Oklahoma

We will present a mathematical model composed of a system of ordinary differential equations describing the immune-mediated dynamics of cancer cell populations with exponential growth. The model will include laser-initiated cancer destruction by means of several classes of immune cells. The primary cells in the immune response for this treatment are Dendritic cells, Cytotoxic and Helper T cells, and B cells. Also included in the model are antibodies and tumor antigen, which play a central role in the success of the treatment. We will show successful treatment and the conditions under which this may occur. In addition, we will describe conditions under which failed treatment may occur. Whenever possible, the results of the model will be compared to experimental results from our collaborators, to show the relative accuracy of the model. To expand the model, treatment of cancer cell populations with non-exponential growth will be discussed. We will show that the ultimate success of laser immunotherapy of cancer is highly related to immunoadjuvants represented by parameters of our model.

91. Evidence of Purifying Selection in Humans

John Long MIT PRIMES
Advisor(s): Angela Yen, MIT

The Human Genome Project completed in 2003 gave us a reference genome for the human species. Before the project was completed, it was believed that the primary function of DNA was to code for protein. However, it was discovered that only 2

92. Degree of Regularity of Linear Homogeneous Equations

Kavish Gandhi Newton North High School
Noah Golowich Lexington High School
Advisor(s): Laszlo Miklos Lovasz, Massachusetts Institute of Technology

An equation is r-regular if, for every r-coloring of the positive integers, there exist positive integers of the same color, or monochromatic, that satisfy the equation. We define a linear homogeneous equations to be strongly r-regular if, when a finite number of inequalities is added to the equation, the system of the equation and inequalities is still r-regular. In our project, we show that if a linear homogeneous equation is r-regular, then it is strongly r-regular. In 2009, Alexeev and Tsimerman introduced a family of equations, each of which is $(n-1)$-regular but not n-regular, verifying a conjecture of Rado from 1933. These equations can be shown to be strongly $(n-1)$-regular as an immediate corollary of our results. We also derive conditions which are sufficient to show the r-regularity of a linear homogeneous equation for any integer r.

93. Generalizations of Tokuyama's Formula to the Hall-Littlewood Polynomials

Uma Roy Boston University PROMYS
Roger Van Peski Boston University PROMYS
Advisor(s): Lucia Mocz, Princeton University

The Schur polynomials are an important class of symmetric functions indexed by partitions, which are used to calculate the characters of irreducible representations of general and special linear groups. We study the Hall-Littlewood polynomials, a class of symmetric functions which generalize the Schur polynomials through a deformation parameter. There exists a result known as the *Weyl Character Formula* which expresses Schur functions algebraically. A later result due to Tokuyama gives an alternative expression for Schur functions, as a sum over combinatorial objects known as *Gelfand-Tsetlin Patterns*: triangular arrays of nonnegative integers subject to certain restrictions. Though much work has been done to generalize Tokuyama's result to representations of other groups, little research exists in generalizing the formula to other symmetric functions, such as the Hall-Littlewood polynomials. We provide several generalized formulae which express Hall-Littlewood polynomials as a sum over Gelfand-Tsetlin patterns of a specific top row, providing further connections between special classes of symmetric functions and combinatorial objects.

94. Minimum Sumset Size

Karenna Genzlinger Gettysburg College
Advisor(s): Bela Bajnok, Gettysburg College

We are interested in finding the minimum possible size of an h-fold restricted sumset of an m-element subset of $\mathbb{Z}_n$, denoted by $\rho\hat{}(n,m,h) = min\{|h\hat{}A| \mid |A| = m\}$. A good upper bound for $\rho\hat{}(n,m,h)$ is $u\hat{}(n,m,h)$, which is found by comparing a few strategically chosen m-subsets of $\mathbb{Z}_n$.

95. The Colored Cubes Problem

Rachel Katz University of Chicago
Daniel Condon Georgia Tech
David Cervantes Nava SUNY Potsdam
Advisor(s): Ethan Berkove, Lafayette College

Given a palette of $m \leq 6$ colors, we say a cube is m-colored if each face is painted with a single color, and each color appears on at least one face. The number of distinct m-colorings depends on m; for example, $m = 6$ gives rise to 30 distinct cubes up to rigid isometry. An interesting question is to determine the minimum number $k(n)$ so that given an arbitrary collection of $k(n)$ m-colored cubes, it is always possible to assemble a larger $n \times n \times n$ cube where each $n \times n$ face is a single color. In this talk we will discuss work on this problem. We will also discuss related findings on how the 6-coloring relates to the symmetric group on 6 letters.

96. Comparing Methods for Classifying Open Knots

Nicole Lopez University of St. Thomas
Elizabeth Annoni University of St. Thomas
Advisor(s): Eric Rawdon, University of St. Thomas

The recent discovery of knotted behavior in proteins has stimulated discussion about how to classify knots in open chains. Topologically speaking, all open arcs are equivalent to a straight line. However, there may still be entanglement present in an open chain that intuitively resembles a knot. The goal of this project is to compare two methods for classifying that entanglement. The first method connects each of the two endpoints of the open chain to points distributed on a sphere to close the knot and calculate which knot type is most prevalently found. The second method creates random arcs that connect the two end points, which again closes the knot and enables the calculation of the knot type found in the entanglement. These two methods are compared to determine the most precise and computationally efficient method for classifying an open knot.

97. An Exploration of Methods of Solving the Combinatorial Game Flood-it

Jason Jaeckel Colorado State University
Rachel Popp Colorado State University
Amanda Rose Colorado State University
Christie Burris Colorado State University
Advisor(s): Patrick Shipman, Colorado State University

Flood-It is a game consisting of an $n \times n$ grid where each entry is one of k colors. The player starts with one block whose color he controls. The objective is to turn the board all one color. The player switches the color of his controlled block(s) to a color of his choosing, any of the adjacent blocks of the same color are then under his control. The player floods the board by repeating this process. Our team has created a version of Flood-It in Matlab in order to develop and analyze algorithms which flood the board. With this research, we have considered generalizations of this game and questions on the space of all boards. We also consider the problem of minimizing the number of moves necessary to flood the board. To address this question, we apply our algorithms to a sample from the space of all possible boards and develop a heuristic for choosing the best algorithm to solve a given board. In analyzing the board space we appeal to methods of data analysis, including recently developed topological methods.

98. Approximating the products of parameterized functions

Alexander Goldstone George mason university
Advisor(s): Habir Antil, George Mason University

We consider a newly proposed algorithm for approximating products of parameterized functions. The algorithm applies the reduced basis method to construct basis which generate a subspace representative of the parameterized function. The algorithm applies a two-step procedure for evaluating products with multiple parameters. The cost and error of the algorithm are analyzed across multi-parameter cases.

99. Dimensionality Reduction of Video Data using SVD

Brian Notarianni George Mason University
Advisor(s): Tim Sauer, George Mason University

We give a quick overview of Singular Value Decomposition SVD and how it can be used for dimensionality reduction. Our approach separates out temporal and spatial data from videos by breaking it down into subimages. We apply our approach to track a Brownian ball.

100. Interpolation Using the Min Kernel

Martin Dillon Illinois Institute of Technology
Advisor(s): Greg Fasshauer, John Erickson, Illinois Institute of Technology

We find a relationship between the second derivative of a function $f \in C^2[0,b]$, such that $f(0) = 0$, and the coefficients c_j which are obtained from solving the interpolation problem $S_f(x_j) = f(x_j)$ at points x_j, $j = 1, \ldots, n$ where $x_j \in (0,b]$, $S_f(x) = \sum_{j=1}^{n} c_j K(x, x_j)$, and K is the min kernel $K(x,z) = \min(x,z)$. We derive the discretization of this relationship easily by using linear algebra, and then interpret it as an integral representation formula using Green's functions among other methods. Since, under appropriate boundary conditions, the min kernel

is the Green's function for the Laplacian operator and also the covariance of two Brownian processes, this suggests that the well known link between the Laplacian and Brownian motion is also related to solving interpolation problems. This is the first step to generalizing this result to the fractional Laplacian operator and fractional Brownian motion using the generalized min kernel $K_H(x,z) = \frac{1}{2}(\|x\|^{2H} + \|z\|^{2H} - \|x-z\|^{2H})$, with Hurst parameter $H \in (0,1)$. This work is being performed the supervision of John Erickson and Greg Fasshauer and supported by the grant NSF-DMS–1115392.

101. Supervised STM Image Segmentation of Self-Assembled Molecule Layers

Daniel Lander Pepperdine University
Advisor(s): Dominique Zosso, University of California, Los Angeles

Self-Assembled monolayers (SAMs) of cage molecules are useful in the construction of nanostructures and nanodevices. For the purposes of nanofabrication, it is useful to monitor the formation of defects in these SAMs and differences in molecular lattice orientation. Through autonomous and supervised image processing, this project aims to segment STM images of SAMs of cage molecules into domains of similar lattice orientation. Due to the translation invariance of the power spectra of the Fourier domain, it is possible to obtain a measure of the similarity or difference between the lattice orientations in two regions in an image and implement traditional clustering methods such as K mean and spectral clustering or more advanced methods such as Markov random fields. The results from the application of these methodologies are promising and could possibly replace manual segmentation as the industry standard.

102. Persistent Homology on Natural Language Text

Sky Hester Western Washington University
Advisor(s): James Johnson, WWU

Given an alphabet A, consider a string of text $D = \{\alpha_i\}_{i=1}^{L}$ where each $\alpha_i \in A$, also referred to as a *document*. Given a *space* character $\_ \in A$ and a *sentence stop* character $. \in A$, separate this document into 1-grams $G_1 = \{\ \{\alpha_i\}_{i=\ell}^{m} \mid \alpha_{\ell-1} = \_\,, \alpha_{m+1} \in \{\_, .\}\}$. From these, construct the n-gram transition probability matrix P_D, defined by $P_{D_{ij}} = \mathbb{P}(g_{\ell+n+1} | g_\ell g_{\ell+1} \dots g_{\ell+n})$ where $g_{\ell+n+1}$ is the ith 1-gram and $g_\ell g_{\ell+1} \dots g_{\ell+n}$ is the jth n-gram (an n-gram is a consecutive sequence of 1-grams of D). Then a metric between documents C, D is $\langle vec P_C, vec P_D \rangle$ and thus there is a finite metric space of the set of all documents D_A employing the alphabet A. The homology of a sequence of Vietoris-Rips simplicial complexes on this space indexed by distance is persistent within a range, and this feature can be exploited for information retrieval. This poster discusses theory and implementation of the process on an archive of scientific documents.

103. Finite Geometry in the Card Game SET

Yumi Li Willamette University
Claire Frechette Brown University
Jordan Awan Clarion University
Advisor(s): Liz McMahon, Lafayette College

The card game SET serves as an excellent model for the ï¬nite geometry AG(4,3). Using that model, previous researchers have found partitions of AG(4,3) into 4 disjoint maximal caps along with a distinguished point/card. We define a new geometric object, a demicap — a maximal cap can be written as the union of two disjoint demicaps. We will present results about these demicaps, which provide insight into the maximal cap partitions of AG(4,3).

104. The Electronic Computation of Simple Perfect Squared Squares

Joseph DiNatale Armstrong Atlantic State University
Advisor(s): Casey Mann, University of Washington Bothell

"Squaring the square" is the problem of dissecting squares of integer side lengths into several smaller squares also of integer side lengths. It is known that an ordinary square may be dissected into at minimum 21 squares. In a similar manner, quotients of squares may be squared. For example, it is known that a cylinder formed as a quotient of a square can be squared with as few as 20 squares. In this project, we developed an algorithmic approach for finding squared quotients of squares, extending the results of S.J. Chapman.

105. Applying Graph Connectivity Indices to the study of Chagas Disease Vectors

Pamela Guerron Montclair State University
Zeyad Boodoo Rutgers Univeristy
Advisor(s): Aihua Li, Montclair State University

Using graph index techniques to analyze DNA data has become a new method in recent years. Our research applies the Randić Connectivity Index, together with various generalized forms of it, to analyze DNA sequence data of Chagas disease obtained from biology research. It aims to better understand the evolutionary relationships of the insect vectors. We apply the graph index method on converted DNA line graphs, perform normalization and principle component analysis on them, and build phylogenetic trees from the data in order to answer scientific questions about the disease. In this presentation we will report the results from this on-going research. This research is funded by NSA (grant H98230-13-1-0270) and NSF (grant DMS-1156582) through MAA NREUP.

106. Forecasting Latent Business Conditions Using Macroeconomic Factors and the Kalman Filter

Rachel Gosch The University of Texas at Austin
Sonia Mahop Howard University
Kimberly McCarty University of Redlands
Jerin Kurien City College of New York
Advisor(s): Marcel Blais, Worcester Polytechnic Institute

Defining and measuring a financial concept such as the current state of the economy is a difficult task. The ability to reliably forecast business conditions to hedge against adverse future market movements poses an even greater challenge, particularly in portfolio management. We consider issues involved with this type of estimation, including the use of numerous interrelated macroeconomic factors and varying information flow frequencies, and we develop an underlying structure for business conditions. Our model builds on the Kalman filter-based Aruoba-Diebold-Scotti Index, proven to be effective in tracking the state of the economy. We extend this framework to accurately forecast major shifts in the economy based on real-time data. In particular, we focus on the creation of multinational indices for relative comparison of business conditions across a set of countries. With these indices, we aim to effectively predict movements in exchange rates for optimal portfolio management strategies.

107. Solar Verification Project

Ariel Sitler West Virginia University
Advisor(s): Chris Clack, NOAA ESRL Boulder, CO

As we drain the Earth's fossil fuels, climate change increasingly puts pressure on scientists to find a cost-efficient renewable energy solution. A nation-wide system using weather resources already in place in combination with the construction of transmission lines and new electricity producing sites is proposed by mentor Chris Clack and associates. The model created optimizes this system with the projected electric load from 2006–2008. Real world publicly accessible solar data was collected and analyzed from ten sites across the contiguous U.S. Corresponding estimation data was computed by the model for solar and collected for comparison. The observation data had been recorded multiple times per hour, and was averaged over the top of the hour to be compared with the hourly model data. Results showed that observation data generally fit well with model data. Correlation coefficients range from .85 to .96 for Global shortwave broadband irradiation. The model was found to perform more effectively in summer months compared to winter months. This was to be expected, as more cloud interference is present throughout the winter compared to summer. The investigation has built accreditation for the wind and solar model, and has also provided a larger pool of data to use in the regression.

108. Comparison of Compressed Sensing Techniques

Trevor Steil Michigan State University
Jonathan Lai University of Texas at Austin
Advisor(s): Leslie Hogben, Namrata Vaswani, Iowa State University

Compressed sensing recovers a sparse vector $x \in \mathbb{R}^n$ from the underdetermined system $y = Ax$ given A and y, where $A \in \mathbb{R}^{m \times n}$ with $m < n$ and $y \in \mathbb{R}^m$. Compressed sensing via basis pursuit is a technique of recovering x by performing an ℓ_1 minimization over all solutions z to the equation $y = Az$. Variations on basis pursuit can be used if a set T is known to approximate the set of indices corresponding to the nonzero entries in x. LS-CS uses least-squares approximation to get an initial estimate for x and then performs basis pursuit on the residual. Modified-CS performs an ℓ_1 minimization on T^c (the complement of T) over all z subject to the constraint $y = Az$. Weighted-CS performs the ℓ_1 minimization on T^c with a small weight added to the contribution on T subject to the same constraints as before. These basis pursuit variations are compared using theoretical results concerning sufficient conditions to guarantee exact reconstruction of x, as well as through numerical simulations. Compressed sensing techniques are of particular interest for applications to image processing.

109. Understanding the Dynamics of Drug-Related Crimes in Chicago Across Time and Space Using Differential Equations

Joanna Sasara Dominican University
Advisor(s): Marion Weedermann, Dominican University

In this project we develop a model and give a description of the spread of illicit drugs possession and trafficking over time. The model is based on the theory of differential association, which proposes that through social interaction with others, individuals learn the values, techniques, and motives for criminal behavior. The population is split into three distinct groups: those who are not interested at all in committing a crime, those who are susceptible to commit a crime, and those who already committed a crime. The model consists of a system of differential equation that describes the flows between these groups, which affect the evolution of drug-related crimes. In this project we show that an approach to understanding the dynamics of crime is similar to the approach used in mathematical biology to model the spread of containment of disease. The results we illustrated using data from drug-related crimes from Chicago, 2006–2010.

110. Effects of Vertical Flux on Nutrient-Phytoplankton-Zooplankton Models in Coherent Structures

Craig Gassaway Arizona State University
Advisor(s): Wenbo Tang, Arizona State University

We study the mixing and transport of ecological species in a chaotic ocean. Our focus is on the mesoscale where convection is predominantly two-dimensional and with coherent structures. With weak coupling to the third dimension, we study the effect of vertical flux of the ecological species on a Nutrient-Phytoplankton-Zooplankton interaction model subject to advection and diffusion processes. This reflects more realistic conditions in the ocean. We present our analyse of the system based on direct numerical simulations using pseudo-spectral methods.

111. Application of The Kalman Filter to Predict Tumor Size in Cancer Patients

Stephanie Reed Arizona State University
Advisor(s): Eric Kostelich, Arizona State University

Cancer is a disease that affects the great majority of people directly or indirectly. Although much effort has gone into finding a cure for cancer, not much is known on how to combat this disease. One issue doctors face is uncertainty in the data they acquire regarding their patients' current conditions. Without accurate data, the act of treating cancer resembles that of guesswork. Models to predict cancer growth are one tool that has been employed to help take the guesswork out of treating cancer. These cancer models however helpful are also flawed in that they are difficult to individualize and they become less and less accurate as time goes on. The Kalman Filter has long been used as a tool of state prediction in weather modeling and other fields. We will try to apply the Kalman Filter to cancer growth models in order to give a more accurate representation of the state of a patient. In particular, we will be focusing on prostate cancer using a common cancer model such as the Gompertz or von Bertalanffy models. Use of the Kalman Filter will allow for more individualized and accurate care rather than relying on cancer models or data alone.

112. Downturn in economy: long or short term?

Teng Zhang Miami University Ohio
Advisor(s): Anna Ghazaryan, Miami University Ohio

The Keen model is a known mathematical model that describes the evolution of wage, employment, and debt. It consists of three nonlinear first order ordinary differential equations. There exists an equilibrium state in the Keen model, which is related to a downturn in wages and employment when debt overwhelms the economy. In the

literature, that equilibrium state is conjectured to be unstable and therefore, in part, economically meaningless. We have been able to identify reasonable mathematical conditions on constant parameters in the system under which this equilibrium is stable. The work is based on the work of Grasselli and Lima.

113. Applying Machine Learning Techniques to Baseball Pitch Prediction

Corey Stafford Columbia University
Michael Hamilton Rutgers University
Advisor(s): Hien Tran, North Carolina State University

Major League Baseball, a professional baseball league in the US and Canada, is one of the most popular sports leagues in the world. Partially because of its popularity and the wide availability of data from games, baseball has become the subject of significant statistical and mathematical analysis. Pitch analysis is especially useful for helping a team better understand the pitch behavior it may face during a game, allowing the team to develop a corresponding batting strategy to combat the predicted pitch behavior. We apply several common machine learning classification methods to PITCH f/x data to classify pitches by type. We then extend the classification task to prediction by utilizing features only known before a pitch is thrown. By performing significant feature analysis and introducing a novel approach for feature selection, moderate improvement over former results is achieved.

114. PBPK Modeling of Hazardous Chemicals in Maternal and Fetal Tissues

Camille Zerfas North Dakota State University
Jasmine Jackson Norfolk State University
Ariel Nikas Meredith College
Advisor(s): Hisham El-Masri, North Carolina State University

Toxic chemicals that are found in the environment affect humans and animals. These can be absorbed, inhaled, or otherwise ingested at safe levels by adults. However, chemicals that pregnant women are exposed to also transfer into their fetuses. Unsafe environmental levels of toxic chemicals, like bisphenol - A (BPA), could potentially lead to birth defects, behavioral abnormalities, and diseases. In this project, a mathematical model was developed to determine the levels of chemicals that are absorbed, distributed, metabolized, and eliminated in specific tissues in the body. From this model, the dose that a developing child receives due to a mother's environmental exposure is also examined. For specific chemicals, this may mean that the current acceptable environmental levels must be lowered.

115. Automated Dancer Pose Recognition

Glo Adelyn Mercado California State University, Channel Islands
Advisor(s): Kathryn Leonard, CSU Channel Islands

We are looking for a better way to record and recognize different ballet choreography. Taking an image of a dancer we use the k-means clustering algorithm to obtain a border of the largest darkest area of an image. We create a skeleton for the dancer shape, like that of the Blum medial axis, by using the Delaunay Triangulation and connect the centers of those circles to create the medial axis. We prune the skeleton using the Extended Distance Function and a combination of Erosion Thickness and Shape Tubularity. Using the Extended Medial Axis to find the centers of the dancer, and we use those centers to align two dancer images, which can be used in a graph matching algorithm to match images.

116. Sheet Music Encryption Schemes

Katelyn Pitts University of North Georgia
Advisor(s): John Holliday, Robb Sinn, University of North Georgia

Mathematicians have often been intrigued by the complexities of the relationship between mathematics and music. Consider encrypting a passage to a sheet of music. Note that we can use the following properties as part of our encryption scheme: time signature, key signature, title, and the well-known Circle of Fifths. The particular scheme explored in âœSheet Music Encryption Schemesâ is generated using a form of a one-time pad (OTP), where each letter maps to a pair of notes as follows. Each letter in the plaintext is assigned an arbitrary ordered pair that is passed through an encryption key that uses a variety of mappings, for example mapping to the circle of fifths or the unit circle, along with the use of a rotation matrix, a key cipher, and a multiplicative cipher. âœSheet Music

Encryption Schemesâ also explores the idea of using two different encryptions at once, thus making the scheme even more difficult to break. Cryptographic possibilities are endless, and this piece explores one of its many applications. Additional development of sheet music encryption schemes is ongoing.

117. Mathematical Modeling for Orexin Neurons Firing Dynamics

Ziyi Xu Gettysburg College
Advisor(s): Cecilia Diniz Behn, Colorado School of Mines

Orexin (also known as hypocretin) neurons, located in the lateral hypothalamus, project to many regions of the brain and are involved in diverse functions including arousal, addiction, and metabolism. There are two types of orexin neurons, Type D and Type H cells. They may be distinguished by electrophysiological features, such as a low-voltage-activated potassium current (A-current), and by their response to glucose: although both types are hyperpolarized by glucose, Type D neurons exhibit adaptive responses to glucose inhibition, and Type H neurons remain hyperpolarized in the presence of glucose. To investigate the link between electrophysiology and function, we developed Hodgkin-Huxley type models for Type H and Type D orexin neurons. We compared the effects of post-synaptic currents of varying amplitudes and durations on each cell type by generating appropriate phase response curves, and we analyzed the implications of these phase response curves for neuronal excitability and firing dynamics.

118. Modeling Intervention Strategies for United States TB Control

Colin Pawlowski Yale University
Matthew McDermott Harvey Mudd College
Emma Hartman Wheaton College
Advisor(s): Dylan Shepardson, Mount Holyoke College

Epidemiological models offer insight into the structure of disease outbreaks and enable comparisons of various interventions. Some common epidemiological models are compartmental differential equation models, in which populations move between various health states according to predetermined rates. The Hill Model is a complex compartmental model for tuberculosis (TB) in the United States. In this work, the Hill Model was extended to include additional tracking capabilities for infection source, sourced infection costs, number of raw TB cases, and TB deaths. To compare the cost effectiveness of various interventions, economic parameters were added to track the cost burden of TB on the US health care system. To capture statistical data about this system, an agent-based stochastic version of the model was implemented, which constitutes the first population level agent-based model of tuberculosis in the US.

119. Truncated Total Least Squares for Ill-posed Problems

Jarom Hogue Arizona State University
Advisor(s): Rosemary Renaut, Arizona State University

We consider an ill-posed matrix representation of a microbial fuel cell inverse problem. We investigate the behavior of the Truncated Total Least Squares(TTLS) technique to obtain the distribution of relaxation times from given impedance data. Specifically we consider visual and analytical formulations of filter factors for the TTLS method in order to minimize error in the solution.

120. The Longevity of Wild and Captive Orcas

Taylor McClanahan University of Arkansas
Advisor(s): Lakeshia Legette Jones, University of Arkansas, Little Rock

Cetaceans are some of the most specialized, distinctive marine mammals in the world. They are characterized into two subgroups: Odontoceti and Mysticeti. There is one group of Odontoceti, the orca, that has received lots of attention. Orcas are also known as killer whales, the largest of all dolphins and are very intelligent. As such, they are known to be great for human attraction. Generally, orcas are seen as exquisite marine animals capable of performing in extravagant water shows. Zoos, amusement parks, sea worlds, aquariums, and other facilities have recognized a profit behind these animals. Orcas are great for increasing human knowledge and entertainment; however, there may be a darker side to the story. Holding the animals captive has been a continuously discussed topic. Leaders of the marine mammal industry either breed orcas and hold them captive, or capture them from the wild. Many studies

have shown that the survival rate of captive cetaceans is much lower than the survival rate of those in the wild. Upon statistical testing, my project shows that there is no difference between survival rates of females and males in captivity. Younger captured orcas seem to live longer in captivity than older captured orcas. Because of poor survival, should orcas be kept in captivity?

121. Modeling Sensitivity Analysis of Hepatitis C Virus Dynamics

Jennifer Houser East Tennessee State University
Yesenia Cruz East Tennessee State University
Advisor(s): Ariel Cintron-Arias, East Tennessee State University

Basic deterministic models are used to describe the interactions between Hepatitis C virus, immune responses (antibodies and CTL’s), and target cells. We employ an application called âœplot digitizerâ to digitize scanned graphs of longitudinal data. In this way we generate several datasets that are employed in model calibration. We compute numerical solutions of relative sensitivity equations to determine which model parameters are most influential and in what time intervals they provide most information. In light of parameter sensitivity information we outline how to formulate an optimal control problem.

122. Mathematical Modeling of College Student Enrollment

Zach Helbert East Tennessee State University
Cecilia Dorado East Tennessee State University
Advisor(s): Ariel Cintron-Arias, East Tennessee State University

Mathematical models with demographic effects, such as emigration, immigration, births, deaths, are explored. In particular, Leslie models with discrete group or stage structure are implemented. The relation between dominant eigenvalue elasticity and orbits (time solutions) is addressed by numerical means. Characteristic polynomials and stable age distributions are also calculated. Demographic datasets, obtained from the US Census Bureau and the US Department of Education, are incorporated with some of the models. Particular consideration is given to modeling student enrollments.

123. Expansion of a Stochastic Model for Anelosimus Studiosus Movement during Prey Capture

Alex Quijano East Tennessee State University
Advisor(s): Ariel Cintron-Arias, East Tennessee State University

We discuss how we updated an earlier version of a model, which simulated the movement of Anelosimus Studiosus during prey capture. The previous model was made to successfully match the data for movement of the spider taken from three video feedings. However, when the model was tested against a new set of data for the same spider species, the model did not match well. During the interaction between the spider and prey, the spider tends to have an error in direction, together with varying velocities and a varying number of pauses (with different durations). We suspect that the error in direction might have been the cause of the discrepancy between the data and simulations of the model. Since the previous model did not fully incorporate the velocities and pauses, we analyzed the new data more closely and develop an expanded model merging the velocities and pauses to simulate the movement more precisely. It is observed that this sub-social species spaces out on their web during certain times of the day. The improved model can be used to answer the question of whether the sub-social spider species spaces out optimally to cooperate during prey capture.

124. Spatio-Temporal Model for Burglary Reports in Tennessee

Kristen Bales East Tennessee State University
William Frazier East Tennessee State University
Advisor(s): Ariel Cintron-Arias, East Tennessee State University

We consider the dispersal of crime waves in a similar manner as to how epidemics spread in time and space. Longitudinal datasets, for household burglaries, corresponding to ninety-five counties of Tennessee are employed here. Geographical distance among counties is incorporated into a force of infection that depends on time and space. These prevalence-like datasets are fitted to an epidemiological model, formulated by a system of nonlinear ordinary different equations, by means of an ordinary least squares procedure. We discuss the effects of underreporting.

125. Asymmetric Intraguild Predation Between Protoperidinium and Heterocapsa in the Presence of a Mutual Predator

Laura Asaro East Central University
Advisor(s): Baojun Song, Montclair State University

The azaspiracid toxin contaminated the harvest of the mussel, *Mytilus edulis*, off the coast of Ireland in 1995. Investigation showed that the genus *Protoperidinium*, previously thought to be harmless, was to blame for a new condition brought about by the azaspiracid toxin. To address this concern, we use nonlinear ordinary differential equations to study the dynamics of two dinoflagellate species, as well as their common predator. An asymmetric intraguild predation model with a mutual predator is introduced, in which the toxin producing *Protoperidinium* preys on the non-toxic *Heterocapsa*, while both ingest the nutrients available in the system and are preyed upon by a higher predator. The equilibria were found and global and local stability was determined. We then found the mode of coexistence of the system in an equilibrium. After the initial analysis, we considered interventions, such as modifying nutrient flow, to reduce the levels of the azaspiracid toxin and observe their effect on the persistence of the system.

126. Improved Contig Filtration for De Novo Genome Assembly

Sarah Bogen Capital University
Advisor(s): Hau Man Yeung, Chinese University of Hong Kong

De novo assembly using next-generation sequencing utilizes short DNA reads to assemble complete genomes without the use of a reference. Initial reads are overlapped to create longer sequences called contigs, contigs are arranged into scaffolds, and gaps within and between scaffolds are filled. Current scaffolding algorithms search for linear paths in the contig relation graph. The repetitive nature of DNA often emerges in graphical ambiguities that lead to inaccurate assembly. Scaffolders handle this problem by detecting repeat-containing contigs and removing them from the data set before scaffolding. Current methods, however, often fail to remove contigs that are repeats and erroneously remove contigs that are not repeats. In this project, we present a new method for identifying repeat contigs in which principal component analysis is used to combine four new contig indices. This method appears to be more successful in accurately identifying repeat contigs. Significant improvements to scaffold length or accuracy were not observed when we tested our method on a prokaryotic genome within the framework of current algorithms. Further tests on more complex, repeat-rich genomes are required to fully understand the impact of this new filtration method on assembly quality.

127. Difference Equation Approximations of Agent Based Models for Solving Optimization Problems

Herbert Susmann State University of New York at Geneseo
Advisor(s): Matthew Oremland, Virginia Bioinformatics Institute

Agent based models (ABMs) are a type of simulation model in which individual agents interact with each other and their environment according to defined rules. While ABMs are a powerful tool for investigating emergent behaviors of interacting agents, they are often computationally intensive and suffer from a lack of mathematical analytical tools. Conversely, mathematical modeling approaches are analytically tractable, but their construction requires high level knowledge of the system to be modeled. We developed a method of capturing the benefits of both modeling paradigms by constructing a system of equations that describes the high level behavior of an agent based model known as SugarScape. We posed a multiobjective optimization problem this ABM and found a solution through the equation model. Our problem was to find an optimal sugar tax schedule that minimizes deaths while maximizing the sugar collected. A system of difference equations was derived analytically and computationally to describe the movement of the ants, their sugar accumulation, and their population size. The optimal tax schedules, discovered through the system of equations, were compared with those from the original agent based model in order to determine the descriptive effectiveness of the mathematical model.

128. Mathematical Analysis of Social Dominance in the Green Anole Lizard

Jordan Bush Trinity University
Advisor(s): E. Cabral Balreira, Michele Johnson, Trinity University

Male dominance is an important aspect of the social organization of many species, and often results in successful territory defense and/or access to potential mates. We developed a series of ranking systems to identify the behavioral

and morphological traits associated with dominance in green anole lizards (*Anolis carolinensis*). We first performed a tournament of arena trials using pairs of 18 male lizards to simulate aggressive interactions. We used these data to rank the lizards using several ranking methods, including original ranking models developed for this application. Our results showed that behavioral displays and relative head length were the most predictive of rank in the majority of ranking systems. We then sought to validate our results by comparing male rank to territory size, a proposed proxy of social dominance, by comparing the ranking obtained through a series of arena trials with territory sizes measured in an artificial enclosure over one week. Although we hypothesized that higher ranked males would have larger territories, we found no correlation between rank and territory size. Thus the traits associated with successful long-term defense of a large territory in green anoles may be independent from those needed to win short-term confrontations.

129. Epithelial Cell Migration in Wound Healing as a Two-Step Diffusion Model

Crystal Nguyen Trinity University

Advisor(s): Eduardo Cabral, Trinity University

Epithelial cell migration in wound healing can be associated with three processes: cell relaxation due to a release of tension, actual migration, and cell proliferation. We developed a diffusion model to predict how cell migration is affected by the knockdown of a tight junction protein, ZO-1, with a source term to model proliferation. In addition we considered a two-step process, where the first step is a diffusion model and the second step is a linear progression of movement. We validated our proposed model using Madin-Darby Canine Kidney (MDCK) cells in a scratch wound assay. We then acquired images using a Nikon confocal microscope to create a time-lapse video over a course of fifteen hours, taking images every thirty minutes. A MATLAB code produced model parameters using the finite difference method then determined the best fitting parameters by minimizing the sum of square errors. Our model produced good fit and may support the idea that post scratch, cells first undergo a relaxation into the empty wound space, modeled by a diffusion, before entering a migration phase, expressed linearly.

130. The Regulatory Effect of microRNAs on the DNA Mismatch Repair Pathway

Madeleine Weinstein Harvey Mudd College

Advisor(s): Reinhard Laubenbacher, Virginia Bioinformatics Institute

Although failure of DNA Mismatch Repair (MMR) is associated with microsatellite instability and colorectal cancer, little is known about MMR except for its biochemical pathway. We introduce a novel gene regulatory network of MMR by assembling known regulatory interactions. Our network is modeled with the Stochastic Discrete Dynamical Systems framework, a recently introduced time- and state-discrete modeling framework that accounts for the cell's inherent stochasticity. This model provides phenotypic predictions for MMR's response to hypoxia and DNA damage. We also substantiate the hypothesis that microRNAs can stabilize network dynamics, thus enhancing genomic stability, by showing that overexpressing microRNAs increases robustness while knocking them out seems to have the opposite effect. We quantify network robustness using two measures: Derrida values, which measure the change in Hamming distance of two states input into our network, and Average Maximal Transition Probabilities. In addition to providing a gene regulatory network of MMR, our model yields experimentally verifiable predictions and enables further analysis of the potential stabilizing effect of microRNAs on dynamics of biological networks.

131. The Algebraic Connectivity of Laplacian Matrices: Fielder's Theorems and Applications to Bioinformatics

Rudy Dehaney Morgan State University

Advisor(s): Asamoah Nkwanta, Morgan State University

A Laplacian Matrix is a matrix in graph theory. Laplacian matrices have several important properties derived from its second eigenvalue which is defined as the algebraic connectivity. The notion of algebraic connectivity is part of a bioinformatics algorithm called RNAmute. In this poster we present theorems of Miroslav Fielder that are used to prove properties of the matrices. We then apply RNAmute to HIV-1 RNA sequences to predict possible mutations in the sequences.

132. Analyzing Knotting in Folded Proteins

Joseph Spitzer University of St. Thomas
Christopher Baldwin University of St. Thomas
Advisor(s): Eric J. Rawdon, University of St. Thomas

Only in the past 20 years have researchers discovered knotting in proteins. Still much is to be understood about their significance. The latest research suggests that knotting in protein chains is critical to their cellular function. We continue work to discover knots in the latest proteins deposited into the Research Collaboratory for Structural Bioinformatics (RCSB) protein data bank with the hopes of understanding why some proteins contain knots and slipknots. We provide background on how to classify knotting in open chains (like proteins) and show the results of our latest search for knotted proteins.

133. The Exploration of 3D Printed Environments for Caenorhabditis elegans

Jeffrey Kopsick James Madison University
Advisor(s): Eva Strawbridge, James Madison University

Using the 3D printers in the JMU Maker Lab, we have developed a protocol for printing 3D environments for the study of C. elegans swimming in different geometries and fluids of different viscosities. C. elegans offers an unprecedented middle ground between the macro and microscopic. This project involves experimental and theoretical components as well as techniques in image segmentation and processing. From these techniques, we aim to quantify how significant the role of environment is to the swimming gait of nematodes. If we can quantify a significant difference between an isolated droplet and a droplet within our environment, we will begin to model the worm's fluid mechanics.

134. A within-host mathematical model of HIV infection during combination therapies

Candace Baker University of Central Oklahoma
Advisor(s): Sean Laverty, University of Central Oklahoma

Mathematical models provide us with a quantitative description of the immune system and its interactions with viruses and other pathogens. We model HIV infection dynamics within a host to study the effects of drug treatments, specifically those that alter the ability of the virus to infect susceptible cells and to produce infectious viruses. The two drugs considered in the model are Reverse Transcriptase inhibitors and Protease inhibitors which block the ability of HIV to successfully infect a cell, and cause the production of non-infectious viral particles, respectively. Our current model has four equations describing the behavior of target T-cells (T) that are susceptible to infection, infected T-cells (I), infectious virus (Vi), non-infectious virus (Vni). Changes in the efficacies of these drugs can cause large fluctuations in host cell and virus dynamics. We will expand the model to examine the effects of novel drug combinations and more detailed T-cell population structure on host and virus dynamics.

135. Social aggregation in pea aphids: An experimentally inspired random walk model

Olivia Warner Macalester College
Christa Nilsen Macalester College
Advisor(s): Chad Topaz, Macalester College

We model aggregation of the pea aphid using experimental data of aphids walking in a circular arena. We deduce individual-level rules and observe that aphids transition stochastically between moving and stationary states. Moving aphids follow a correlated random walk. The probabilities of motion state transitions and the random walk parameters depend strongly on distance to an aphid's nearest neighbor. At large distances, aphids are essentially isolated and move faster, turn less, and are less likely to stop. The opposite is observed for short distances; this behavior constitutes an aggregation mechanism. From the experimental data, we estimate the state transition probabilities and correlated random walk parameters as functions of nearest neighbor distance. We then assess whether the individual-level model reproduces the group-level patterns of movement by considering three distributions: distance to nearest neighbor, angle to nearest neighbor, and percentage of population moving at any time. For each, we compare the data to numerical simulations of our nearest neighbor model, and of a control model in which aphids do not interact. Our stochastic, social nearest neighbor model reproduces salient features of the experimental data that are not captured by the control.

136. Patient Specific Modeling of Cerebral Blood Flow Velocity

Theresa Scarnati Indiana University of Pennsylvania
Miranda Henderson Benedictine University
Advisor(s): Adam Mahdi and Mette Olufsen, North Carolina State University

The human brain accounts for only two percent of human body mass, yet over fifteen percent of the body's blood supply goes to the brain. Cerebral autoregulation (CA) is the brain's local ability to maintain stable cerebral blood flow during changes in arterial blood pressure. Numerous studies have been dedicated to developing computational models in order to understand the physiological mechanisms associated with CA. However, the majority of previous models only supply us with a qualitative understanding of CA. Here, we aim to create a simple, quantitative, and patient speci c CA model. Because of the limited data available and the complex physiological processes involved in CA, we predict cerebral blood flow given only systemic arterial blood pressure. The model is made patient speci c by using sensitivity and covariance analyses to obtain a subset of parameters that can be estimated given available, quantitative data.

137. Understanding the scales of locomotion for *Caenorhabditis elegans* in a viscous fluid

Katie Sipes James Madison University
Advisor(s): Eva Strawbridge, James Madison University

Viscosity is a measurement of a fluid's resistance to the rate of deformation. An example of a liquid with high viscosity is tree sap, or a homogeneous mixture of mud. Both of these liquids run very slowly when acted upon by gravity. In contrast, water has a very low viscosity and flows readily. So how does a liquid's characteristics effect the locomotion of a swimming organism? Do higher viscosities change the dynamics that an organism implements in order to move in a solution? The Reynolds number is defined as the ratio of inertial to viscious forces and is given by $\frac{\rho V L}{\mu}$ where ρ is the fluid density, V is characteristic velocity, L is the characteristic length of the system, and μ is kinematic viscosity. In particular, I am interested in measuring these different scales in a system where the worm *C. elegans* is swimming in fluids of different viscosities. We will compare these measurements to different modes of locomotion.

138. Modeling Homeostatic Expansion of Various T Cell Populations

Molly Lynch College of the Holy Cross
Advisor(s): David Damiano, College of the Holy Cross

The immune system is comprised of the innate and the adaptive immune system. Our project uses a ten compartment non-autonomous system of ordinary differential equations to model the various T cell interactions within the adaptive immune system. By coupling the compartments for CD4+ and CD8+ T cells, we incorporate the concept of density dependent homeostatic expansion and density dependent death. We include a term for time dependent decay of thymic output in our model in an attempt to model the healthy immune system as accurately as possible. In order for the model to match a set of target populations for each of the compartments, three optimization methods are explored: a simplex method, simulated annealing, and Latin hypercube sampling. While it appears the model is a good fit to existing biological data, it is based on estimates from the literature. Therefore, local and global sensitivity analyses were performed to examine the sensitivity of the model to any one or group of parameters. This is part of long-term project to model and investigate the interaction of HIV with the immune system.

139. Mathematical Modeling in Ecology: What Killed the Mammoth?

Anneliese Slaton Mary Baldwin College
Michael Frank Simpson College
Teresa Tinta University of Maryland Eastern Shore
Advisor(s): Alex Capaldi, Valparaiso University

One extinction theory of the Columbian mammoth (*Mammuthus columbi*), called overkill, hypothesizes that early humans overhunted the animal. We will employ two different approaches to test this theory mathematically: analyze the stability of the equilibria of a differential equations system and secondly, simulate a stochastic extension of this ODE system into a metapopulation model. The first system of ODEs is a modified predator-prey model that also includes immigration and emigration. The simulation is a high-dimensional ODE system on a rectangular grid

designed to represent North America at the end of the last ice age. Using this simulation, we model the migration of humans into North America and the response in the mammoth population. These approaches show evidence that human-mammoth interaction would have affected the extinction of the Columbian mammoth during the late Pleistocene.

140. Ticks, Deer, Mice, and a Touch of Sensitivity: A Recipe for Reducing Lyme Disease

Matthew Jastrebski Northeastern Illinois University

Advisor(s): Leon Arriola, University of Wisconsin Whitewater

Borrelia burgdorferi sensu stricto is a bacterial spirochete prevalent in the Northeastern United States that causes Lyme disease. Lyme disease is the most common arthropod-borne disease in the United States; affecting mice, deer, humans and other mammals. The disease is spread by *Ixodes Scapularis*, a species of tick whose primary food source are deer and mice. Reducing the population of ticks feeding on both large and small mammals below some critical threshold can decrease the prevalence of Lyme disease among humans. A simplified, six-dimensional Susceptible–Infected, SI, model is used to capture the mice-deer-tick dynamics while considering the impact of varying population-specific death rates on infected population size. We analyzed the stability of the models two equilibria, the unstable disease free equilibrium and the endemic equilibrium. Static forward sensitivity analysis is conducted on the basic reproduction number and the endemic equilibrium. A dynamic approach was explored to observe change in the sensitivity of the death rates over time. These analyses were conducted to determine the efficacy of changing death rates in order to reduce prevalence of Lyme disease.

141. Modeling Cell Arrangement in Epithelia

Michelle Andersen State University of New York, The College at Brockport

Advisor(s): Sharon Lubkin, North Carolina State University

We developed an off-lattice, three-dimensional, particle-based model to simulate cell rearrangement in epithelial sheets. This model renders cells that have no inertia, exhibit fluid-like behavior, and form tissues that resist deformation. Our model is novel in that it assigns cells orientation and polarization in addition to position, volume, and shape. Cells are given an orientation when they are polarized, allowing cell-to-cell interactions to be simulated as forces determined by the relationships between the orientations and positions of neighboring cells. Including orientation as a cell attribute also allowed us to calculate and impose curvature on sheets and other surfaces of cells, creating hollow spheres, cylinders, and bent sheets. Controlling the time and location of cell polarization allowed us to transform a solid sphere of cells into a hollow sphere, mimicking a morula developing into a blastula. Including orientation and polarization in our model not only allowed us to add another facet of realism to individual cells and to cell-to-cell interactions, but it also allowed us to more realistically simulate important developmental processes in collections of cells.

142. Continuous and Discrete Modeling of Tumor Growth and Invasion

Jessica Perez The College of New Jersey

Advisor(s): Jana Gevertz, The College of New Jersey

Cancer is a highly complex and multi-faceted disease that is far from being fully understood. While many tumor-host interactions influence cancer progression, our goal is to understand the interaction between the tumor vasculature (blood vessels) and invasive cancer cells that are the precursor to metastasis. The relationship between these two aspects of cancer progression is a complicated one, as regions with limited vasculature can promote cancer cell invasion by encouraging cancer cells to seek out tissue space with more oxygen. On the other hand, blood vessels provide a track upon which cancer cells can invade, and therefore encourage invasion and metastasis. In this project, two models representing the growth and invasion of cancer were developed: a continuous and deterministic differential equation model, and a discrete and stochastic cellular automaton model. The parameters in the two models were then optimized to quantitatively calibrate model output. By having two calibrated models with different strengths, we can gain a more complete picture of the impact the vasculature has on both tumor growth and invasion.

143. Modeling Illicit Drug Use: Analyzing the Spread of Methamphetamine Through Urban and Rural Areas

Lilyana Staight California State University, Monterey Bay
Bernadette Bucher University of Alabama
Robin Mabe Grove City College
Danielle Williams East Stroudsburg University
Advisor(s): Alun Lloyd, North Carolina State University

While the use of other illicit drugs such as cocaine and heroine is declining, the use of methamphetamine continues to rise, in part due to ineffective current treatment strategies. To date, mathematical models have not been used to explore the dynamics of methamphetamine use in a population. We propose two mathematical models that can predict, evaluate, and simulate methamphetamine use in urban and rural populations. Characteristically, rural areas are hot spots for home based labs, while in urban areas methamphetamine is mostly bought from dealers who are not part of the urban drug using population. Consequently, we created separate compartmental models for urban and rural areas. Similar to techniques often used in infectious disease modeling, the interaction between susceptible, using and recovered individuals in our drug using population acts as a mechanism for the spread of methamphetamine use. Furthermore, we found optimal control strategies for both models to show how the spread of methamphetamine use reacts to implementations of government controls across a population.

144. Modeling Mixing Effects on Harmful Algal Blooms in Eutrophic Lakes

Nicholas Myers University of Wisconsin-Milwaukee
Jason Zellmer University of Wisconsin-Milwaukee
Kimberly Siegler University of Wisconsin-Milwaukee
Advisor(s): Gabriella Pinter, Istvan Lauko, Todd Miller, University of Wisconsin - Milwuakee

Microcystis has been gaining recognition for contributing to harmful algal blooms due to its negative effect on recreational and potable water. Literature suggests that in a eutrophic environment, water column mixing, light intensity, and nutrient density are all influential factors in microcystis growth. Since other species' growth depends on these factors, we consider a competition model which incorporates the interaction of microcystis and other phytoplankton. In this work, we build our mathematical model as a system of nonlinear integro-partial differential equations including convective and diffusive terms and non-local light-limited growth. We simulate the changes in species biomass through the water column incorporating different buoyancy characteristics. In particular, we study the various environmental effects on the formation and persistence of microcystis blooms. Through the analysis of the dynamical behavior of the system, we aim to forecast the onset of harmful algal blooms under a variety of environmental conditions.

145. Predicting the Future of Coral Reefs: A Simulation Approach

Chelsey Beese Coastal Carolina University
Aaron Smith Coastal Carolina University
Advisor(s): Tessa Weinstein, Coastal Carolina University

In the past three decades, Caribbean coral reefs experienced a dramatic decline in health associated with an ecosystem phase shift from coral to macroalgal dominance. Severe environmental disturbances caused previously stable reefs, such as the forereef in Discovery Bay, Jamaica, to destabilize. The lack of significant grazing allowed macroalgae to flourish, threatening the health and very existence of the reef. To inform management strategies for jeopardized reefs, biologists are turning to mathematical models. Here, we present an agent-based spatial simulation model designed in MATLAB to forecast the future of Discovery Bay based on its unique characteristics. Unlike previous works, we use an abnormally small cell area of 1 cm^2, the approximate size of a coral polyp. Additionally, a hexagonal grid, rather than a traditional square grid, is employed. These choices allow neighboring organisms to interact more closely than in previous models. Specific input parameters for representative species are garnered from empirical data. Finally, the model is applied to a number of initial conditions to determine threshold values of grazing and disturbance such that the reef shows significant recovery. These threshold values can be used as goals for ecosystem management.

146. Combinatorial Optimization in Network Alignment

Vikram Saraph University of Notre Dame
Advisor(s): Tijana Milenkovic, University of Notre Dame

The subgraph isomorphism problem is known to be NP-complete. In complex networks, *network alignment* is an approximation to this problem, seeking an injective vertex map $f : G \to H$ (an *alignment*) between networks G and H, so that a highly resembles a graph embedding. We introduce a new algorithm to solve network alignment. Our algorithm accepts a set of candidate alignments, and produces successively better alignments by combining existing ones. Our core mathematical contribution to this algorithm is its *crossover function*, which defines how two alignments are combined to produce a third. Each alignment a is viewed as a vertex in the Cayley graph Γ of the symmetric group S_n, and the crossover function is a binary operation $\otimes$ on S_n, where $a \otimes b$ is a midpoint of a and b in Γ. We show that $a \otimes b$ resembles both a and b, in the sense that in the limit, $a \otimes b$ shares half its aligned pairs with a, and half with b. We can compute $a \otimes b$ in $O(n)$ time Network alignment is often used in bioinformatics. We applied our algorithm to align the yeast protein-protein interaction (PPI) network to the human PPI network, and were able to extract the underlying biological function.

147. Dynamical Systems Modeling of the Cold Shock Response in Saccharomyces cerevisiae

Katrina Sherbina Loyola Marymount University
Advisor(s): Ben G. Fitzpatrick, Kam D. Dahlquist, Loyola Marymount University

DNA microarrays were used to measure the effect of cold shock on gene expression in Saccharomyces cerevisiae, budding yeast. The wild type strain BY4741 and four transcription factor deletion strains were subjected to cold shock at 13°C and allowed to recover at 30°C. A modified ANOVA test was used to detect significant differences in gene expression between the control and experimental conditions and between strains. Expression of each gene in a regulatory network consisting of 21 transcription factors was modeled by a nonlinear differential equation describing the change in expression over time as the difference between the production and degradation rates. The fmincon function in MATLAB compared the differential equation model to the microarray data to find optimized weights and threshold constants by a nonlinear least squares fit criterion. The deletion strains were modeled by removing the gene from the dynamical system. Modeling production by Michaelis-Menten kinetics instead of a sigmoid function more accurately described repression and the case of an OR transcriptional gate. The Michaelis-Menten model accurately predicted that PHD1, the gene with the most connections in the network, is down-regulated in the cin5 deletion strain but up-regulated in the wild type strain.

148. Detecting Associations Between Rare Genetic Variants and Quantitative Traits

Alexander Lasiuk University of Wisconsin-Eau Claire
Advisor(s): Brisbin, University of Wisconsin-Eau Claire

Identifying genetic regions that are associated with diseases and other complex traits is important for understanding the causes of those traits. However, many existing methods do not work well when a particular region contains single nucleotide polymorphisms that put the person at risk, in addition to polymorphisms that protect them from a certain quantitative phenotype. To fix this, we are looking at the covariance between the genotypes of individuals at a particular single nucleotide variant, and a vector containing information on each individual's phenotype, or trait value. Then we sum up the square, or absolute value of all those values to conclude if that set of single nucleotide variants is associated with the individual's quantitative phenotype. We have run this on simulated quantitative data, and returned p values similar to SKAT, a competing method. We will also present the results of our method on real DNA sequences of individuals experiencing chemotherapy-induced peripheral neuropathy.

149. Mathematical Model of Carbapenem-Resistant Enterobacteriaceae with Low and High Prevalence Settings

Hongli Chen Ursinus College
Noelle Sassany Ursinus College
Advisor(s): Mohammed Yahdi, Ursinus College

Antibacterial resistance is a serious and growing threat to hospitalized patients. The treatment of drug-resistant bacterial infections is predicted to become a global crisis due to the scarcity of new antibiotics in the pharmaceutical

industry's pipeline. Carbapenem, one of the most reliable antibiotics, has become increasingly ineffective in the past 10 years. Infections caused by carbapenem-resistant Enterobacteriaceae (CRE) have been associated with extremely high mortality rate (30

150. Modeling Positions Coupled to F508, Site of Chief CF Causing Mutation

Saba Nafees Texas Tech Institution
Advisor(s): Sean Rice, Texas Tech University

Cystic fibrosis is a genetic disease that arises due to misfolding of the protein CFTR. Analyses of the mutation data can show the way the various mutations act to produce disease. Our proposed method to do this is to use multivariate polynomial regression. We use Orthogonal Polynomials and dual bases for multivariate data. The observed distribution of mutations defines a covariant basis and a contravariant basis. Projecting different responses into these and taking the covariance would yield their interrelation. We then build functions that capture the properties of the mutations of CFTR, so that we can assess each mutation's interaction with other mutation(s). This will provide insight into the mechanism of the misfolding problem. The orthogonal and dual spaces approach conserves intrinsic biological properties of any phenomenon being tested while simultaneously capturing its quantitative properties.

151. Characterization of Blink Motion Through Curve Fitting

Joseph Brosch University of Delaware
Advisor(s): Richard Braun, Dept of Mathematical Sciences, U of Delaware

The motion of the upper eyelid during blinking can be important in determining possible diseases and syndromes that affect the eye. In particular, combining machine learning with a low-dimensional model for the blink motion of the upper eyelid might yield an effective diagnostic tool for abnormalities in blinking and potentially for Dry Eye Syndrome (DES). Nonlinear least squares fitting was used to fit the position of the center of the upper lid during blinks of different subjects under four experimentally controlled conditions. The coefficients from the hypothesized lid motion functions are used for classification purposes. Preliminary use of clustering algorithms on the fit coefficients, suggests possible automatic identification of normal or abnormal blinks based solely on the nonlinear fits.

152. Modeling the Dynamics of Prion Proteins with a Birth-Death Process with Catastrophes

Brian Sarracino-Aguilera University of California, Merced
Advisor(s): Suzanne Sindi, University of California, Merced

Prion proteins are responsible for a host of deadly disease in mammals such as "Mad-Cow" disease in cattle, chronic wasting disease in elk and several diseases in human. Prion proteins also exist for yeast, a single celled organism; however, for yeast these diseases are not fatal and sometimes reversible making yeast an ideal model for prion diseases. To understand the mechanisms by which yeast can "cure" themselves of prion diseases (i.e., have the number of prion proteins decrease to 0) we formulated a stochastic birth-death process with catastrophes. Prion proteins within a yeast cell reproduce according to a birth-death process and are transmitted to offspring during cell division. Thus, during cell division the number of prion proteins in a single cell may significantly decrease. Our goal is to understand if prion proteins are more likely to be lost by the birth-death process within a cell or the catastrophe event when prion proteins are transmitted to a daughter cell. We present both theoretical and computational analysis of our population dynamics.

153. Avoiding Permutation Patterns in Ordered Set Partitions

Ruyue Yuan Valparaiso University
Stephanie DeGraaf Iowa State University
Kai Orans Pomona College
Advisor(s): Anant Godbole, East Tennessee State University

We consider the enumeration of ordered set partitions avoiding a permutation pattern. Recently, many results have been published concerning this topic, including enumerative results by Bruner, Chen et al., Godbole et al., and Kasraoui. We specifically consider Kasraoui's summation formula for the number of ordered set partitions that avoid a pattern of length 3, and analyze it in a variety of ways. Through parameterization, we find the value of i for which the maximum value in the sum occurs. We develop a simplified approximation of the formula, and we then obtain a

lower bound for the resulting sum. All results are thus asymptotic but the number of parts in the partition is allowed to grow to infinity with n.

154. Irreducible representations of $SU(n)$ with prime power degree

Sarah Peluse The University of Chicago
Advisor(s): Ken Ono, Emory University

The correspondence between irreducible representations of the symmetric group S_n and partitions of n is well-known. Less well-known is the connection between irreducible representations of the special unitary group $SU(n)$ and partitions with less than n parts. This correspondence is used to classify the irreducible representations of $SU(n)$ with prime power degree.

155. Rank-Unimodality of Binary Partitions

Alexa Ortiz St. Edward's University
Advisor(s): Edward Early, St. Edward's University

This research project focuses on binary partitions, which are ways of writing an integer as a sum of powers of 2. Previous research has shown that with a suitable partial ordering, the set of binary partitions is rank-unimodal for integers less than 30, which means that the number of partitions increases for each rank, and then decreases. It was also discovered that there is no clear pattern regarding whether or not the set of binary partitions for integers greater than 30 is rank-unimodal. We expanded the data from previous work to include integers up to 27,000. We used Maple 17 software to create a program that could recursively generate new data more efficiently. The program then analyzed the data and gave a list of integers for which the set of binary partitions were not rank-unimodal. We discovered that there are only 33 even integers for the set of binary partitions of integers less than 27,000 that are not rank-unimodal. The next phase of this research will be to examine properties and patterns of these 33 even integers. Future research will involve expanding the findings to other types of partitions.

156. Some Don't Sum to Zero: Examination of the Maximum Size of Zero-Sum-Free Subsets of Cyclic and Non-Cyclic Groups

Conor Finn Gettysburg College
Advisor(s): Bela Bajnok, Gettysburg College

Given a finite abelian group G, we say that a subset $A \subseteq G$ is zero-sum-free if the sum of any fixed or limited number of (not necessarily distinct) elements of A is never equal to zero. For a fixed h, the maximum size of a zero-h-sum-free subset is denoted by $\tau(G, h) = \max\{|A| \mid 0 \notin hA\}$. If h takes on a range of values satisfying $h \in [1, t]$, then we write $\tau(G, [1, t]) = \max\{|A| \mid 0 \notin [1, t]A\}$. Here a lower bound on $\tau(G, h)$ is found for non-cyclic groups of the form $\mathbb{Z}_k^r$ that disagrees with a similar result found by Green and Ruzsa for (k, l)-sum-free sets. Also, the value of $\tau(G, [1, t])$ is found for certain cyclic groups $\mathbb{Z}_n$, and upper and lower bounds are constructed for the remaining cyclic groups. Finally, it is shown that our lower bound on $\tau(G, h)$ for non-cyclic groups of the form $\mathbb{Z}_k^r$ holds for $\tau(G, [1, t])$.

157. 4-Equitable Tree Labeling

Zena Coles Bard College
Alana Huszar The College of New Jersey
Jared Miller (institution missing)
Advisor(s): Zsuzsanna Szaniszlo, Valparaiso University

A tree is a vertex-edge graph that is connected and contains no cycles. A 4−equitable labeling of a graph is an assignment of labels $\{0, 1, 2, 3\}$ to the vertices. The edge labels are the absolute difference of the labels of the vertices that they are incident to. The labels must be distributed as evenly as possible amongst the vertices and they must also be distributed as evenly as possible amongst the edges. We study 4−equitable labelings of different trees; we found that all caterpillars, symmetric generalized stars, and complete n-ary trees for all $n \in \mathbb{N}$ are 4−equitable. We believe that proving all trees are 4-equitable will bring us one step closer to proving the famous graceful tree conjecture that has been open for half a century.

158. The Speeds of Families of Intersection Graphs

Jessica Shi Massachusetts Institute of Technology
Advisor(s): Francisco Unda, Massachusetts Institute of Technology

A fundamental question of graph theory lies in counting the number of graphs which satisfy certain properties. In particular, the structure of intersection graphs of planar curves is unknown, and Schaefer, Sedgwick, and Stefankovic have shown that recognizing such graphs is NP-complete. As such, the number of intersection graphs of planar curves poses an interesting problem. Pach and Tóth have previously proven bounds on the number of intersection graphs of string graphs. We investigate the more specific case of the number of intersection graphs on n vertices of systems of segments of parabolas, conic sections, polynomials, and rational functions. We extend the results of Pach and Solymosi, who obtained upper bounds on the number of intersection graphs of line segments, and Fox, who obtained tight lower bounds. For each system we establish a set of polynomials whose sign patterns give an intersection graph. We use Warren's Theorem to obtain an upper bound on the number of sign patterns of this set. We then use a constructive approach to calculate tight lower bounds on the number of intersection graphs. In general, the bounds on the intersection graphs of these systems is $n^{n(f+o(1))}$, where f is the degree of freedom.

159. Optimal Paths in Graphs with Variable Weights

Lisa Gullo Dominican University
Advisor(s): Marion Weedermann, Dominican University

We consider an extension of a basic shortest path problem with applications in the analysis of traffic patterns. Traffic patterns might vary depending on factors such as the day of the week and the time of day. In this project, we focus on analyzing the different traffic patterns between Gary, Indiana and O'Hare Airport. We investigate directed, acyclic, weighted graphs where each weight varies according to a normal distribution. After determining the optimal path from Gary, Indiana to O'Hare Airport based on mean travel times, we use Monte Carol simulations in conjunction with Dijkstra's algorithm to analyze the effect that standard deviations have on the optimal path.

160. Radio Number of Wheel and Gear Graphs

Daniel Winney California State University, San Bernardino
Cyrus Jose California State University, San Bernardino
Kathleen Orque California State University, San Bernardino
Sehee Hwang California State University, San Bernardino
Advisor(s): Min-Lin Lo, California State University, San Bernardino

Inspired by the Channel Assignment Problem of dividing the broadcasting spectrum among neighboring radio stations to minimize interference, radio-labeling was introduced to use graph models to find channel distributions with the smallest frequency range possible. Using vertices in a connected graph, G, to represent the radio stations, a radio-labeling is a coloring function, f, so any vertices u and v in G satisfy the inequality $|f(u) - f(v)| \geq diam(G) + 1 - d(u, v)$ where $d(u, v)$ indicates the distance between u and v and $diam(G)$ is the largest distance in the graph. An optimal radio-labeling then is one whose range is equal to the graph's radio number, defined as the minimum span of of all radio-labelings of the graph. In this presentation we will discuss the optimal radio-labeling patterns we found for wheel and gear graphs during a 2013 summer research program as part of the NSF PRISM grant DMS-1035120 (Proactive Recruitment in Introductory Science and Mathematics)

161. On the Crossing Number of the Cartesian Product of a Vertex-transitive Graph with a Cycle

Junho Won Massachusetts Institute of Technology PRIMES
Advisor(s): Chiheon Kim, Massachusetts Institute of Technology

The minimum number of crossings for all drawings of a given graph G on a plane is called its crossing number, denoted $cr(G)$. Computing the exact crossing number of a given graph is an old problem but very difficult in practice, and such computation has been shown to be NP-hard. Exact crossing numbers are known only for a few families of graphs, and even the crossing number of a complete graph K_m is unknown. Wenping et al. showed that $cr(K_m \square C_n) \geq n \cdot cr(K_{m+2})$ for $n \geq 4$ and $m \geq 4$. We adopt their method to find a lower bound for $cr(G \square C_n)$ where G is a vertex-transitive graph of degree at least 3. This reduces the problem into computing the crossing number of a smaller graph G'. We also discuss how this applies to well-known vertex transitive graphs, e.g. the Petersen graph $G(5, 2)$ and the regular complete bipartite graph $K_{m,m}$.

162. Dynamically Random Graphs

Alexis Byers Wittenberg University
Mallory Reed Earlham College
Laura Rucci Cabrini College
Elle VanTilburg University of Texas at Austin
Advisor(s): Dan Pritikin, Miami University

We introduce the idea of a weighted graph that has edges with associated probabilities of being available at discrete time instants. We attempt to solve problems such as the Chinese Postman Problem, finding Eulerian tours, and finding spanning trees in such graphs with the added challenge of minimizing the time spent waiting for edges to become available. We look at a related problem in which we are given a set of n tasks, each with a probability of being available and a completion time, and we provide a strategy for completing k of the n tasks with shortest expected time. Finally, we consider the computation times of our strategies and discuss avenues for further research.

163. Accessibility Polynomials of Sandpile Graphs

Reuben Tate University of Hawaii at Hilo
Armando Salinas Arizona State University
Andrew You Duke University
Advisor(s): Luis David Garcia Puente, Sam Houston State University

Sandpiles are a mathematical object where we assign each non-sink vertex of a directed graph a non-negative integer denoting the number of grains of sand. When the number of grains of sand at a vertex is at least its out-degree, it is unstable so we topple it, sending each grain of sand at that vertex to its neighbors. We add sandpiles together by doing simple component-wise addition at the vertices and then continuously toppling until we end up with a stable sandpile. In this presentation, we focus on what we call accessibility polynomials. For any sandpile graph, there is an associated accessibility polynomial that contains embedded information about how sandpiles can access each other through sandpile addition. We discuss the very first findings of these polynomials which include such polynomials for various types of graphs and the discovery of the super accessible transient, a special sandpile with very interesting properties. We look at these properties in addition to how graph connectivity affects the existence of such a sandpile. Lastly, we discuss how the implications of our findings are significant in the development of sandpile theory.

164. Newman's conjecture in various settings

Alan Chang Princeton University
Advisor(s): Steven J. Miller, Williams College

Polya introduced a deformation of the Riemann zeta function $\zeta(s)$, and De Bruijn and Newman found a real constant Λ which encodes the movement of the zeros of $\zeta(s)$ under the deformation. The Riemann hypothesis is equivalent to $\Lambda \leq 0$. Newman made the conjecture that $\Lambda \geq 0$ along with the remark that "the new conjecture is a quantitative version of the dictum that the Riemann hypothesis, if true, is only barely so." Newman's conjecture is still unsolved, and previous work could only handle the Riemann zeta function and quadratic Dirichlet L-functions, obtaining lower bounds very close to zero (for example, for $\zeta(s)$ the bound is at least $-1.14541 \cdot 10^{-11}$). We generalize the techniques to apply to a wider class of L-functions, including automorphic L-functions as well as function field L-functionsEach type of "family" of function field quadratic L-functions gives a different version of Newman's conjecture. These variations have connections to other fields, including random matrix theory and the Sato–Tate conjecture. In particular, the recent proof of Sato–Tate for elliptic curves over totally real fields allows us to prove a version of Newman's conjecture involving fixed $D \in \mathbb{Z}[T]$ of degree 3.

165. A Generalization of Fibonacci Far-Difference Representations and Gaussian Behavior

Philippe Demontigny Williams College
Advisor(s): Steven Miller, Williams College

A natural generalization of base B expansions is Zeckendorf's Theorem, which states that every integer can be uniquely written as a sum of non-consecutive Fibonacci numbers. If we allow the coefficients in the decomposition to be zero or ± 1, the resulting expression is called a far-difference representation. Alpert proved that a Fibonacci far-difference representation exists and is unique when two adjacent summands of the same sign are at least 4

indices apart and those of opposite signs are at least 3 indices apart. We prove that a far-difference representation can be created using sets of k-Skipponacci numbers, which are generated by recurrence relations of the form $S_{n+1} = S_n + S_{n-k}$ for $k \geq 0$. Now every integer can be written uniquely as a sum of $\pm S_n$'s such that every two terms of the same sign are at least $2k+2$ indices apart, and every two terms of opposite signs are at least $k+2$ indices apart. Additionally, we prove that the number of positive and negative terms in given k-Skipponacci decompositions converges to a Gaussian. We conclude by proving that for any choice of k, the probability of finding a gap of length $j \geq 2k+2$ decays geometrically, with decay ratio equal to the largest root of the given k-Skipponacci recurrence.

166. Reconstruction of Video Using SVD With Delays

Wonjun Lee George Mason University
Advisor(s): Tim Sauer, George Mason University

Singular Value Decomposition (SVD) is a matrix factorization that can be used to compress or filter data. We apply this idea to reconstruct video acquired with noise. Using delay of time frames will be able to interpret the correlation of the time frames of the data and cancel the noise.

167. Application of Leapfrog method with 4-th order time filter to shallow water equations

Christopher Barton Arizona State University
Lee Burke Arizona State University
Lauren Johnson Arizona State University
Advisor(s): Mohamed Moustaoui, Eric Kostelich, Arizona State University

We propose a new time stepping scheme for the shallow water equations and similar partial differential equations. It is based on the leapfrog method and includes a fourth-order time filter. Our analysis shows that it is significantly more accurate and remains stable for larger time steps than the standard (Robert-Asselin) approach. It is also stable when used in conjunction with the semi-implicit scheme. I will present some numerical results for the shallow-water equations.

168. Dimension of Smooth Bivariate Splines on Hexagonal Partitions

Larry Allen Towson University
Advisor(s): Tatyana Sorokina, Towson University

Bivariate polynomial splines are considered on a hexagonal tiling of a plane region. Traditional Bernstein-Bézier techniques cannot be applied in this case since the underlying partition is not a triangulation. Using homological algebra methods, we compute the Hilbert polynomial for the space of splines on hexagonal partitions. In turn, the Hilbert polynomial defines the dimension of the splines of any polynomial degree and any smoothness. We use hexagons whose vertices are integers, and begin by investigating the so-called Alfeld split given by three rays emanating from the origin. We obtain a formula for the generators of the spline space on the Alfeld split. By adding lines to this split, we create a partition of the plane that includes non-enclosed hexagons. For this partition, some conjectures are made for the dimension formulae. Our main result deals with the partition consisting of an arbitrary number of enclosed hexagons, where we make some conjectures regarding the dimension of spline spaces and confirm the conjectures computationally by using a code in Macaulay software package.

169. Cryptanalysis of Encrypted Search with Lucene Transform

Eric Jones Colorado School of Mines
Huijing Gong University of California, Berkeley
Minh Pham University of Georgia
Advisor(s): George Schaeffer, University of California, Los Angeles

This poster explores a method of performing secure searches in an encrypted cloud server. Specifically, we consider encrypting the searching data structure—we call this approach "wrapped encrypted search." We explain this theoretical scheme and explore the information leakage associated with it. Lastly, we consider an actual implementation of this scheme in which we use the open-source *Lucene*, which provides search functionality, and a slightly modified CTR mode of operation (implemented by *Lucene Transform*) to provide encryption for *Lucene*. In our analysis, we

found that *Lucene Transform*'s efficiency is determined by the encryption and compression mechanisms used. Further, we found the ratio of encrypted search time to unencrypted search time shows a dependency on the encryption block size. Lastly, since AES is used for encryption, the program can be parallelized for better performance.

170. Animating Still Images

Michael Pilosov State University of New York at Geneseo
Advisor(s): Douglas Baldwin, SUNY Geneseo

We present a means of developing digital image transformations that allow a still image to be turned into a short and visually pleasing animation. Rather than manually altering successive frames to create the illusion of motion, the method presented here requires only the input of a few parameters for each transformation. We developed a mathematical framework wherein we defined animations as sequences of still images, and "transformations" as composable functions on such sequences. To implement this work, we have built a MATLAB library of composable functions that streamline the process of turning still images into novel animations. Examples include manipulation of contrast, intensity, and colors of pixels, as well as warps of contours, positions, and size of select regions. The transformations allow for easy animation of regions of interest, giving some semblance of life to still images by turning them into animated GIFS.

171. A Network Based Method for Ranking NBA Players

Milica Hadzi-Tanovic University of San Francisco
Advisor(s): Stephen Devlin, University of San Francisco

We introduce a network structure on NBA players where individuals are connected when they play against each other during a period of time in an NBA game. Using readily available play-by-play data, we give weights to the network edges to allow for head-to-head comparisons between players based on in-game performance. Using this network structure, we formulate and solve a graph diffusion process to produce a ranking of players. We then compare and contrast the diffusion ranking with existing player rankings such as Player Efficiency Rating and Adjusted Plus-Minus, as well as with similar network based ranking systems used in other contexts including the methods of Keener and Colley, and Google PageRank.

172. Toric Surface Codes Over The Field F9

Lauren Buckley The College of the Holy Cross
Advisor(s): John Little, The College of the Holy Cross

Coding theory is the field of study concerned with the transmission of information across noisy channels using a variety of encoding and decoding techniques to improve the reliability of information transfer. To do so, redundancies are incorporated so that errors occurring in transmission can be detected and corrected. Toric surface codes are a class of linear error-correcting codes that are constructed from a set S of integer lattice points in R2 and a finite field Fq. The error-correcting capacity of a code is determined by a parameter called the minimum distance—the smallest Hamming distance between any two distinct codewords. The greater the minimum distance, the greater the error-correcting capacity is. The minimum distance of a toric code is determined by the geometry of subsets of lattice points in S in subtle ways. Our focus was on toric surface codes over the field F9. From extensive calculations and results about the geometry of the sets of integer lattice points, we determined the best minimum distance for each dimension between 1 and 8 and compared these results to the best-known examples in the codetables.de online database. Our main result was a computer-assisted proof that toric surface codes of dimension 8 over F9 cannot have minimum distance greater than 45.

173. Magic Fano Planes

R Miesner Le Moyne College
Advisor(s): David A Nash, Assistant Professor/Le Moyne College

Taking the concept of magic squares, we investigate whether or not the shape of the Fano Plane can ever be made to be magic. That is, as each point on the plane is assigned a numerical value, we investigate to see whether or not the sums along each of the lines on the plane can ever be made to equal all of the others. We are able to answer this question after first addressing the question of when the Fano Plane can be made to be quasi-magic. After showing

this, we investigate different generalizations of the problem. For example, is the Fano Plane ever magic after having certain sum-lines removed? Or, is the Fano Plane ever magic modulo n?

174. Rotation Number of Outer Billiard with Polygonal Invariant Curves

Zijian Yao Brown University
Advisor(s): Sergei Tabachnikov, ICERM / Penn State University

In this paper, we discuss rotation number on the invariant curve of an one parameter family of outer billiard tables. Given a convex polygon η, we can construct an outer billiard table $\mathcal{T}$ by the method of cutting out a fixed area $\mathcal{A}$ from the polygon. $\mathcal{T}$ is piece-wise hyperbolic and the polygon η is an invariant curve of $\mathcal{T}$ under the billiard map T. We will show that if $\beta \in \eta$ is a periodic point under T with associated rational rotation number $\tau = \frac{p}{q}$, then T^q is not locally conjugate to rotation of a circle at β. This proves that the rotation number τ as a function of the parameter $\mathcal{A}$ is a devil's staircase function.

175. Dynamics and Control of an Invasive Species: The Case of the Rasberry crazy ant Colonies

Octavious Talbot Morehouse College
Victor Suriel Stony Brook University
Agustin Flores Northeastern Illinois University
Valerie Cheathon Arizona State University West Campus
Advisor(s): Octavious Talbot, Morehouse College

This project is motivated by the costs related with the documented risks of the introduction of non-native invasive species of plants, animals, or pathogens associated with travel and international trade. Such invasive species often have no natural enemies in their new regions. The spatiotemporal dynamics related to the invasion/spread of Nylanderia fulva, commonly known as the Rasberry crazy ant, are explored via the use of models that focus on the reproduction of ant colonies. A Cellular Automaton (CA) simulates the spatially explicit spread of ants on a grid. The impact of local spatial correlations on the dynamics of invasion is investigated numerically and analytically with the aid of a Mean Field (MF) model and a Pair Approximation (PA) model, the latter of which accounts for adjacent cell level effects. The PA model approach considers the limited mobility range of N. fulva, that is, the grid cell dynamics are not strongly influenced by non-adjacent cells. The model determines the rate of growth of colonies of N. fulva under distinct cell spatial architecture. Numerical results and qualitative conclusions on the spread and control of this invasive ant species are discussed.

176. Random Walks on Spheres and Harmonic Functions

Annie Brunelle Belmont University
Ryan Gallagher University of Connecticut
David Wegscheid Michigan State University
Jeff Dzugan Samford University
Advisor(s): Igor Nazarov, Michigan State University

We are examining a classical Kakutani result on the relationship between Brownian motion, a form of random movement, and harmonic functions, which are solutions to Laplace equation. We will use this result to numerically solve Laplace's equation in certain regions with various boundary conditions via the Random Walks on Spheres method. Looking at several regions, we will discuss the distribution of the point of first encounter with the boundary. We will also approximate the solutions using Gaussian quadrature, a method which allows us to have incomplete data concerning the boundary conditions. Lastly we consider the accuracy of our numerical solutions.

177. Predictors in Determining Recurrence of Pancreatic Cancer

Lesly Almanzar Kean University
Advisor(s): Louis Beaugris, Kean University

Pancreatic ductal adenocarcinoma is the tenth most common cancer and fourth leading cause in cancer related death. This disease is often lethal because of its very high recurrence rate after the initial removal surgery. The most common recurrence site is the liver. We will present a statistical analysis of a data set previously collected by Aeen Asghar (BS, BA) and James J. Mezhir (MD) beginning in 1996 and ending in 2013. 207 patients were followed from diagnosis through operation and continued follow-up. Our goal was to find significant predictors of overall

recurrence, liver metastasis, and cancel related death. Predictors previously reported to be significant varied from patient age, tumor size, grade of cancer progression and concentration of cancer specific antigen. Yet none of these predictors produced statistically significant result in our analysis. However, a smaller number of variables from our dataset were found to be significant predictors for overall recurrence, liver recurrence or death in Cox univariate analysis. Theses predictors included tumor and nodal biopsy result, R status, neoadjuvant therapy and percent of positive node. These predictors were then combined in a multivariate Cox regression to determined if some of the variables were linked.

178. A Statistical and Geographical Analysis on the Demographics of a City and its Effects on Crime Rates on Campuses

Sherod-Malik Davis Morehouse College
Lorena Maxwell Rose-Hulman Institute of Technology
Malachi Morgan Morehouse College
Elaine Perrin Miami University
Travis Sellers Maryville College
Advisor(s): Monica Jackson, American University

Campus crime rates are generally lower than the national average; however thousands of crimes take place on college campuses daily. Cities that are notoriously dangerous would likely be undesirable locations for a college campus. This study examined the crime rates on campuses throughout the United States and whether or not they were significantly affected by surrounding cities. A multivariate regression analysis was performed to investigate which characteristics of a city, along with a few chosen demographics of a school, impacted the crime rate on a college campus. Our study suggested that the percentage of males of a population, and the diversity index of a city affected the number of crimes that occur on campuses. Whether a city was considered to be urban or rural played a role in determining crime rates as well. Our study also showed the higher percentage of students involved in Greek life, the more crimes were likely to occur. We also examined if a geographical pattern existed among the crime rates and among whether a city was urban or rural.

179. Predicting Galaxy Redshifts

Cathleen Gillette University of Pittsburgh
Cary Morgan Allegheny College
Advisor(s): Chad Schafer, Carnegie Mellon University

This presentation will begin with a discussion of astronomy and spectroscopy, which provides essential background to our project. We used photometric data from 2,230 galaxies, obtained from Kitt Peak National Observatory, to create both parametric and non-parametric regression models. Various statistical regression techniques were implemented, including general additive models, linear models, and smoothing splines, to create and evaluate models predicting galaxy redshifts. Our models were generated with R, a statistical programming software. Over the course of seven weeks, we experimented with different regression concepts, continually improving the models. We would like to share our results, along with the process of how we obtained and tested them, and why this project is so important to the fields of statistics and astronomy.

180. On the optimization of 1-cycle persistence under the Vietoris-Rips complex

Walter Cai Cornell University
Advisor(s): Glen Wilson, Rutgers University

Given a finite discrete point set $X \subset \mathbb{R}^n$ and distance parameter δ, we may generate the Vietoris-Rips Complex $R_\delta(X)$; the simplicial complex where a simplex $[x_i]_{i\in I} \in R_\delta(X)$ if $|x_i - x_j| \leq \delta$ for all $i, j \in I$. Given arbitrary 1-cycle σ appearing in the Rips Complex, we may define the the birth and death of σ as $\alpha = \min(\delta : \sigma \in C_1(R_\delta(X)))$ and $\gamma = \min(\delta : \sigma \equiv 0 \in H_1(R_\delta(X)))$ respectively. We seek to maximize γ with respect to α. In order to do so we consider a specific class of finite point sets $X = \{x_i\}_{0\leq i\leq n}$ where the cycle in question is $\sigma = \sum_{i=0}^{n-1}[x_i, x_{i+1}] + [x_n, x_0]$. We conjecture a possible optimal configuration occurring when the vertices of X are equally spaced on the circle. Our algorithms focus on the manipulation of the original X, incrementally increasing γ while maintaining a constant α. These processes operate by finding subsets of X and through reflection and angular splitting, spread out the vertices in the pursuit of a more circular distribution.

181. Invariants of Virtual Operator for Knot Groups

Mayra Lopez University of North Texas at Dallas
Advisor(s): Noureen, University of North Texas at Dallas

The fundamental groups of knot complement is a strong invariant of classical knots. In this paper we extend the notion of fundamental groups for virtual knots. We introduce virtual operator “T” for a virtual crossing and calculate Wirtinger relations for virtual link diagrams. Moreover, we present combinatorial approach for virtual knot groups to show that virtual crossings have impact on the constructed objects.

182. The Topological Microstructure of Murine Tumors

Melissa McGuirl College of the Holy Cross
Advisor(s): David Damiano, College of the Holy Cross

Using methods of computational topology, we have developed a method of analyzing CT-SPECT images of experimentally induced solid tissue tumors in mice. With a focus on Morse theory, we extract and analyze critical points on each image, and represent their structure with contour trees. Through algorithmic development in MATLAB, we create data array and binary tree representations of the critical points and level surfaces for each image. We will utilize a phylogenetic metric on trees and clustering techniques to discriminate between treated tumors and control-group tumors. Mouse images and image extraction software have been provided courtesy of inviCRO.

183. A New Approach to Enumerating Statistics Modulo n

William Kuszmaul MIT PRIMES
Advisor(s): Darij Grinberg, MIT

We find a new approach to computing the remainder of a polynomial modulo $x^n - 1$; such a computation is called modular enumeration. Given a polynomial with coefficients from a field of characteristic zero, our first main result constructs the remainder simply from the coefficients of residues of the polynomial modulo $\Phi_d(x)$ for each $d|n$. Since such residues can often be found to have nice values, this simplifies a number of modular enumeration problems; indeed in some cases, such residues are already known while the related modular enumeration problem has remained unsolved. We list six such such cases where our novel technique effortlessly yields new results. Our second main result is a formula for the unique polynomial a such that $a \equiv f \mod \Phi_n(x)$ and $a \equiv 0 \mod x^d - 1$ for each proper divisor d of n. We make progress on a problem in modular enumeration on subset sums posed by Kitchloo and Pachter. We also find a formula for remainders of q-binomial coefficients and for remainders of q-Catalan numbers modulo $q^n - 1$, reducing each problem to a finite number of cases for any fixed n, and leading to the discovery of a new cyclic group operation on certain lattice paths.

184. Maximum number of arcs on a digraph for a given zero forcing number

Cora Brown Carleton College
Advisor(s): Leslie Hogben and Adam Berliner, Iowa State University

A digraph Γ is a set of vertices along with a set of arcs whose elements are ordered pairs of vertices. If all vertices of Γ are colored either white or blue, the color change rule states that a blue vertex v can force an adjacent white vertex w to become blue if w is the only white out-neighbor of v. The zero forcing number of Γ is the minimum number of blue vertices needed to force all the vertices of Γ to become blue by the color change rule. The maximum nullity of a digraph, $M(\Gamma)$, is the maximum nullity over the set of matrices described by Γ. We will present results on the maximum number of arcs for a digraph of order n with a given zero forcing number k, and provide a construction for a digraph $\Gamma_{n,k}$ that realizes this bound. We will also examine the maximum nullity of $\Gamma_{n,k}$.

185. On the minimum size of a 2-fold restricted sumset of an m-element subset of a non-cyclic group

Derek Bloom Gettysburg College
Advisor(s): Bela Bajnok, Gettysburg College

I have established a method for finding an upper-bound on the minimum size of a 2-fold restricted sumset of an m-element subset of a non-cyclic group $Z_{n_1} \bigoplus Z_{n_2}$, denoted $\rho\hat{}(G, m, h)$. This upper-bound, referred to as $u\hat{}(Z_{n_1} \bigoplus Z_{n_2}, m, 2)$ is eqaul to the $min\{|2\hat{}B_{d_1,d_2}|\}$ where d_1 is a divisor of n_1, d_2 is a divisor of n_2, and B_{d_1,d_2} is the set of m elements determined by method.

186. The Lattice of Set Partitions and Transition Matrices of Symmetric Functions

Alexandria Burnley University of Illinois at Urbana-Champaign
Simone Thiry University of Maine at Farmington
Aquia Richburg Morehouse College
Advisor(s): Rosa C. Orellana, Dartmouth College

In 1991, O. Eğecioğlu and J. Remmel studied the transition matrices between several bases for the commuting symmetric functions. In our poster we describe an alternative way to obtain these transitions matrices using symmetric functions in non-commuting variables and the lattice of set partitions. In particular, we study the functions $\mathrm{N}_\lambda(\mu)$, the number of set partitions of type μ that are coarser than a set partition of type λ, and $\mathrm{n}_\lambda(\mu)$, the number of set partitions of type μ that are finer than a set partition of type λ. In addition, if $\hat{0}$ is the minimal set partition, we also study $J_\lambda(\mu)$ which is defined as the number of set partitions σ of type μ such that $\pi \wedge \sigma = \hat{0}$ for a given set parition π of type λ. Finally we show how these functions relate to the transition matrices.

187. Games, Genomes, and Graphs

Erik Holmes Boise State University
Advisor(s): Marion Scheepers, Boise State University

Ciliates, single celled organisms, upgrade their genome by reordering the encrypted DNA of their micronuclei into readable strands. The decryption process uses context guided operations which can be modeled on permutations. Using a graphical representation of the permutation we have characterized which strings can be decrypted using these operations. Strands that cannot be decrypted by this operation can be analyzed using finite, determined games between two players. Utilizing graphical representation, we have found criteria for deciding which of the two players has the winning strategy for certain permutations.

188. Equivalence Classes of Permutations Modulo Replacements Between 123 and Two-Integer Patterns

Vahid Fazel-Rezai Phillips Exeter Academy
Advisor(s): Tanya Khovanova, Massachusetts Institute of Technology

We explore a new type of replacement of patterns in permutations that does not preserve the length of permutations. In particular, we focus on replacements between 123 and a pattern of two integer elements. We apply these replacements in the classical sense; that is, the elements being replaced need not be adjacent in position or value. For each replacement, the set of all permutations is partitioned into equivalence classes consisting of permutations reachable from one another through a series of bi-directional replacements. We break the eighteen replacements of interest into four categories by the structure of their classes and fully characterize all of their classes.

189. Constructing 4 dimensional tops and analyzing Fano polygons

Alexa Syryczuk University of Wisconsin-Eau Claire
Christopher Magyar University of Wisconsin- Eau Claire
Advisor(s): Ursula Whitcher, University of Wisconsin-Eau Claire

The polar duality transformation takes a polytope with integer lattice points to its polar dual. If the polar dual is also a lattice polytope then we refer to the polytopes as reflexive polytopes. Reflexive polytopes have been classified in 2, 3, and 4 dimensions, with 16, 4,319 and 473,800,776 classes respectively. The goal of these classes is to get to a better understanding of their relationship to string theory and extending to higher dimensions. A lattice polytope that contains the origin is known as a top. Bouchard and Skarke have classified the 3 dimensional tops corresponding to each class of reflexive 2-dimensional base polytopes. We describe two related projects examining reflexive polytopes in several dimensions. In 3 and 4 dimensions our examinations delve into combinatorial questions, while in 2 dimensions the investigation is mainly number theoretic. We use triangulations of 3-dimensional reflexive polytopes to construct new, "exceptional} examples of tops. We specifically analyze exceptional tops from different reflexive simplices.. We construct polynomials from these polar dual Fano polygons, the two dimensional reflexive lattice polytopes, and show there exists a symmetry between the solutions of these polynomials over finite fields.

190. Factor Groups of Knot and LOT Groups

Renata Gerecke Pomona College
Bryan Oakley University of Georgia
Advisor(s): Jens Harlander, Boise State University

It is difficult to determine whether, given a finite, balanced group presentation, the group is finite or infinite. We study this problem in the context of knot groups and label orientated tree (LOT) groups. More specifically, we are looking at factor groups of knot & LOT groups by powers of meridians. This is in the spirit of Coxeter's work on the factor groups of braid groups. Indeed, our findings generalize Coxeter's work from the three-strand braid groups to knot groups.

191. On Binary Formations and Sequence Extremal Functions

Rohil Prasad MIT PRIMES
Advisor(s): Jesse Geneson, Massachusetts Institute of Technology

An (r, s)-formation is a concatenation of s permutations on r distinct letters. We define Form(u) for some pattern of letters u to be the smallest s such that every (r, s)-formation contains u for some r. We begin by finding Form$((a_1 a_2 \dots a_c)^k) = 2k - 1$ and discussing the use of this result by Geneson et al. to improve the bound on the number of edges in simple k-quasiplanar topological graphs. We conjecture Form$(uau') \leq$ Form$(uu') + 1$ given patterns u, u' and a letter a and how assuming this conjecture gives a tighter general upper bound on Form(u). Then, we expand on a previous technique for bounding Form(u) and classify Form(u) for every pattern u on two distinct letters. We find Form$((a_1 a_2 \dots a_c)^k (a_c a_{c-1} \dots a_1)) = c + 2k - 1$. We also bound Form$(I_c^2 D_c^2)$ and Form$((a_c a_{c-1} \dots a_1)(a_1 a_2 \dots a_c)^k (a_c a_{c-1} \dots a_1))$. In addition, we find a lower bound for the Form(u) value for any formation of arbitrary length in which each pair of adjacent permutations are in reverse order. Finally, we create an algorithm for calculating Form(u) and use it to partially classify the sequences for which Form$(u) = 4$.

192. Permutation Patterns for Real-Valued Functions

Gustavo Melndez Ros University of Puerto Rico, Río Piedras Campus
Alicia Arrua California State Polytechnic University, Pomona
Lynesia Taylor Spelman College
Advisor(s): Rosa Orellana, Dartmouth College

Let $[x, f(x), f(f(x)) = f^2(x), \dots, f^{(n-1)}(x)]$ be the sequence where f is a real-valued function and $n \geq 2$. We can associate a permutation to every such sequence by comparing it with $x_1 < x_2 < \dots < x_n$, where $x_i = f^{(j-1)}(x)$ for some $j = 1, 2, \dots, n$. Permutations that occur are allowed permutations; otherwise they are called forbidden permutations. We focus on enumerating and identifying permutations of real-valued functions such as the logistic map ($L_r(x) = rx(1-x)$ with $0 < r \leq 4$), the half-circle map ($H(x) = \sqrt{x - x^2}$), the sine map ($S(x) = sin(\pi x)$), the castle map ($C(x) = -16x^4 + 32x^3 - 24x^2 + 8x$), and variations of the logistic map ($L_r(x^k) = rx^k(1 - x^k)$ where $k \geq 2$). In the case of the half-circle map, we completely enumerated all of its permutations. Furthermore, we have conjectured that the sine and castle maps are order isomorphic to the $L_4(x)$, i.e., they have the same allowed and forbidden permutations. We have also conjectured that several variations of the logistic map are order isomorphic to each other.

193. Winning strategies for the game upset-downset

Charles Petersen San Jose State University
Advisor(s): Timothy Hsu, San Jose State University

Let P be a finite poset. For $x \in P$, recall that the *downset* and *upset* of x are defined to be $\check{x} = \{y \in P \mid y \leq x\}$ and $\hat{x} = \{y \in P \mid y \geq x\}$, respectively. We define the *upset-downset game* $G(P)$ to be the game with the following possible moves: For any $x \in P$, **Left** may remove the downset of x, leaving the game $G(P - \check{x})$; and for any $x \in P$, **Right** may remove the upset of x, leaving the game $G(P - \hat{x})$. The first player unable to move loses, i.e., $G(\emptyset) = 0$. By standard results, we see that upset-downset is a partisan game whose values are all infinitesimal ("all small"). This talk describes our work in two special cases of upset-downset. In the case where the Hasse diagram of P is a disjoint union of complete bipartite graphs, we exhibit a winning strategy for any winnable position. We also describe preliminary results in the case where P is a rank 2 poset where the downset of every rank 2 element

x contains exactly two elements of rank 1. (Such posets may be thought of as graphs, where the rank 2 elements come from edges and the rank 1 elements come from vertices.)

194. The New Mastermind Game

Thomas Simmons Virginia Wesleyan College
Samantha Eanes Virginia Wesleyan College
Advisor(s): Audrey Malagon, Virginia Wesleyan College

Mastermind is a popular code-breaking board game in which a player attempts to determine a hidden sequence of colored pegs by laying out guesses. In the traditional board game, one player creates the hidden code using 4 pegs available in 6 colors, and the other player tries to guess the hidden code in fewer than twelve guesses. In 1977, computer scientist Donald Knuth published his now famous paper detailing a strategy that would win the original Mastermind game with 4 pegs and 6 colors in five of fewer moves. Lately new versions of the game have been appearing on iPad apps that vary components of the game, including number of pegs, number of colors, whether or not repetition of colors is allowed, and feedback given to the player. In this project, we examine the effects of these variations and explore new bounds on the number of moves required to win.

195. Enumeration of Subclasses of (2+2)-free Partially Ordered Sets

Nihal Gowravaram Massachusetts Institute of Technology PRIMES Program
Advisor(s): Wuttisak, Trongsiriwat

We investigate avoidance in (2+2)-free partially ordered sets, posets that do not contain any induced subposet isomorphic to the union of two disjoint chains of length two. In particular, we are interested in enumerating the number of partially ordered sets of size n avoiding both 2+2 and some other poset α. For any α of size 3, the results are already well-known. However, out of the 15 such α of size 4, only 2 were previously known. Through the course of this paper, we explicitly enumerate 7 other such α of size 4. Also, we consider the avoidance of three posets simultaneously, 2+2 along with some pair (α, β); it turns out that this enumeration is often clean, and has sometimes surprising results. Furthermore, we turn to the question of Wilf-equivalences in (2+2)-free posets. We show such an equivalence between the Y-shaped and chain posets of size 4 via a direct bijection, and in fact, we extend this to show a Wilf-equivalence between the general chain poset and a general Y-shaped poset of the same size. In this paper, while our focus is on enumeration, we also seek to develop an understanding of the structures of the posets in the subclasses we are studying.

196. Semantic Similarity between Topic Pairs

Shiqi Zheng Wheaton College
Advisor(s): Roja, University of California, Los Angeles

In our project we explored novel ways to measure the semantic similarity between topic pairs. We investigated this problem for both intra-language (English) and inter-language (English and French) topics. The data used are text files from English Wikipedia French wikipedia. For the intra-language problem, we developed an algorithm that could detect the semantic similarity between English topic pairs. We have named our new algorithms SARAL and Topo-SARAL. Cosine similarity is adopted as the base-line method. Based the evaluation, our algorithms outperform Cosine similarity in terms of lower standard deviations and higher precisions. Our approach to inter-language problem is generating three topic-models:an English, a French, and an inter-language topic model. To train the interlanguage topic model, we combined documents that are independently generated from English and French topics and form a training set. The similarity between a topic in English and a topic in French is then measured by inserting French documents and English documents into the mixed language model. Cosine similarity is then applied to measure the distance between the vectors. To evaluate the quality of our methods, we compared the results to human judgment.

197. Bootstrapping for Text Extraction in Cyber Security

Nikki McNeil University of Maryland, Baltimore County
Advisor(s): Robert A. Bridges, Oak Ridge National Laboratory

As the Internet accumulates an enormous amount of information, a variety of domains may benefit from automated extraction of key concepts; however, lack of training data inhibits supervised machine learning. Specifically, in the

domain of cyber security, for which little labeled data exists, we have applied the method of bootstrapping with minimal seeds and demonstrated its advantages. Bootstrapping is a semi-supervised method of extracting entities and relations from unstructured text. It is a cyclical text extraction process, which, given a set of seed instances and patterns, iteratively compares instances to find new patterns, and uses patterns to search for new instances. Previous techniques have shown that a challenge in bootstrapping is a program's learning spurious results, which compound accuracy errors over time. In order to optimize precision, we have tested and compared different metrics for deciding which instances and patterns found by a program should be used in its future iterations. Our goal is that, months before security information is organized into databases and vendors release patches, analysts will be able to extract relevant knowledge from unstructured sources such as news sites, mailing lists, and social media, for the purpose of timely cyber-defense.

198. Scanning for Complementary Code Matrices

Brooke Logan Rowan University
Advisor(s): Hieu D. Nguyen, Rowan University

Complementary code matrices (CCMs) are a generalization of binary complementary codes (Golay pairs) and were first studied by G. Coxson and W. Haloupek for arbitrary dimensions. CCMs are useful in many radar applications due to their low peak sidelobe levels. A subset of these CCMs consists of the Hadamard matrices. We present our results for 4x4 and 6x6 quad-phase CCMs based on an exhaustive search and a partial classification in terms of primitives and their equivalency classes.

199. Fooling Functions

Martha Razo Illinois Institute of Technology
Advisor(s): Fred J. Hickernell, Illinois Institute of Technology

We often rely on numerical algorithms to compute definite integrals that cannot be expressed in terms of elementary functions. Such algorithms are convenient and fast, and they claim to provide the requested integral with an error no greater than the user-specified tolerance. Unfortunately, nearly all such algorithms can be fooled. That is, there exist integrands for which the automatic quadrature algorithm returns an answer with an error that exceeds the tolerance. We have investigated the characteristics of integrands that fool automatic quadrature algorithms. We have found that the error estimates used by most automatic algorithms are flawed because these error estimates are based on just two approximations to the integral. Using this insight, we have constructed fooling functions that are not particularly spiky, but for which the automatic algorithm is grossly incorrect. We therefore propose that typical error estimation formulae taught in standard numerical methods texts should be used with caution.

200. Converging on the Area of the Mandelbrot Set

Long Cheong Rowan University
Daniel Bittner Rowan University
Advisor(s): Hieu Nguyen, Rowan University

Determining the exact area of the Mandelbrot set remains an open problem. There exist two primary methods for calculating this area: the pixel counting method, which gives an approximation of 1.5066 by Forstemann, and the Laurent series method, which from Ewing and Schober's work approximates the area at 1.72 using 500,000 terms in the series. Our work seeks to extend Ewing and Schober's calculations to over 1,000,000 terms by finding more efficient algorithms to compute a recursive two-dimensional series $\beta_{n,m}$ that lies behind their area formula. We report on a new upper bound of 1.704 for the area of the Mandelbrot set, a new parallel method of calculation using multithreading, and new results concerning fractal properties of $\beta_{n,m}$.

201. Multilevel Monte Carlo for Financial Option Pricing

Kole Reddig George Mason University
Wonjun Lee George Mason University
Advisor(s): Timothy Sauer, George Mason University

The expected value of financial options can be computed with an approximation of the Black-Scholes Stochastic Differential Equation and the Monte Carlo method. Multilevel Monte Carlo is a computational method that achieves an

improved accuracy over the normal Monte Carlo method, while having the same computational cost. This research includes application of the Multilevel Monte Carlo algorithm to various financial options and other stochastic processes, along with error analysis and comparison of the Monte Carlo and Multilevel Monte Carlo methods.

202. Applications of Gersgorin Disc Theory to the Detection of Hopf Bifurcations

Morgan Swaidan California Lutheran University
Advisor(s): Christopher Brown, California Lutheran University

Systems of differential equations are useful in constructing mathematical models of natural phenomena. In the absence of complete solutions, the eigenvalues of a system are used to qualitatively describe its behavior. To determine the full range of possible behaviors a system can exhibit, different combinations of parameter values must be analyzed. However, as systems get larger, computing eigenvalues gets to be time consuming. We attempt to use the theory of Gersgorin's discs to quickly eliminate the possibility of purely imaginary eigenvalues (and hence periodic behavior, often of special interest to scientists) for given parameter values, which would eliminate the need for difficult eigenvalue calculations. Timed computation tests in Python indicate that calculating the Gersgorin discs becomes significantly more efficient than solving for eigenvalues when the dimension of a system exceeds 200. In further work we attempt to apply Lie theory to alter the discs in hopes of speeding up eigenvalue computations.

203. Measuring Open Water Fraction and Floe Size Distribution In The Arctic Ocean Ice Pack Using High Resolution Satellite Images

Lauren Sommers California State University, Monterey Bay
Advisor(s): Timothy P. Stanton, Naval Postgraduate School

The Interplay between upper ocean heat content, sea ice cover, and incoming solar radiation determines the perennial ice cover in the Arctic Ocean. Because of this balance, the local open water fraction strongly influences summertime upper ocean heat flux. Determining floe size distribution is important for modeling the incoming solar radiation, since solar radiation heats the ocean in the absence of ice cover. Very high-resolution visible images are now available that show ice floes in great detail. With algorithms that determine open water fraction and area of ice floes, we will be able to apply quantitative measures to assess the solar heat flux entering the ocean. We developed algorithms in MATLAB to extract open water fraction data and floe size distribution at 1m â\ 10,000m scales from visible wavelength images taken around oceanographic buoys deployed in the Arctic. The algorithms combine image-processing techniques and a manual threshold selection to accurately identify boundaries of ices floes within an image. A MATLAB routine is used to remove melt ponds within large floes to allow the floe size distribution and open water fraction to be determined. These methods are being applied in an ongoing research project studying the marginal ice zone in the arctic.

204. The Dynamics of Nucleation in Stochastic Cahn-Hilliard Systems

Mahmoud Namazi George Mason University
Advisor(s): Evelyn Sander, George Mason University

The stochastic Cahn-Hilliard partial differential equation describes the process of phase separation in a binary alloy. One common phenomena that occurs in this model is nucleation where one metal forms droplets in the other. This research utilizes various homological tools such as Betti numbers and the Euler characteristic in order to analyze th stochastice Cahn-Hilliard model with Neumann and periodic boundary conditions. We show that these tools are effective in counting droplets and proving certain qualities about stochastic Cahn-Hilliard model.

205. Exterior Point Method for Support Vector Machines

Ben Howard George Mason University
Advisor(s): Igor Griva, George Mason University

Support vector machines (SVMs) have been an important concept in machine learning for many years. SVMs require solving a nonlinear optimization problem with quadratic objective function and linear constraints. Our approach utilizes an exterior-point method (EPM) that stems from nonlinear rescaling and augmented Lagrangian methods that enables iterations to approach the solution of a constrained nonlinear optimization problem from the exterior

of a feasible set. Further, each iteration solves a well-conditioned system of linear equations, avoiding numerical-inaccuracies that can occur solving ill-conditioned systems. This makes the EPM a feasible alternative to existing quadratic programming solvers for training SVMs.

206. Parallel Computing Improvements to Adaptive Metropolis Hastings Algorithm

Darryl Johnson Shepherd University
Advisor(s): Qing Wang, Shepherd University

We propose an adaptive Metropolis-Hastings algorithm that makes use of parallel computing to estimate the co-variance of the posterior distribution. The adaptive model, as proposed by Klinke(2009) uses the covariance of previously sampled space to modify the proposal distribution for more rapid convergence. Implementation of parallel computing allows for even further improvements to convergence rates. A toy model based off of Lotka-Volterra predator-prey relationships and data from Yellowstone National Park is used to demonstrate improvements in the speed of convergence over previous adaptive models.

207. A Geometric Interpretation of Knot Complement Gluings

Martin Bobb Carleton College
Advisor(s): Rolland Trapp, California State University, San Bernardino

A knot is an embedding of a circle in a sphere. Many knots, including 2-bridge knots, have complements with a hyperbolic structure determined by the knot. We explore the hyperbolic volumes of knot complements for 2-bridge links obtained by Dehn fi llings. We build on work by Purcell to more accurately explore how the geometry of universal covers and gluing operations affect hyperbolic volumes.

208. Developing an Algorithm to Compute the Discrete LS Category of a Simplicial Complex

Brian Green Ursinus College
Advisor(s): Nicholas Scoville, Ursinus College

Let K be a simplicial complex and suppose that K collapses into L. Define n to be 1 less than the minimum number of collapsible sets it takes to cover L. Then the discrete Lusternik–Schnirelmann category of K is the smallest n taken over all such L. We introduce an algorithm to estimate the discrete LS category of an n-dimensional simplicial complex. We estimate the discrete category of a simplicial realization of several topological spaces with interesting topological features. In particular, we show the discrete category of the smallest triangulation of the dunce cap is 2, whereas the smooth category is 0.

209. Realistic modeling and simulation of influenza transmission over an urban community

Jiechen Chen University at Buffalo
Advisor(s): Gino Biondini, Ling Bian, University at Buffalo

Infectious diseases that are spread through human contact can progress very rapidly in a population. One of the key factors in the spreading of contagion, and a main concern in attempting to stop the spread of illness, is the particular configuration of links among individuals in local communities within the larger population. This study uses a detailed individual-based, three-partite model comprising about 245,000 individuals located in an urban area in the Northeastern United States. Interactions among individuals are divided into family, workplace and pastime (service places, shopping, etc.), each occurring during a separate time period (daytime, pastime, and nighttime). Thus, the network allows one to model the spatial and temporal heterogeneity in the transmission of communicable diseases and to capture the differences between various individuals' vulnerability to infection. We performed Monte-Carlo simulations of the spreading of influenza through this network. Simulation results correspond well to the reported epidemic information. We expect that the findings will offer a valuable platform to devise spatially and temporally oriented control and intervention strategies for communicable diseases.

210. An initial modeling of fractal nets for the Sierpi ński gasket

Barrett Leslie Illinois Institute of Technology
Advisor(s): Gregory, Fasshauer

The Sierpiński gasket is obtained by applying an iterated function system (IFS) method to a regular tetrahedron. We give a sketch and an algorithm for a folding method that allows us to construct the nth iteration of the Sierpiński gasket. We also provide an initial exploration of construction methods for minimal volume objects that retain their surface area under refinement. This idea has potential applications at the nano-scale.

211. Curvature homogeneity of type (1,3) in pseudo-Riemannian manifolds

Cullen McDonald Beloit College
Advisor(s): Corey Dunn, California State University San Bernardino

We construct two new families of pseudo-Riemannian manifolds which are curvature homogeneous of type $(1,3)$. The first family has signature $(2k, 2k+1)$ and is curvature homogeneous of type $(1,3)$ but not curvature homogeneous. The second family has signature $(1,2)$ and is curvature homogeneous of type $(1,3)$ of all orders but not locally homogeneous, showing there is no finite Singer number for this type of curvature homogeneity.

212. The Isoperimetric Problem in the Plane with Density $e^{-1/r}$.

David Hu Georgetown University
Advisor(s): Frank Morgan, Williams College

We study the isoperimetric problem in the plane with weighting or density $e^{-1/r}$. The isoperimetric problem seeks to enclose prescribed weighted area with minimum weighted perimeter. For unit density (the Euclidean case), the answer is of course any circle. For density $e^{-1/r}$, isoperimetric curves are conjectured to pass through the origin.

213. Convex Region Isoperimetric Conjecture

Maggie Miller University of Texas at Austin
Advisor(s): Frank Morgan, Williams College

We study the Convex Body Isoperimetric Conjecture in the plane, which states that it takes more perimeter to enclose given area in a unit-area disk than in any other unit-area convex region. Previously this was known only for area $1/2$ (Esposito et al., 2010) but we have shown that the claim holds for arbitrarily small area, given a convex region (Gallagher et al., 2013). We discuss further the difficulty of this problem in higher dimensions, where a convex body and a ball must not always intersect in the boundary of a spherical cap.

214. Rigid Tilings of Quadrants by L-ominoes and Notched Rectangles

Samantha Fairchild Houghton College
Samuel Simon Carnegie Mellon University
Advisor(s): Viorel Nitica, West Chester University

We examine rigid tilings of the four quadrants in a Cartesian coordinate system by tiling sets consisting of L-shaped polyominoes and notched rectangles. The first tiling sets we consider consist of an L-shaped polyomino and a notched rectangle, appearing from the dissection of an $n \times n$, $n \geq 3$, square, and of their symmetries about the first diagonal. In this case, a tiling of a quadrant is called rigid if it reduces to a tiling by $n \times n$ squares, each of the squares in turn tiled by an L-shaped polyomino and a notched rectangle. We further determine the rigidity of tilings of the quadrants with tiling sets appearing from similar dissections of $mn \times n$, $m \geq 1$, rectangles. Our technique of proof is to use induction along a staircase line built out of $n \times n$ squares and to show that the existence of a tile in an irregular position propagates further towards the edges of the quadrant, and eventually leads to a contradiction. Further, we look at sets of tiles appearing from rectangles of coprime dimension. These tilings are never rigid.

215. Unilateral and Equitransitive Tilings by Squares of Four Sizes

Emily Pierce Baylor University
Joseph DiNatale Armstrong Atlantic State University
Ellen Vitercik Columbia University
Advisor(s): Casey Mann, University of Washington Bothell

D. Schattschneider proved that there are exactly eight unilateral and equitransitive tilings of the plane by squares of three distinct sizes. This project extends Schattschneider's methods to determine a classification of all such tilings by squares of four different sizes. It is determined that there are exactly 39 unilateral and equitransitive tilings by squares of four different sizes.

216. Generalization of Tournament Graphs

Kyle Webb Wake Forest University
Advisor(s): Kenneth S. Berenhaut, WFU

In this poster we consider some preliminary work on generalizations of standard tournament graphs. For an n node tournament graph, edges are determined by the results of $n(n-1)/2$ games for n players (where each player plays each of the $n-1$ other players once). A directed edge from x to y results from Player x winning over Player y; each tournament graph has $n(n-1)/2$ edges. We consider scenarios where players compete in the $\binom{n}{k}$ possible k-player games (where $k \leq n$) and are interested, for instance, in maximal and minimal numbers of edges (under different edge (domination) constructions).

217. Determining Critical Locations in a Road Network

Sara Reed Simpson College
Casey Croson Simpson College
Louis Joslyn Simpson College
Advisor(s): Debra Czarneski, Simpson College

Critical locations in infrastructure are roads that, if damaged, would cause a large disruption in the ability of vehicles to navigate a city. In this talk, we consider critical locations in the road network of Indianola, Iowa. The presence of cut vertices and values of betweenness for a given road segment are used in determining the importance of that given road segment. We present a model that uses these critical factors to order the importance of separate road segments. Finally, we explore models that focus on betweenness to improve accuracy when discovering critical locations.

218. Identifying the (t, r) broadcast domination theory of one-way street grids

Christie Mauretour Florida Gulf Coast University
Advisor(s): Erik Insko, Florida Gulf Coast University

In this presentation, we explore a concept that we call the (t, r) broadcast domination theory of directed street grids. We call a $m \times n$ directed grid $G_{m,n}$ a *directed street grid* if edges are directed like a grid of one-way streets (i.e., the edges alternate directions every block). On this grid, we suppose that a police officer can protect the vertex she is stationed at with weight t and every vertex within distance $d < t$ with weight $t-d$. We wish to have every vertex of $G_{m,n}$ protected with a weight of r or higher. We call the minimum number of police officers needed to protect the one-way street grid $G_{m,n}$ in this fashion, the (t, r) broadcast domination number of $G_{m,n}$. In this presentation, we present closed formulas for the (t, r) broadcast domination number of grids $G_{m,n}$ when $m = 3, 4, 5$, $n \geq 1$, and $r \leq t \leq 3$.

219. Towards generalizing thrackles to arbitrary graphs

Jin-Woo Oh The Stony Brook School
Advisor(s): Rik Sengupta, MIT

In the 1950s, John Conway came up with the notion of *thrackles*, graphs with embeddings in which no edge crosses itself, but every pair of distinct edges intersects each other exactly once. He conjectured that $|E(G)| \leq |V(G)|$ for any thrackle G, a question unsolved to this day. In this paper, we discuss some of the known properties of thrackles and contribute a few new ones. Only a few sparse graphs can be thrackles, and so it is of interest to find an analogous notion that applies to denser graphs as well. In this paper we introduce a generalized version of thrackles called *near-thrackles*, and prove some of their properties. We also discuss a large number of conjectures about them which seem very obvious but nonetheless are hard to prove.

220. Radio Labeling of Path Graphs to the 6th Power

Edward Melendez California State University, San Bernardino
Jesus Mora-Sanchez California State University, San Bernardino
Reyna Hernandez California State University, San Bernardino
Cynthia Salgado California State University, San Bernardino
Advisor(s): Min-Lin Lo, CSU, San Bernardino

Radio labeling is a process used to model the problem of efficiently assigning channels to FM radio stations to avoid interference. Let G be a connected graph. For any two vertices u and v, the distance, $d(u, v)$, between u and v in

G is the length of the shortest $u - v$ path in G. The maximum distance between any pair of vertices of G is called the diameter of G, $diam(G)$. A radio-labeling of G is a function f that assigns a label from the set $\{0, 1, 2, \ldots\}$ to each vertex such that the following holds for any vertices u and v: $|f(u) - f(v)| \geq diam(G) - d(u, v) + 1$. The span of f is defined as $max_{u,v \in G}$â<$\{|f(u) - f(v)|\}$. The radio number of G is the minimum span among all radio-labelings of G. The 6th power of G is a graph constructed from G by adding edges between vertices of distance six or less apart in G. In this presentation we will discuss the progress we made towards finding the radio number for 6th power of paths during a 2013 MAA summer research program funded by NSA (grant H98230-13-1-0270) and NSF (grant DMS-1156582).

221. Exponential Domination of Cycles Connected at One or Two Vertices

Amanda Bright Westminster College
Advisor(s): Erin Martin, Westminster College, MO

The purpose of this research is to explore the behavior of minimum exponential dominating sets of cycles connected at one or wto vertices. A minimum exponential dominating set is a subset of vertices of a graph that gives weight to all other vertices in the graph. It is well known that the size of a minimum exponential dominating set for a cycle C_n of size $n \geq 5$ is $\gamma_e(C_n) = \lceil n/4 \rceil$. When two cycles of size n are connected at one vertex, we are going to prove the resulting minimum exponential dominating set has the size $\gamma_e(Q_n) = 2\lceil n/4 \rceil - 1$ for $n \geq 5$. We will extend this result to the minimum exponential dominating set when two cycles of size $n \geq 5$ are connected by two vertices. We will further look into connecting additional cycles to make a conjecture to generalize the size of the minimum exponential dominating set.

222. Minimum Degrees of Minimal Ramsey Graphs

Raj Raina Novi High School
Advisor(s): Rik Sengupta, Andrey Grinshpun, Massachusetts Institute of Technology

For graphs F and H, we say F is Ramsey for H if every two-coloring of the edges of F contains a monochromatic copy of H. The graph F is H-minimal if every proper subgraph of F is not Ramsey for H. Burr, Erdős, and Lovász defined $s(H)$ to be the minimum degree of F over all Ramsey H-minimal graphs. Such Ramsey minimal problems arise in many practical situations including network connectivity and information transfer. Define $H_{t,d}$ to be a graph on $t + 1$ vertices that contains a K_t and one additional vertex of degree d. The value of $s(H_{t,d})$ was known only for $d = t$ and very recently found for $d = 1$. We determine $s(H_{t,d})$ for all values $1 < d < t$. Next, we generalize results of Burr et. al. and Fox et. al. by determining tight bounds on $s(H)$ where H is the complete equipartite graph. Finally, Szabó et. al. asked what graphs G have $s(G) = 2\delta(G) - 1$. We partially answer this question for a very large and general class of graphs. We use both classical combinatorial arguments and new probabilistic methods and note that these graphs are the first time $s(H)$ has been determined for several well-connected classes of graphs which are not vertex-transitive.

223. Porous exponential domination number of planar triangulations

Felicia Stover Concord University
Mariah Farley Concord University
Josh Beverly Concord University
Advisor(s): Christopher McClain, Concord University

Given a graph G with vertex set V, a subset S of V is a dominating set if every vertex in V is either in S or adjacent to some vertex in S. The size of a smallest dominating set is called the domination number of G. We study a variant of domination called porous exponential domination in which each vertex v of V is assigned a weight by each vertex s of S that decreases exponentially as the distance between v and s increases. S is a porous exponential dominating set if all vertices in S distribute to vertices in G a total weight of at least 1. The porous exponential domination number of G is the smallest possible size of S. We compute bounds for the porous exponential domination number of certain planar triangulations.

224. New Insights into Stock Returns through Clustering

Jared Rohe University of San Francisco
Paul Hundal University of San Francisco
Advisor(s): David Uminsky, Jeff Hamrick, University of San Francisco

Spectral clustering techniques use properties of the spectrum of the similarity matrix of a collection of data in order to reduce the dimensionality reduction. We apply spectral clustering techniques promulgated by Shi and Malik (2000), as well as other clustering techniques, to analyze the log-returns of the constituents of the Standard and Poor's 500 Index for the 2007–2013 period. We use two different measures to diagnose the number of clusters that are latent in the collection of stock returns. The resulting clusters are closely aligned with the sectors associated with the two-digit Standard Industrial Classification (SIC) codes maintained by the Occupational Safety and Health Administration. We also explore the associations between smaller numbers of clusters and various financial metrics, including price-to-earnings ratio, debt-to-equity ratio, and profitability ratios like net profit margin.

225. Random subtrees of complete graphs

Alex Chin North Carolina State University
Kellie MacPhee Dartmouth College
Charles Vincent Lafayette College
Advisor(s): Gary Gordon, Lafayette College

Select a subtree of the complete graph K_n uniformly at random. What is the expected number of edges in the subtree, and what is the probability that the subtree is spanning? We answer both questions and discuss how these statistics change when instead considering a weighted probability measure, in which the probability of choosing a subtree is proportional to the size of the subtree. We then define a generating function polynomial that can help answer some open questions.

226. 2-tone Colorings in the Direct Product of Graphs

Jennifer Loe Oklahoma Christian University
Advisor(s): Kirsti Wash, Clemson University

A variation of graph coloring known as a t-tone k-coloring assigns a set of t colors to each vertex of a graph from the set $\{1, \ldots, k\}$ where the sets of colors assigned to any two vertices distance d apart share fewer than d colors in common. The minimum integer k such that a graph G has a t-tone k-coloring is known as the t-tone chromatic number. We study the 2-tone chromatic number of the direct product of graphs. In particular, we give a formula for the 2-tone chromatic number of the direct product of two complete graphs and give bounds for the 2-tone chromatic number of the direct product of any two graphs $G \times H$.

227. Extreme Zero Forcing, Maximum Nullity, and Minimum Rank for Oriented Graphs

Katrina Jacobs Pomona College
Joshua Carlson Iowa State University
Advisor(s): Leslie Hogben, A. Berliner, T. Peters, N. Warnberg, Iowa State University, St. Olaf, Culver-Stockton

A graph G is a collection of vertices and edges, where an edge is an unordered pair of vertices. A digraph $\Gamma = (V, E)$, is a vertex set, V, and an arc set of ordered pairs, E. A simple digraph is a digraph which contains no arcs from a vertex to itself or multiple arcs pointing in the same direction between the same two vertices. Given a graph G, an orientation $\vec{G}$ of G is the digraph obtained by replacing each edge $\{u, v\}$ by exactly one of the arcs (u, v) and (v, u) (so a graph G has $2^{|E(G)|}$ orientations). An oriented graph describes the off-diagonal zero-nonzero pattern of a family of non-symmetric square matrices. Several parameters of an orientation and its corresponding family of matrices are zero forcing number, maximum nullity, and minimum rank. We investigated whether there are minimum and maximum values of these parameters for specific types of oriented graphs, specifically tournaments, oriented cycles, and oriented paths. We also considered the relations between these parameters for specific types of oriented graphs, and the possibility that there are bounds on all of these parameters which are satisfied for all orientations of a graph.

228. Orientations that realize zero forcing number and path cover number of underlying graphs

Jason Hu University of California, Berkeley
Advisor(s): Leslie Hogben, Adam Berliner, Michael Young, Iowa State University

Let G be a simple graph. An *orientation* $\vec{G}$ of G is a digraph that assigns a single arc to each edge of G. The *path cover number*, $P(G)$, is the minimum number of vertex disjoint induced paths that include all vertices of G. The *color-change rule* states that if each vertex of G is colored white or blue, u is a blue vertex, and v is a unique white neighbor of u, then change the color of v to blue. A *zero forcing set* for G is a subset of vertices B such that if the vertices in B are colored blue and all other vertices white, then after iteratively applying the color-change rule, all vertices are blue. The *zero forcing number* $Z(G)$ is the size of a minimum zero forcing set $B \subseteq V(G)$. Related definitions exist for simple digraphs, with neighbors restricted to out-neighbors in the color-change rule and Hessenberg paths permitted in a path cover. We prove that for every G, there is always an orientation $\vec{G}$ such that $P(\vec{G}) = P(G)$. However, there exist graphs G such that for all orientations $\vec{G}$, $Z(\vec{G}) \neq Z(G)$. If $P(G) = Z(G)$ (e.g. for a tree), we prove that there exists an orientation $\vec{G}$ of G such that $Z(\vec{G}) = Z(G)$. Related results on oriented graphs are included.

229. The Discrete Lusternik--Schnirelmann category of 1-dimensional simplicial complexes

Matthew Belle Ursinus College
Advisor(s): Nicholas Scoville, Ursinus College

The discrete Lusternik–Schnirelmann (LS) category of a finite graph G is 1 plus the minimum number of subtrees of G it takes to cover G. In this poster, we compute the discrete LS category for a variety of graphs including complete graphs and wedges. We study the discrete category of the product of graphs and share a general upper bound, analogous to the classical upper bound for products in the smooth case, and prove a Ganea-like conjecture for some classes of graphs. We conclude by proposing a Ganea conjecture for all graphs

230. Graph Connectivity Indices of Octagonal Systems

William Burke Montclair State University
Rob Rexler Baello Montclair State University
Advisor(s): Aihua Li, Montclair State University

In this paper we will report the results of our study on Randić Connectivity Indices (RCI) of certain octagonal systems derived from chemistry and biology problems. A graph is called an octagonal graph if it is connected and it is made of regular octagons of the same size without overlapping octagonal regions and every two octagons are either disjoint or share exactly one edge. Formulas for RCI values of octagonal graphs are given and the graphs with maximum or minimum RCI values among all octagonal graphs of the same size are identified. Special properties are given for âœtreesâ of octagons and âœcyclesâ of octagons. As one of the most successful molecular descriptors for structural-property and structural-activity relationship studies in Chemistry, the RCI values may reveal certain connectivity and related properties of the considered graphs This research is funded by NSA (grant H98230-13-1-0270) and NSF (grant DMS-1156582) through MAA NREUP.

231. An Empirical Study of Social Clustering in R-MAT Generated Graphs

Timothy Goodrich Valparaiso University
Advisor(s): Blair D. Sullivan, North Carolina State University

We lay a foundation for the mathematical characterization of degeneracy in the popular Recursive MATrix (R-MAT) generator through the collection and analysis of empirical evidence. Currently used for the Graph500 supercomputer benchmark, R-MAT is expected to generate graphs that emulate real-world data; we examine this claim mathematically. While previous studies have limited their analysis to popular data ranges, we examine all possible parameter values in the symmetric matrix case, revealing beautiful symmetries and phase changes in the entirety of the data. We prove this symmetry holds regardless of the graph's density or skew, and conjecture about possible implications of the phase change. Specifically, results from the data strongly suggest that the graph's connection density largely determines the distribution of isolated vertices, maximum degree, and the entire graph's degeneracy.

232. Robust Graph Ideals

Timothy Duff New College of Florida
Karl Schaefer Rice University
Laura Lyman Reed College
Bryan Brown Pomona College
Advisor(s): Adam Boocher, University of California, Berkeley

Let I be a toric ideal. We say I is *robust* if its universal Gröbner basis is a minimal generating set. We show that any robust toric ideal arising from a graph G is minimally generated by its Graver basis. We then completely characterize all graphs which give rise to robust ideals. Our characterization shows that robustness can be determined solely in terms of graph-theoretic conditions on the set of circuits of G.

233. Distinguished-Color Rainbow-Connected Graphs and Their Applications

Katherine Boligitz La Salle University
Advisor(s): Janet Fierson, La Salle University

We look at rainbow-connected graphs (edge-colored graphs for which there exists a path between any two vertices with uniquely-colored edges). The number of colors needed to make a graph rainbow-connected is denoted as $rc(G)$. We extend this idea by forcing a distinguished edge coloring. Not only do we require that a rainbow-colored path exist for each pair of vertices; this path must also include an edge of a distinguished color. This coloring scheme cannot always be completed with $rc(G)$ colors, so we introduce the $drc(G)$, which is the minimum number of colors needed for a rainbow connected graph that satisfies the distinguished-color edge condition. We look at applications of this in various environments, such as amusement parks and museums, to see if certain graph layouts are more effective in achieving the best experience for visitors. We look at the benefits of, and possible ideas behind, existing designs.

234. When does the Pure Nash Equilibrium of a Graph Coloring Game achieve an Optimal Coloring?

Rob Spahn State University of New York at Potsdam
Advisor(s): Derek Habermas, SUNY Potsdam

Panagopoulou and Spirakis, authors of "Playing a Game to Bound the Chromatic Number," designed a graph coloring game where a graph begins with each vertex/player having its own unique color, then the vertices/players work their way to a proper coloring with fewer and fewer colors. They show that a Pure Nash Equilibrium of the game corresponds to a proper coloring to satisfy four known upper bounds on the chromatic number. We investigate the results obtained through such colorings of several classes of graphs.

235. Uniquely Decomposable Gluings of Three-Component Trees

Elizabeth Field Southern Connecticut State University
Advisor(s): Robert DeMarco, Amanda Redlich, DIMACS, Bowdoin

This is an extension of research done by Bobby DeMarco and Amanda Redlich (2012) in which they study the asymptotic distribution of the number of copies of a given disconnected graph modulo q within the random graph $G(n, p)$. Their research was motivated by a result of Kolaitis and Kopparty (2009) which showed that for any integer q, the number of copies of a given connected graph in $G(n, p)$ is asymptotically uniformly distributed modulo q. This result was used by Kolaitis and Kopparty to prove an extension of a zero-one law to the setting of first-order logic on the random graph augmented with a parity quantifier. DeMarco and Redlich completely characterize the asymptotic distribution of all two-connected graphs and show that nearly all two-connected graphs are uniformly distributed by giving constructions of uniquely decomposable gluings. We determine which three-component forests are uniquely decomposable and give constructions for those that are.

236. When Things Get a Little Edgy: Finding the Grundy Number of Line Graphs

Matthew DeVilbiss University of Dayton
Advisor(s): Peter Johnson, Auburn University

A Grundy coloring of the edges of a graph is a proper coloring with positive integers such that every edge colored with an integer $c > 1$ is adjacent to the edges colored $1, \ldots, c - 1$. The Edge Grundy number of a graph G is the

greatest number if colors appearing in a Grundy coloring of the edges of G. We find the Edge Grundy number for G in a number of classes, including the complete graphs, the complete bipartite graphs, the hypercubes, and the grids as well as a range for the Edge Grundy number for several classes of complete multipartite graphs.

236. Optimizing Kidney Paired Donation: Optimal and Equitable Matchings in Bipartite Graphs

Robert Montgomery St. Lawrence University
Advisor(s): Patti Frazer Lock, St. Lawrence University

If a donor is not a good match for a kidney transplant recipient, the donor/recipient pair can be combined with other pairs to find a sequence of pairings that is more effective. The group of donor/recipient pairs, with information on how strong a match each donor is with each recipient, forms a weighted bipartite graph. The Hungarian method allows us to find an optimal matching for such a graph.However, the outcome which is optimal for the group might not be the most equitable for the individual patients involved. We present several modifications to the Hungarian method which consider a balance between the optimal score for the group and the most uniformly equitable score for the individuals. We examine the strengths and weaknesses of these modifications within the current climate of kidney allocation in the United States. Finally, we expand these findings to other fields where these revised algorithms may also hold particular significance.

237. Cayley Graphs: Separating the wheat from the chaff

John Eallonardo Le Moyne College
Advisor(s): David Nash, Le Moyne College

Cayley graphs, or graphical representations of groups and generating sets, are specific types of digraphs and gaining insight on these specific graphs might lead to further conclusions depending on what a given digraph represents. As there are an infinite number of digraphs, we'd like to be able to conclude whether or not a digraph is a representation of a group and a generating set for that group. After observing group properties, different small sized Cayley graphs and their adjacency matrices, a list of required characteristics was created for which all digraphs must follow in order to be considered Cayley. Looking only at digraphs that satisfy the requirements, we then classify small graphs and certain infinite families of graphs as either Cayley or not Cayley.

239. Geometric Variations on the Euclidean Steiner Tree Problem

Jack Holby St. Lawrence University
Advisor(s): Sam Vandervelde, St. Lawrence University

The Euclidean Steiner Tree Problem (ESTP) attempts to create a minimal spanning network of a set of points by allowing the introduction of new points, called Steiner points. This poster discusses a variation on this classic problem by introducing a single "Steiner line} in addition to the Steiner points, whose weight is not counted in the resulting network. For small sets, we have arrived at a complete geometric solution. We discuss heuristic algorithms for solving this variation on larger sets. We believe that in general, this problem is NP-hard.

240. On Weakly Diameter-m-Critical Graphs

Claire Gibbons Morningside College
Aaron Morse SUNY Potsdam
Advisor(s): Joel Foisy, SUNY Potsdam

A graph G is diameter-m-critical if the diameter of G is m and the deletion of any edge increases the diameter. Diameter-2-critical graphs have been studied extensively in connection with a conjecture of Murty and Simon. We show that a consequence of this conjecture holds. We then generalize the diameter-m-critical graphs to include graphs of diameter less than m. We identify several infinite families of these graphs, classify all such paths and cycles, and discuss some results and conjectures regarding the number of such graphs on n vertices.

241. New Bounds on Extremal Numbers in Acyclic Ordered Graphs

Qinru Shi Massachusetts Institute of Technology
Advisor(s): Jacob Fox, Rik Sengupta, MIT

This paper is mainly concerned with the upper and lower bound of the number of edges an ordered graph can have avoiding a fixed forbidden ordered subgraph H. The only case where a sharp bound has not been discovered is when H has interval chromatic number 2, where H can be represented as a 0-1 matrix P. Let $ex_{<}(n,n,P)$ be the maximum weight of an n by n 0-1 matrix avoiding P. When P contains a cycle, the corresponding bound of $ex_{<}(n,n,P)$ is also known. Hence, the interesting case is when P is acyclic. In this paper, we construct a family of patterns $\mathcal{P}$ such that for a positive integer m, there exists $P \in \mathcal{P}$ with $ex_{<}(n,n,P) = \Omega(n \log n \log\log n \cdots \underbrace{\log\log\cdots\log n}_{m \text{ iterations}})$. This result suggests an improved lower bound for the least upper bound of extremal numbers in acyclic ordered graphs. In addition, we suggest a new method for attaining an upper bound of $ex_{<}(n,n,P)$ for a special set of patterns.

242. The Abelian Sandpile and it's Avalanche Polynomials.

Andy Fry Western Oregon University
Advisor(s): Luis Garcia Puente, Sam Houston State University

Considering a directed graph Γ with a global sink, we define a sandpile σ as a vector of non-negative integers indexed by the non-sink vertices of Γ, where the vth entry of σ, $\sigma(v)$, represents the number of grains of sand at vertex v. A sandpile σ is stable if, every vertex v, $\sigma(v)$ is less than the number of edges emanating from the vertex v; if not, σ is unstable. An unstable sandpile σ may be stabilized to σ° by a sequence of topplings. If a stable sandpile σ can be accessed from any other stable sandpile via a sequence of sand additions and topplings, we call it recurrent. Given σ a recurrent sandpile, define a principal avalanche on Γ as a sequence of topplings required to stabilize the sandpile $\sigma' = \sigma + 1_v$, obtained by adding a grain of sand at some vertex v. Then one can define the avalanche polynomial of a graph Γ as the polynomial $A_\Gamma(x) = \sum c_k x^k$, where c_k is the number of principal avalanches of size k. Our work examines this idea on various graphs.

243. Chromatic Symmetric Functions for Special Types of Graphs

Damien Gonzales University of California, Berkeley
Arman Green Morehouse College
Caprice Stanley The George Washington University
Advisor(s): Rosa Orellana, Candice Price, Dartmouth College

Given any simple graph, there is a corresponding symmetric function called the chromatic symmetric function (CSF). Introduced by Richard Stanley in 1995, the CSF of a graph $G = \big(V(G), E(G)\big)$ is defined in terms of a summation of the product of monomials obtained from proper colorings of G. A proper coloring is a labeling of a graph such that no two adjacent vertices have the same label. In Geoffrey Scott's senior thesis, published in 2008, several open problems in graph theory were presented. In our poster, we investigate these open problems and generalize some of his results. Our ultimate goal is to give necessary conditions for any graphs that will ensure that they have the same CSF. We consider two special types of graphs: trees and unicycles and write a program in Sage to compare the CSF for any simple graph, and thus, compile a library of graphs with a small number of vertices alongside their CSFs. Speci cally, we prove some results about squids and stars to help identify when unicycles have the same chromatic symmetric function.

244. Classifying Large Data Networks

Skyler Stasiewicz Hood College
Tarang Hirani Hood College
Advisor(s): Jill Dunham and Ann Stewart, Hood College

We will present an analysis of the Air Traffic network and the Yeast Genome for comparison, which was studied using methodologies from graph theory. Using a result from the formulas for calculating average shortest path (Fronczak, Fronczak, and Holyst) and the maximum clique (Bollobás and Erdős) of both ős|Rényi random models and Barabasi Albert models, we developed algorithms to calculate a new measure of the randomness in the structure of each data

set. To implement these algorithms, we utilized MATLAB for computation and statistical simulations, and Gephi for visualization.

245. Utilizing graph theory to see the unseen in nuclear data

John Hirdt St. Joseph's College
Advisor(s): S. Jane Fritz, Dr. David Brown, St. Joseph's College, Brookhaven National Lab

Nuclear databases can contain hundreds of thousands of entries, with many different forms of data. Many of these databases include specific measurements which have been standardized due to importance and physical applications. A unique approach to analyze these databases is by creating a graph of the database using graph theory. Nodes taken from measurements in data files and edges become connections from different properties such as similar isotopes or from something more specific like isomer math. Once the graph was complete we were able to analyze things such as which measurements were the most connected, an important measure of centrality in the database. Then we took the most connected nodes and were able to analyze those further using methods such as Eigen decomposition and betweenness to determine if there were any reactions in the database that were central enough that they would warrant future standardization efforts. The other thing that we looked to analyze was the distance that a measurement was from standard. By applying graph theory to a nuclear data library was something that no one had ever done before and the results will help pave the way for future standardization efforts, which until previously, were solely decided based upon speculation.

246. Number Sequences for 3-D Regular Polytopes, Variations, and Regular Maps

Nathaniel Rupprecht Grove City College
Mary Collins Grove City College
Annie Marie Holfelder Grove City College
Advisor(s): Michael Jackson, Grove City College

Polytope numbers are sequences of numbers generated based on the geometric properties of polytopes, which are generalizations of polygons and polyhedral in n-dimensional space. We extended the idea of polytope number sequences to include regular maps, analogies to polytopes on different topological spaces, and polytope transformations of regular maps. We derived number sequence and interior sequence formulas based on the genus, Schlafli symbol, and facet numbers of regular maps and polytope transformations of regular maps.

247. Polygonal Numbers and Fermat's Last Theorem

Aseem Bhagwat Indian Statistical Institute
Advisor(s): B. Sury, Professor, Indian Statistical Institute

We begin with the elementary Diophantine $x^2 + y^2 = z^2$ in positive integers, which we know has in finite solutions. Fermat's Last Theorem does not let us generalize this for higher powers. But we can generalize this for polygonal numbers; we can in fact prove that there are infinitely many n-gonal numbers which can be represented as a sum of m n-gonal numbers, for all m and n. Now, if we consider the above Diophantine for higher dimensional regular convex polytope numbers (squares above being two-dimensional regular convex polytopes), we notice that there are special cases in each dimension where the solutions do not exist. As we see where the solutions exist and where they do not, we gain some new insights into Fermat's Last Theorem. We observe that Fermat's Last Theorem does not simply give us a family of Diophantine equations having no positive integer solutions, but something much more significant. Lastly, based on our insights, we ask a few questions, which if answered, could actually explain in a different way why Fermat's Last Theorem holds!

248. Generalizing Zeckendorf's Theorem to f-decompositions

Umang Varma Kalamazoo College
Advisor(s): Steven Miller, Williams College

A beautiful theorem of Zeckendorf states that every positive integer can be uniquely expressed as a sum of non-consecutive Fibonacci numbers. For sequences $\{G_n\}$ satisfying linear recurrence relations with nonnegative coefficients, there is a notion of a legal decomposition which again leads to a unique representation. The number of summands in the representations of $m \in [G_n, G_{n+1})$ converges to a Gaussian as $n \to \infty$ Given a notion of legal

decomposition, we ask if there exists a sequence $\{a_n\}$ such that every positive integer can be uniquely decomposed as a sum of terms from $\{a_n\}$. Let $f : \mathbb{N}_0 \to \mathbb{N}_0$. We say that if a_n is in an "f-decomposition" of a number x, then the decomposition cannot contain the $f(n)$ terms immediately before a_n in the sequence. We prove that for any $f : \mathbb{N}_0 \to \mathbb{N}_0$, there exists a sequence $\{a_n\}$ such that every positive integer has a unique f-decomposition using $\{a_n\}$. If f is periodic, then the unique increasing sequence $\{a_n\}$ induced by f satisfies a linear recurrence relation. For some class of functions f, we prove that the number of summands in the f-decomposition of integers in a suitable growing interval converges to a normal distribution.

249. On the residue classes of $\pi(n)$ modulo t and bounded gaps between products of special primes

Ping Ngai Chung Massachusetts Institute of Technology
Shiyu Li University of California, Berkeley
Advisor(s): Ken Ono, Emory University

We shall present two results: On the residue classes of $\pi(n)$ modulo t. The prime number theorem is one of the most fundamental theorems of analytic number theory, stating that the prime counting function, $\pi(x)$, is asymptotic to $x/\log x$. However, it says little about the parity of $\pi(n)$ as an arithmetic function. Using Selberg's sieve, we prove a positive lower bound for the proportion of positive integers n such that $\pi(n)$ is r mod t for any fixed integers r and t. Moreover, we generalize this to the counting function of any set of primes with positive density. Bounded gaps between products of special primes In their breakthrough paper in 2006, Goldston, Graham, Pintz, and Yildirim proved several results about bounded gaps between products of two distinct primes. Frank Thorne expanded on this result, proving bounded gaps in the set of square-free numbers with r prime factors for any $r \geq 2$, all of which are in a given set of primes. His results yield applications to divisbility of class numbers and triviality of ranks of elliptic curves. In this paper, we relax the condition on the number of prime factors and prove an analogous result using a modified approach. We then revisit each of Thorne's applications and give a better bound in each case.

250. Inequalities For Positive Rank and Crank Moments of Overpartitions

Alexa Rust University of Washington
Acadia Larsen Whittier College
Advisor(s): Holly Swisher, Oregon State University

In recent work, Andrews, Chan and Kim extend a result of Garvan about even rank and crank moments of partitions. In a similar fashion we extend a result of Mao about even rank moments of overpartitions. We study positive Dyson-rank, M_2-rank, first residual crank, and second residual crank moments of overpartitions. We denote these positive k-th rank and crank moments by $\overline{N}_k^+(n)$, $\overline{N2}_k^+(n)$, $\overline{M}_k^+(n)$, and $\overline{M2}_k^+(n)$, respectively. We prove a number of inequalities involving the rank and residual crank moments. In particular, we prove a conjecture of Mao which states that: for $n \geq 2$ and k a positive integer,

$$\overline{N}_k^+(n) > \overline{N2}_k^+(n).$$

251. Generalizations of the Giuga Number and Some of Its Properties to Number Fields

Katie Casey Cornell University
Jamaris Burns Johnson C. Smith University
Advisor(s): Gregory Johnson, Carnegie Mellon University

In 1950, Giuseppe Giuga famously conjectured that only prime numbers n could satisfy: $\sum_{k=1}^{n-1} k^{n-1} \equiv -1$ (mod n). In this research project, we analyzed Giuga's conjecture under the broader framework of number fields and rings of algebraic integers. We generalize to ideals because rings do not have unique factorization. To show an equivalent condition for the criteria of a Giuga number, a number which satisfies Giuga's conjecture but is not prime, we first examined past research on the topic (Borwein) and then proceeded to generalize Giuga's conjecture using ideals by writing original proofs. We also showed an equivalent characterization using ideals of several of the famous results regarding Giuga numbers- a weak Giuga number, a strong Giuga number and a Carmichael number. Our findings show that Giuga's conjecture can be satisifed by not only integers. Now that we have these implications we can go further to generalize a third Giuga equivalence using the sum - product of primes and also, to generalize a Giuga sequence. Gaussian integers also spark interest in finding a concrete example of our generalized Giuga number- a Giuga ideal.

252. Powers in Lucas Sequences via Galois Representations

Isabel Vogt Harvard University
Jesse Silliman University of Chicago
Advisor(s): Ken Ono, David Zureick-Brown, Emory University

Let u_n be a nondegenerate Lucas sequence. We generalize the results of Bugeaud, Mignotte, and Siksek, 2006 to give a systematic approach towards the problem of determining all perfect powers in any particular Lucas sequence. We then prove a general bound on admissible prime powers in a Lucas sequence assuming the Frey-Mazur conjecture on isomorphic mod p Galois representations of elliptic curves.

253. Periodicity of Third-Order Linear Recursive Sequences

Shan Shan Agnes Scott College
Advisor(s): Alan Koch, Agnes Soctt College

In 2011, Franzel, Psalmond, and Tobiasz provided a necessary and sufficient condition for a number k to be the period length of an integer sequence described by a second-order recurrence relation modulo a prime p. In this poster, we extend their techniques to sequences described by third-order recurrence relations. We show that any such sequence modulo p has period length dividing p^3-1, p^2-1 or $p(p-1)$. Conversely, any divisor of these three numbers can be realized as a period length for some such sequence.

254. The Lucas Numbers and other Gibonacci Sequences mod m

Jeremiah Southwick Le Moyne College
Advisor(s): Jonathan Needleman, Le Moyne College

It has been categorized which m have the property that the Fibonacci sequence contains all residues mod m. When m has this property, we say m is complete in the Fibonacci sequence. We extend this work to all Gibonacci sequences, giving special attention to the Lucas numbers. We conclude by determining which m are complete in a given Gibonacci sequence containing the relatively prime consecutive terms a, b.

255. Solvability of systems of polynomial equations over finite fields

Ramon Collazo University of Puerto Rico, Rio Piedras
Julio de la Cruz University of Puerto Rico, Rio Piedras
Daniel Ramirez University of Puerto Rico, Rio Piedras
Advisor(s): Ivelisse Rubio, Francis Castro, University of Puerto Rico, Rio Piedras

An important problem in mathematics is to determine if a system of polynomial equations has or not solutions over a given set. A diagonal equation over a finite field F_q, $q = p^r$, p prime is an equation of the form $a_1 X^{k_1} + \cdots + a_n X^{k_n} = \alpha$, where $a_i, \alpha \in F_q$. We study systems of generalized and deformed diagonal equations over F_p and look for sufficient conditions that guarantee their solvability over the field. Using the covering method of [1] we get conditions that allow us to construct families of systems that have exact p-divisibility of the number of solutions of the system and therefore guarantee solvability over the finite field. This type of results can be used to compute the covering radius of certain codes. [1] F. Castro and I. Rubio. Exact p-divisibility of exponential sums via the covering method, accepted in the Proceedings of the AMS.

256. The Moebius Power Series

Hang Yang University of Illinois
Advisor(s): A. J. Hilderbrand, University of Illinois

We investigate the power series $f(z) = \sum_{n=1}^{\infty} \mu(n) z^n$, where $\mu(n)$ is the Moebius function and $|z| < 1$. The corresponding Dirichlet series, $\sum_{n=1}^{\infty} \mu(n) n^{-s}$, is the reciprocal of the Riemann zeta function and has been extensively investigated. By contrast, very little is known about the behavior of the power series $f(z)$ as z approaches the unit circle. Hardy and Littlewood mentioned this series in one of their papers, but did not pursue it further because of the difficulties involved. In the 1960s Froberg carried out extensive numerical investigations of $f(z)$ that revealed intriguing features that have yet to be fully explained; in particular, the behavior of $f(z)$ at complex values z near the unit circle remains largely a mystery. We present results of large-scale computations, carried out at the Illinois

Geometry Lab, which significantly extend those of Froberg, and we provide heuristic explanations for many of the features observed.

257. The Geometry of Gauss Sums

Yiwang Chen University of Illinois, Urbana-Champaign
Advisor(s): A.J. Hildebrand, University of Illinois

Given an odd prime p, the (quadratic) Gauss sum modulo p, $G(p)$, is given by either of the two equivalent formulas, (1) $G(p) = \sum_{n=0}^{p-1} e^{2\pi i n^2/p}$, and (2) $G(p) = \sum_{n=1}^{p-1} (\frac{n}{p}) e^{2\pi i n/p}$, where $(\frac{n}{p})$ is the Legendre symbol. These sums were first introduced by Gauss in 1801, who in 1805, after four years of effort, succeeded in exactly evaluating $G(p)$. The partial sums of the representation (1) of $G(p)$ have been extensively studied in the literature by D.H. Lehmer and others, and their graphs in the complex plane are approximated by certain smooth curves whose behavior is well-understood. By contrast, very little is known in the literature about the geometric behavior of the partial sums of (2), i.e., the sums $S(N, p) = \sum_{n=1}^{N} (\frac{n}{p}) e^{2\pi i n/p}$. The graphs of these sums show many random-like features, but also some remarkable symmetries. In this presentation we report on results of large-scale computations and visualizations of the partial sums of (2) carried out at the Illinois Geometry Lab, we formulate conjectures suggested by these experimental results, and we put these into the context of known results and conjectures on related questions.

258. Number-theoretic Random Walks

Tong Zhang University of Illinois at Urbana-Champaign
Advisor(s): A.J. Hildebrand, University of Illinois

Exponential sums of the form (1) $S(N) = S(N, f, \alpha) = \sum_{n=0}^{N-1} f(n) e^{i\alpha n}$, where $f(n)$ is some number-theoretic function and α a real number number, arise in many problems in number theory and form a key tool in understanding the properties and behavior of the function $f(n)$. In this presentation, we focus on the *geometric* behavior of these sums, interpreting the sums (1) as a random walk in the complex plane whose nth step is given by $f(n)e^{i\alpha n}$. We call such a random walk a number-theoretic random walk. Specifically, we consider the case when $f(n) = e^{2\pi i s_b(n)/b}$, where $s_b(n)$ denotes the sum of digits of n in base b. We obtain exact formulas for the *scaling exponent* $\mu = \limsup_{N\to\infty} \log |S(N)| / \log N$ in the case when α is a rational multiple of 2π, and numerical values in many other cases. These results suggest that such number-theoretic random walks, while exhibiting many random-like features, grow at a much slower rate than a true random walk. This research was carried out at the Illinois Geometry Lab.

259. A New Algorithm For Computing Inverses in Modular Arithmetic

Shen Lu Lebanon Valley College
Christina Doran Lebanon Valley College
Advisor(s): Barry Smith, Lebanon Valley College

A well-known result states: If n/a is a reduced fraction with symmetric continued fraction expansion, then a^2 is congruent to ± 1 modulo n. The converse is also true. We provide analogues of this theorem and its converse for certain “almost symmetric} continued fractions. The work also leads to a new algorithm procedure for computing multiplicative inverses in modular arithmetic. To compute the inverse of a modulo n, perform the Euclidean algorithm with n^2 and a^{n+1}. The first remainder less than n is a multiplicative inverse for a.

260. Maximal Varieties over Finite Fields arising from Algebraic Groups and ℓ-adic Representations of their Symmetry Groups

Dylan Yott Boston University
Advisor(s): Jared Weinstein, Boston University

In the theory of algebraic groups, there are certain projective varieties defined over $\mathbb{F}_q$ that arise naturally with the maximal number of points with respect to their Betti numbers. Let X be a projective curve defined over $\mathbb{F}_q$ with genus g. Then the Weil conjectures give the bound $X(\mathbb{F}_q) \leq q + 1 + 2g\sqrt{q}$. If X attains this bound, then we say X is maximal. Since maximal curves have lots of points, we might expect them to have interesting symmetry groups. Grothendieck’s theory of ´etale cohomology allows us to study these symmetry groups by looking at their action on the cohomology of X with coefficients in $\mathbb{Q}_\ell$, where $\ell \nmid p$. Using the Grothendieck-Lefschetz trace formula,

we can compute the character of the representation of the symmetry group on $H^1(X, \mathbb{Q}_l)$. We also consider an interesting family of hypersurfaces with Abelian symmetry group, $\mathbb{F}_q$, which also have an action of $O(n)$ for some n. These hypersurfaces have cohomology up to degree n, and we use the tools as mentioned above to understand the representation theory associated to these hypersurfaces, as well as their Weil zeta functions.

261. Associating Finite Groups with Dessins d'Enfants

Ahmed Tadde Howard University
Advisor(s): Edray Goins, Purdue University

Each finite, connected planar graph has an automorphism group; such permutations can be extended to automorphisms of the Riemann sphere. In 1984, Alexander Grothendieck, inspired by a result of Gennadii Belyi from 1979, constructed a finite, connected planar graph via certain rational functions by looking at the inverse image of the interval from 0 to 1. The automorphisms of such a graph can be identified with the Galois group of the associated rational function. In this project, we investigate how restrictive Grothendieck's concept of a Dessin d'Enfant is in generating all automorphisms of planar graphs. We discuss the rigid rotations of the Platonic solids (the tetrahedron, cube, octahedron, icosahedron, and dodecahedron), the Archimedean solids, and the Catalan solids via explicit Belyi maps. Conversely, we enumerate groups of small order and discuss which groups can — and cannot — be realized as Galois groups of Belyi maps.

262. It Just Doesn't Add Up: The Frobenius Number of Three Numbers

Ryan Matzke Gettysburg College
Advisor(s): Peter Johnson, Auburn University

The Frobenius Number of a set of numbers is the greatest number that cannot be made from a linear combination, with non-negative coefficients, of those numbers. For two numbers a and b, the value of the Frobenius Number, $F(a, b)$ is infinite if a and b are not relatively prime, but if they are, then $F(a, b) = ab - a - b$. Though there is no general explicit formula for the Frobenius Number of three numbers, the value can be found. If c can be written as a linear combination of a and b with positive coefficients, $F(a, b, c) = F(a, b)$. However, if it cannot, and a, b, and c are relatively prime, we find that we can produce the value of the Frobenius Number through an algorithm using modular arithmetic, and often in a small number of steps.

263. Congruence properties of Taylor coefficients of modular forms

Hannah Larson Harvard University
Advisor(s): Ken Ono, Emory University

In their work, Serre and Swinnerton-Dyer study the congruence properties of the Fourier coefficients of modular forms. We will examine similar congruence properties, but for the Taylor series coefficients of modular forms about a CM point τ. These coefficients can be shown to be the product of a power of a constant transcendental factor and an algebraic integer. In our work, we give a condition on τ and a prime number p that, if satisfied, implies that p^m divides all the Taylor coefficients of f of sufficiently high degree. We also give effective bounds on the largest n such that p^m does not divide the nth Taylor coefficient of f at τ that are sharp under certain additional hypotheses.

264. Multiplication of Polynomials in Bernstein-B©zier Form

Rachael Mady Towson University
Advisor(s): Tatyana Sorokina and Alexei Kolesnikov, Towson University

When dealing with piecewise polynomials, it is useful to write the pieces in the so-called Bernstein-Bézier form. This form uses barycentric coordinates instead of Cartesian coordinates. In order to work with generators for spline spaces, it is necessary to perform multiplication of polynomial pieces in the Bernstein-Bézier form. We developed an algorithm that computes the coefficients of the product in the Bernstein-Bézier form, given the corresponding coefficients of the factors. The algorithm makes use of the so-called "domain points} that are essential in the field of multivariate splines. The algorithm proves that multiplication of polynomials in the Bernstein-Bézier form is local, i.e. only a few coefficients are used. This is in contrast to the multiplication of polynomials with Cartesian coordinates.

265. The Numerical Solution of the Exterior Robin Problem for the Helmholtz's Equation: Super Ellipsoid

Hy Dinh Roger Williams University
Advisor(s): Warnapala Yajni, Roger Williams University

Many scattering and radiation problems are concerned with finding the solutions of the Helmholtz's equation in the exterior domain. Finite element methods and finite difference methods are the most popular methods for solving elliptic partial differential equation. But for the Helmholtz's equation, there is a fundamental difficulty in using these methods. The difficulty is that the region of interest is of infinite extent and any solution must satisfy the radiation condition at infinity. Integral equation methods avoid these difficulties. In this poster, I use the Galerkin Method to solve the exterior Robin problem for the Helmholtz's equation for the Super Ellipsoid.

266. Tangent Line/Plane Approximation of Definite Integrals

Meghan Peer Saginaw Valley State University
Advisor(s): Emmanuel Kengni Ncheuguim, Saginaw Valley State University

The purpose of this presentation is to share a new method of approximate integration utilizing tangent lines and tangent planes in both two-dimensional and three-dimensional spaces. This method will be compared with already existing methods in terms of efficiency and error. This presented method of definite integral approximation will likely be of value to mathematics and the applied sciences, as the majority of integrals cannot be solved computationally. Despite limitations, the tangent line/plane approximation demonstrates that integrals can be approximated to nearly exact solutions.

267. A Numerical Analysis of the S I R Model for Modeling Epidemics

Casey Howren University of Mary Washington
Advisor(s): Leo Lee, University of Mary Washington

In this research project, we use numerical approximation techniques to model the projected course and severity of epidemics, while focusing specifically on the 2009 H1N1 flu pandemic. Our model is based off of the commonly used S-I-R model for epidemics, which models the relationships between the Susceptible, Infected, and Removed classes of a population that a disease is affecting. An exact solution is derived for the model using elementary algebra and calculus techniques. An algorithm is then developed using the numerical approximation method of Euler's Method and a corresponding MATLAB code is created to compute numerical solutions and plots that give an approximated model of an epidemic. Our methods are then applied to real world data in order to model the behavior of the 2009 H1N1 pandemic in the United States. This work seeks to show that using mathematical methods to model epidemics can accurately predict the behavior and severity of a disease in a manner that is more efficient and less complicated than methods for exact solutions, which can then allow for the creation of efficient prevention and treatment plans that will in turn ensure that an epidemic has the least impact possible on a population.

268. Identifying Numerical Oscillations in Parabolic Partial Differential Equations

Mitchell Main George Fox University
Advisor(s): Richard Corban Harwood, George Fox University

This paper investigates conditions for oscillation-free stability of numerical methods for linear and nonlinear parabolic partial differential equations. Not clearly understood, numerical oscillations can create infeasible results. Since oscillation-free behavior is not ensured by stability conditions nor is a positive definite matrix required, a more precise condition would be useful for accurate solutions. Using Von Neumann and spectral analyses, we and and explore oscillation- free conditions for several finite difference schemes. Further relationships between oscillatory behavior and eigenvalues is supported with numerical evidence and proof. Also, evidence suggests that the oscillation-free stability condition for a consistent linearization is sufficient to provide oscillation-free stability of the nonlinear solution. These conditions are verified numerically for several example problems by visually comparing the analytical conditions to the behavior of the numerical solution for a wide range of mesh sizes.

269. Numerical Analysis of Dynamic Elastic Rods

Charles Crook James Madison University
Advisor(s): Eva Strawbridge, James Madison University

We numerically study the dynamics of a thin rod in a viscous fluid using the Kirchoff rod model and resistive force theory. We will be analyzing the dynamics of rod as it bends and rotates. We expect to apply this knowledge to the 1 mm worm C. elegant to predict or model how it moves in different environments.

270. Validity of down-sampling data for regularization parameter estimation when solving large-scale ill-posed inverse problems

Michael Horst Arizona State University
Advisor(s): Rosemary Renaut, Arizona State University

Many physical systems are modeled using solutions to integral inverse problems. These solutions are found using some form of regularization, which requires one to determine the best regularization parameter, but this can be computationally expensive for problems with large data sets. One method for finding the regularization parameter is the Generalized Discrepancy Principle (GDP). For an integral kernel which is square integrable, the discrete Singular Value Decomposition (SVD) for the discrete kernel reveals information about the continuous Singular Value Expansion (SVE). With this information, convergence of the GDP parameter estimate with increasing resolution can be obtained. Hence one can down-sample the data and use the GDP to find a regularization parameter that will solve the full-scale problem.

271. Numerical Solution of the Radiosity Equation via Galerkin Method: Neumann Condition

Liuting Lu Roger Williams University
Advisor(s): Yajni Warnapala, Roger Williams University

In this paper, a global Galerkin method is used numerically to solve the exterior Neumann problem for the Radiosity equation for the sphere, ellipsoid and the oval of Cassini in three dimensions. The Radiosity equation is a mathematical model for the brightness of a collection of one or more surfaces when their reflectivity and emissivity are given. Theoretical and computational details of the method for small reflectivity values are presented.

272. Application of Akima Method to Cubic Spline

James Hughes Towson University
Advisor(s): Tatyana Sorokina, Towson University

The Akima Method is a useful tool in approximating the derivative at a given point of a curve. We developed a computational module that applies Akima method to a series of points on a curve and collects the necessary information needed to generate a cubic spline. The points and their derivatives are then used to construct a differentiable cubic spline. This module then uses the univariate cubic spline to create a smooth surface of revolution. With the help of the module, we constructed a model of a baseball bat, determined the dimension of the spline space used in the construction, and printed the baseball bat a using 3D-printer.

273. Classifications of Definable Sets

Abigail Raz Wellesley College
Rebecca Scanlon Wellesley College
Advisor(s): Karen Lange, Wellesley College

We examine questions in computable structure theory, specifically regarding classifications of definable subsets. We are concerned with what subsets are Σ_1, d-Σ_1, and Σ_2-definable in fixed computable equivalence structures and classifications of these formulas. A subset, A, is C-definable in a model, M, if there exists a C-formula, such that $x \in B$ if and only if the formula is true in the model. All the definable subsets in an equivalence structure are Σ_2-definable. A C-classification is a computable list of C-formulas where every C-definable subset is defined by exactly one formula in the list. Lange and Wallbaum proved there exists a computable equivalence structure with no Σ_1-classification. We explore the links between Σ_1, d-Σ_1, and Σ_2-classifications. We show that a computable equivalence structure with unbounded class sizes has a Σ_1-classification if and only if it has a d-Σ_1-classificatio.

We also prove that if a computable equivalence structure with unbounded class sizes has a Σ_1-classification then it has a Σ_2-classification; yet, there exists a computable equivalence structure with a Σ_2-classification and no Σ_1-classification.

274. The MLB All Star Challenge

Nicholas Yaeger Simpson College
Ruth Roberts Simpson College
Advisor(s): William Schellhorn, Simpson College

We developed logistic models to determine which batters in each position should make the Major League Baseball All Star teams. We collected 36 different statistics on players from 2009 to 2012 and analyzed them using the statistical software JMP. Our models are based on a feature subset selection of only 3 of these statistics and they are effective at predicting the 2008 and 2013 All Star batters. Our results suggest that although players make the All Star team based on their on-field performance, other factors seem to determine which are the starters versus the reserves.

275. Long-Square-Free Colorings of Graphs

Joseph Antonides Susquehanna University
Claire Spychalla Taylor University
Nicole Yamzon The University of Texas at Austin
Advisor(s): David Milan, The University of Texas at Tyler

A *long square* is a word w such that $w = uu$ where $|u| \geq 3$. We prove that every graph admits a subdivision so that the subdivided graph can be long-square-free 2-colored. Then, we construct a tree called a *Tyler tree* that cannot be long-square-free 2-colored. We finish by showing that certain classes of trees can always be 2-colored to be long-square-free.

276. Number of permutations with same peak set for signed permutations

Rita Zevallos Swarthmore College
Francis Castro Massachusetts Institute of Technology
José Pastrana University of Puerto Rico
Advisor(s): Rosa Orellana, Dartmouth College

A signed permutation $\pi = \pi_1\pi_2\dots\pi_n$ in the hyperoctahedral group B_n is a word such that each $\pi_i \in -n,\dots,-1,1,\dots,n$ and $|\pi_1|,|\pi_2|,\dots,|\pi_n| = 1,2,\dots,n$. An index i is a peak of π if $\pi_{i-1} < \pi_i > \pi_{i+1}$ and $P_B(\pi)$ denotes the set of all peaks of π. Given any set S, we define $P_B(S,n)$ to be the set of signed permutations $\pi \in B_n$ with $P_B(\pi) = S$. In this paper we are interested in the cardinality of the set $P_B(S,n)$. In 2012, Billey, Burdzy and Sagan investigated the analogous problem for permutations in the symmetric group, S_n. In this paper we extend their results to the hyperoctahedral group; in particular we show that $\#P_B(S,n) = p(n)2^{2n-|S|-1}$ where $p(n)$ is the same polynomial found in by Billey, Burdzy and Sagan which leads to the explicit computation of interesting special cases of the polynomial $p(n)$. In addition we have extended these results to the case where we add $\pi_0 = 0$ at the beginning of the permutations, which gives rise to the possibility of a peak at position 1, for both the symmetric and the hyperoctahedral groups.

277. Solution Behavior of a Periodic Boundary Value Problem

Claire Zajaczkowski Gettysburg College
Advisor(s): Bela Bajnok, Gettysburg College

The authors study a type of second order nonlinear discrete periodic boundary value problems. The existence and uniqueness of positive solutions are discussed. The parametric dependence of the solutions are also investigated. Two examples are given as applications of the results.

278. A Proposed Digital Signature for Direct Embedded Elliptic Curve Cryptography

Kelly Isham Skidmore College
Advisor(s): Gove Effinger, Skidmore College

An efficient and secure public key cryptographic scheme can be created by defining a binary operation on elliptic curves. This scheme allows messages to be embedded directly onto the curve over some finite field F_p and its security relies on the intractability of the "elliptic curve discrete logarithm problem." We propose a simple digital signature method to accompany this scheme which also relies on the above intractability. Briefly, the signer (Alice) sends to the recipient (Bob) a prime factor p of her secret multiplier k, and a "signature point} $PS = (k/p) * P$ where P is the public "initial point} on her curve. Bob then computes $p * PS$ and checks to see if it matches Alice's public "target point} PT. If they do match, the message must have come from Alice.

279. Representations of Magic Squares and their transformations

Brianna Melus Fitchburg State University
Advisor(s): Peter Staab, Fitchburg State University

An nth-order magic square is an $n \times n$ grid of distinct integers arranged such that each row, column, and diagonal sum to the magic constant, denoted μ. I will focus on 4th-order magic squares; particularly the subset known as Type I. Using basic Linear Algebra principles, I will find the basis and explain the representation of this group of magic squares. With the basis I will also find the group of corresponding transformation matrices and examine common patterns and characteristics they share.

280. Factoring in Cryptology

Sarah Armstrong University of Redlands
Advisor(s): Tamara Veenstra, University of Redlands

The goal of this project has been to transfer Maple programs into Sage, write new Sage programs, and to research cryptology. I have researched various factoring techniques which are used to crack the RSA cipher created by Rivest, Shamir, and Adleman. The techniques range from the fairly simple Fermat's Method to the more complicated and more modern Quadratic Sieve. This summer, many programs were made based of my independent projects, while the others were based off of my advisor, Dr. Tamara Veenstra's Conjecture and Proof in Discrete Mathematics course with an emphasis in cryptology.

281. The Power Series of $e^{\sin(x)}$

Ye Yuan St. Olaf College
Advisor(s): Paul Humke, St. Olaf College

In this research, we investigate the coefficients of the power series of $e^{\sin(x)}$ from two aspects: finding an efficient way to calculate its coefficients and determining whether there are other zero coefficients after the 3rd term. We address a way to calculate the coefficients recursively at first, and find that the Nth coefficient has a relation to the set partition of $\{1 \ldots N\}$ into odd-sized blocks. We also study Kruchinin's work on generating function for the coefficient of $e^{\sin(x)}$, and use his generalized formula along with our previous findings to prove that certain coefficients are non-zero.

282. Generalization of the Whirlpool Hash Function

Andrew Albertson Boise State University
Alex Hegedus Alma College
Henry de Kergorlay Wesleyan University
Advisor(s): Liljana Babinkostova, Boise State University

In the fast-paced world of cryptographic innovation, it is essential to anticipate attacks that will result in insecure electronic communication. Hash functions are used for password storage, message integrity verification, pseudorandom number generation, and non-repudiation in digital security. A hash function takes a string of arbitrary length and returns a string of a fixed length. Whirlpool is a hash function developed in 2003 and accepted by the New

European Schemes for Signatures, Integrity, and Encryption (NESSIE) for widespread use. Whirlpool uses a Rijndael-like block cipher in a Merkle-Damgard construction to create a hash function. This internal block cipher has a rich algebraic structure, which is vital to the security of the function We generalize the standard version of Whirlpool and explore these algebraic properties. In particular, we investigate the conditions under which the set of encryption functions used in Whirlpool form a group under functional composition. Our analysis of the component functions has diverse applications, especially in solving combinatorial problems from other areas, such as computer science and neuroscience.

283. Anomaly Detection Using Dictionary Learning

Sara Staszak Macalester College
Mengjie Pan Bryn Mawr College
Zach Siegel Pomona College
Mark Eisen University of Pennsylvania
Advisor(s): Alicia Johnson, Macalester College

Recent developments in signal processing and machine learning focus on representing high-dimensional data sparsely, or as linear combinations of few basis elements. Sparse coding and dictionary learning aim to find such a representation of data Y with respect to a learned basis, or dictionary D. Specifically, iterative learning algorithms are used to solve the minimization problem $\min_{X,D} \|Y - DX\|_2^2 + \lambda\|X\|_0$, where X is a set of coefficients and λ controls the sparsity of X. Sparsity helps assign semantic meaning to individual dictionary elements based upon their use in reconstructing data, which in turn highlights natural groupings and relationships among the data points. Thus, though traditional applications of dictionary learning include image denoising, novel methods for identification of *anomalous* or *salient* data points can also be derived from such structural features. To this end, we develop sparsity-informed metrics for defining and identifying anomalies with broad applications. Our results are promising and competitive with previous methods for flagging anomalous data in both images and propagating wavefield video.

284. Comparative Efficiency of Skeletal Models of a Shape

Jillian Glassett California State University, Channel Islands
Advisor(s): Kathryn Leonard, California State University Channel Islands

We are exploring two skeletal models that compress a shape without losing critical information. One model, the Blum Medial Axis, is created using a series of maximally inscribe circles, where each circle generates a point on the medial axis. This method is efficient under particular conditions but is inflexible: the Blum axis of a shape is uniquely determined by that shape. A more flexible model, the General Medial Axis, is created with two circles per point of the medial axis which sacrifices uniqueness for potentially greater efficiency. We are evaluating these models for efficiency, where efficiency refers to using the smallest number of computer bits. The Blum axis shows increased efficiency when it satisfies certain geometric constraints. Using this fact, we create and work with a General Medial Axis satisfying these constraints and compare its bit number to the bit number of the Blum Medial Axis.

285. Constructively Coloring the Line

Loren Anderson North Dakota State University
Advisor(s): Peter Johnson, Auburn University

Kemnitz and Marangio showed that for any list of k distances, the real line can be colored with $k + 1$ colors so that all k of the distances are “forbidden,” meaning that two points any of those distances apart must be colored differently. Their proof is non-constructive, as it appeals to a famous theorem of de Bruijn and Erdős, from the proof of which the Axiom of Choice cannot be excised. Here, we give explicit instructions for obtaining distance-forbidding colorings. Also, we examine extensions to periodic colorings.

286. New Views on the Yoneda Lemma

David Sidi University of Arizona
Advisor(s): Kristen Beck, The University of Arizona

The Yoneda Lemma is among the first deep results encountered in learning Category Theory, yet its proof is usually given with at most several narrowly-related diagrams (for example, see (MacLane 1998), (Awodey 2006)). The Yoneda Lemma can be stated briefly. For **C** a locally small category, fix the hom-functor $\mathrm{Hom}_{\mathbf{C}}(-, C) : \mathbf{C}^{Op} \to \mathbf{Set}$,

and let $P : \mathbf{C}^{Op} \to \mathbf{Set}$ be an arbitrary functor. The Yoneda Lemma says that there is a bijection from the set $P(C)$ to the set of natural transformations from $\mathrm{Hom}(-, C)$ to P. Thus, the Yoneda Lemma gives a universal element that picks out the representations of P by the natural transformations it induces. We examine definitions and propositions important to the proof of the Yoneda Lemma with a variety of diagrammatic notations, some of which are new. For example, modified string diagrams, overlayed sequences of "three-dimensional" colorized mapping diagrams, and diagrams with "magnification" of points are used. We also contextualize Yoneda as a generalization of Cayley's Theorem, and examine the connection between the Yoneda lemma and monoidal categories.

287. Doubly Ensemble Movie Prediction With Social Media Data Using TBEEF

Christopher Rackauckas Oberlin College
Advisor(s): Avery Ching, Hong Kong University of Science and Technology

In the spring of 2013, Baidu, Inc. hosted a competition for teams to develop new algorithms for movie recommendation systems. The purpose of the competition was to develop better models for rating prediction and suggest methods for incorporating social media data into the prediction models. The goal of our project was to use the information disseminated by the top competitors of the contest to develop new algorithms for recommendation systems. Noting the prevalence of ensemble methods employed on factorization models, our team developed a doubly ensemble framework named TBEEF, Triple Bagged Ensemble Ensemble Framework, a software framework with a plugin interface through which factorization and ensemble models aggregated using bootstrap aggregation could be easily developed and ran. Our results showed that TBEEF performed adequately on the Baidu dataset, though it did not perform as well as the models from the top competitors due to dataset limitations.

288. Estimating the Volatility in the Black-Scholes Formula

Josh Matti Indiana Wesleyan University
Rachel Lane Concordia College
Rebecca Keenan Eastern Connecticut State University
Advisor(s): Hui Gong, Valparaiso University

The Black-Scholes formula is one of the most popular option pricing models used in today's financial markets; however, one of the inputs, volatility, is not observable and thus not available for immediate application in the formula. In our research we examine four different approaches for better estimating the volatility: smoothing, deriving the distribution of the volatility, building time series models, and using nonparametric techniques. We then plug the estimated volatility from each approach back into the Black-Scholes formula and use three different methods of numerical comparison to determine which approach is best.

289. Using Independent Bernoulli Random Variables to Model Gender Hiring Practices

Kimberly Hildebrand University of Mary Washington
Advisor(s): Debra Hydorn, University of Mary Washington

Hiring practices based on gender can be modeled using sums of independent Bernoulli Random Variables where p represents the proportion of female applicants. Using the moment generating function (MGF) of the Bernoulli Distribution, it is possible to calculate the expected value (mean) and variance for the number of women hires for n positions. In addition, the probability generating function (PGF) of a sample of size n can be used to find the probability of hiring one or fewer women. A computer program was used to simulate different male/female distributions using recent data on the proportion of women earning a PhD in a variety of disciplines. The simulations were used to represent hiring results for seven faculty positions where the proportion of women applicants ranged from 0.3 to 0.7. Results revealed that hiring one or fewer women in seven positions tends to have a low probability, which could provide evidence of gender bias.

290. Modeling NBA Player Value from Box Score Data

Daniel Persia Denison University
Advisor(s): Matthew Neal, Denison University

Recent findings in basketball analytics reveal the importance of five-man units in modeling player interactions on the court. In a synergistic environment, how much is a shot worth? How much should we value an assist? A turnover? A block? In this session we'll explore how box score data can be used to evaluate NBA player performance. We

introduce an adjusted plus-minus statistic, generated for NBA players across four seasons, and regress against linear and non-linear variations of the box score. We consider models to predict future performance, with an eye toward determining which player types play well together, producing synergistic effects. Can a reasonable definition of player value be determined?

291. Modeling the Retreat of Glaciers in a Changing Climate

Robert Guillette Bridgewater State University
Advisor(s): Irina Seceleanu, Bridgewater State University

Glaciers across the globe have been increasingly losing mass over the last century. The melting of glaciers offers tangible evidence of broader environmental changes as they respond directly to long-term trends in temperature, precipitation and solar radiation. Since glacier retreat provides a barometer of climate change, it is important to better understand the effects of climatic factors on glaciers. In this project we created a mathematical model of glacier retreat, which we applied to study the variation in the total area of the Folgefonna glacier in Norway over the past decades. We used multiple linear regression to study the effects of temperature, precipitation, insolation, wind speed and the local climatic phenomenon NAO on the total area of the glacier, and found that within our model these factors explained 82.7% of the total variation of the Folgefonna glacier area. To simulate the evolution of the glacier in time, we performed a time series analysis and obtained a prediction interval for when the glacier will completely disappear. Moreover, based on different projections of global temperature over the next century, we simulated the evolution of the total area of the Folgefonna glacier under these scenarios.

292. What happens to the Drunkard?

Angie Ibarguen Lozano Benedict College
Advisor(s): Naima Naheed, Benedict College

A Markov chain is a mathematical system that undergoes transition from one state to another, among a finite or countable number of possible states. It is a random process usually characterized as memoryless: the next state depends on the current state and not on the sequence of events that preceded it. This specific kind of memorylessness is called the Markov property. These Markov chains can be useful in many real world applications, including internet applications, economics and finance, gambling, music, baseball and statistical testing. In this project, a drunkard walks along a street in downtown. He walks to the left with probability q and to the right with probability p. He continues until he reaches left corner #1, which is a bar, or right corner #5, which is home, where he must stay. These are absorbing states. This project will focus on absorbing Markov Chains. Using a Markov chain we will find out where the drunkard will end his journey starting at building #2, #3, and #4.

293. How beautiful is the Golden Ratio? An Overlap of Psychology and Mathematics

Jamie Harvey University of North Georgia
Advisor(s): Brad Bailey, University of North Georgia

There are many claims that the Golden Ratio is the most aesthetically pleasing ratio, but most of those claims are purely anecdotal, or the result of analyzing existing art or architecture. We sought to verify that figures containing the Golden Ratio tend to be preferred by audiences over figures that do not contain the Golden Ratio. The student researcher designed a short survey and administered it to over 200 participants; then the data were statistically analyzed. The results included a couple of statistically significant outcomes. We will discuss the survey design, methods of statistical analysis, and the results of the study.

294. Using Markov Chains to Estimate the Length of Board Games

Harris Greenwood St. Edward's University
Advisor(s): Jason Callahan, St. Edward's University

We discuss the expected length of the board game Chutes and Ladders based on "Chutes and Ladders for the Impatient} by Cheteyan et al., which defines a Markov chain to calculate the expected game length with uniformly distributed spinners of all possible ranges. In particular, they show that a uniformly distributed spinner of range 15 yields the shortest expected game length of 23.81 turns. In this presentation, we investigate optimal distributions of non-uniform spinners of ranges 2 and 3. Specifically, we show that the optimal distribution of a non-uniform spinner

of range 2 has probability 86.61% of spinning a 2 and yields an expected game length of 47.28 turns, 13.48 turns less than a uniformly distributed spinner of range 2. For simplicity, non-uniform spinners of range 3 are analyzed on a small 3-by-3 example board on which the shortest game is found when the probability of spinning a 3 approaches 1 and the probabilities of spinning a 1 or a 2 approach 0. However, depending on how the probabilities of spinning a 1 or a 2 approach 0, infinitely many expected game lengths can be approached. The seemingly paradoxical behavior of this non-absorbing Markov chain is discussed.

295. North Georgia Playfulness Scale

Samantha Duchscherer University of North Georgia
Denver Coker University of North Georgia
Katelyn Pitts University of North Georgia
Georgianna Campbell University of North Georgia
Advisor(s): Robb Sinn, University of North Georgia

A 36-item North Georgia Playfulness Scale (NGPS) was developed to measure adult predisposition to play. Playfulness is a construct developed in the counseling literature. We examined the Adult Playfulness Scale (APS) created by Glynn and Webster in 1992 and the Playfulness Scale for Adults (PSA) developed in 1997 by Schafer and Greenburg. Several PSA items were either dated or poorly worded, and much more appropriate for middle-aged adults. We updated or deleted several of the original 35 items and developed new items. Alpha testing began with a 52-item scale. Preliminary analysis of the VIF's and KMO's allowed the deletion of the weakest items. Beta testing was conducted with a revised 36-item scale (n=293) using Principal Components Analysis (PCA) with Varimax rotation which led to a 4-component structure with readily interpretable components. Items must fit both mathematically (according to its loading) and thematically. The goal is for components which have easily interpretable themes and retain strong psychometric properties. Reliability analysis is ongoing. Our research also includes an investigation on whether playfulness correlates to personality variables such as STRESS, ANXIETY, OPTIMISM, COPING HUMOR, and HUMOR STYLES.

296. Protein-Protein Interaction Detection Using Mixed Models

Andrea Ekey Howard University
Sarah Jermeland Simpson College
Alyssa Everding Wheaton College
Advisor(s): Heng Wang, Michigan State University

Membrane protein-protein interactions (PPI) play an important role in biological processes; however, knowledge about membrane proteins is limited. In order to study membrane protein interactions, the mating-based split-ubiquitin system (mbSUS), which utilizes yeast as a heterologous system, is used. The resultant fluorescence is an indicator of PPIs, so the fluorescence scores are a focal point of this study. The observed scores may be affected by various fixed and random effects such as overall mean fluorescence, test versus positive control groups, plate effect, and PPI effect. We propose a statistical model of these effects, a mixed model, and apply it to a simulated data set. From this, we develop a methodology for analyzing the PPI data and determining significant PPIs. The results of the simulation study suggest that the mixed model may be a good fit for the real data.

297. When Tom will grab Jerry

Duanquira Myers Benedict College
Advisor(s): Naima Naheed, Benedict College

A Markov chain is just a random process or series of events that occur by chance and evolve in time with different probabilities. In many real-world situations (for example, values of stocks over a period of time, weather patterns from day to day, results of congressional elections over a period of elections and so forth) analysts like to predict the future, both in the short and in the long term. We will apply the properties of a Markov chain to a specific example about a cat and a mouse as Tom and Jerry in an apartment separated into four rooms with one-way passages between the rooms. Two types of Markov chains will be explored: first is an Absorbing Markov Chain and the second is a Regular Markov Chain. At first, Tom will be chasing Jerry and eventually end up in the same room. In this project we will focus on how many steps it takes for this to happen. After absorbing Jerry, Tom will be alone in the apartment. In the second case, one-way passages between the rooms will be changed to form a regular Markov

chain. We will find out the probability for Tom to be in each room in the long run. We will use simulations to verify these probabilities.

298. Examining the probability of Drawing Integers

Olulade Fasanmade Howard University
Advisor(s): Rondey Kerby, Morgan State University

Since the Monty Hall Paradox, students and professors alike have been wary of probability questions that seem to go against one's intuition. In this experiment, one examines a game as stated: There are two players and a pot of n balls, each having a random integer on the ball. The first player picks a ball, and the second picks a Y number of balls from a pot. Next, the first player picks c clue balls. The purpose of this experiment was to find the strategy that result in highest probability of the first player guessing right. The results were that one can use the amount of clue balls that are greater than or less than the player one ball to maximize player 1's chance of winning. When $Y = 1$, the probability of player 1 guessing correctly as the number of clue ball picked increases toward infinity is $3/4$. Without the clue balls, one is expected to have a $1/2$ probability of guessing correctly. When $Y = 2$, the probability of player 1 guessing correctly as c increases toward infinity is $17/27$. The expected probability without any clue balls is $1/3$.

299. Transfer Attrition: Hazards and Perseverance

Heather Gronewald Southwestern University
Advisor(s): Alison Marr, Southwestern University

We will present a mathematical model exploring the attrition rates within an undergraduate transfer population within a major public university. The model will focus on perseverance and hazard rates, and build towards establishing relationships with significant predictors of individual success within the university.

300. Applying Bayesian Methods to Ground Fuel Estimation

Ryan Karpisz Marist College
Advisor(s): Katharine Gray, California State University, Chico

The ability to accurately estimate quantities of biofuel is of critical importance in fire science, and it is important to understand the benefits and limitations of available sampling methods. Using data collected during a previous sampling method comparison study, we attempt an implementation of Bayesian analysis in an effort to improve the accuracy and precision of fuel load estimates obtained from different sampling methods used for the same load.

301. Torified Rational Links

Karly Brint Pitzer College
Advisor(s): Rollie Trapp, CSUSB

This research examines a certain class of links, called Torified Rational Links, with a focus on bounds for stick number. These bounds are found by supercoiling the Torified Rational tangles and attaching the coil to an outer skeleton, similar to that of a rational link. The bounds obtained using this model are compared to the known upper bound for the stick number of any link, $s(L) \leq \frac{3}{2}(c(L) + 1)$, using relationships between crossing number and the maximal and minimal degrees of the variables in the HOMFLY polynomial.

302. Applying the Bracket Polynomial to Multi-Crossing Projections

Samantha Petti Williams College
Advisor(s): Colin Adams, Williams College

The recently defined n-crossing is a singular point in a projection at which n strands cross so that each strand bisects the crossing. We generalize to n-crossing number the classic result of Kauffman, Murasugi, and Thistlethwaite, which relates the span of the bracket polynomial to the double-crossing number of a link, $span\langle K\rangle \leq 4c_2$. In this paper

we find the following lower bound on the n-crossing number in terms of the span of the bracket polynomial for any n:

$$\mathrm{Span}\langle K\rangle \leq \left(\left\lfloor \frac{n^2}{2} \right\rfloor + 4n - 8\right) c_n(K).$$

303. Stick Numbers in the Hexagonal Lattice

Ryan Bailey University of Texas
Melanie Dennis Middlebury College
Elise McMahon Ave Maria University
Advisor(s): Jennifer McLoud-Mann, University of Washington Bothell

This work is motivated by a paper by Mann, McLoud-Mann, and Milan, in which they prove the minimal stick number of the hexagonal lattice is 11. The authors give a new proof of this result using bridge number. Bridge number is also used to establish a lower bound on stick number of hexagonal lattice knots similar to the work of Rensburg and Promislow in the simple cubic lattice. Constructions for $(p, p+1)$ torus knots are given requiring one more stick than the lower bound guarantees.

304. Enumeration and Projection Dependence of 1-Singular Knots

David Brown Grinnell College
Katherine Tucker Willamette University
Ryan Stees James Madison University
Sarah Nicholson University of Dallas
Advisor(s): Leonard Van Wyk, Laura Taalman, James Madison

We describe our methods of enumerating knots with a lone singularity, methods we used to distinguish these 1-singular knots, and surprising difficulties encountered along the way. These surprises include dependence of the projection of the classic knots from which 1-singular knots are obtained, even when the projections are both minimal in terms of crossing number. We also show that the two standard projections of (p, q)-torus knots yield different 1-singular sets if $p < \frac{3q}{2}$.

305. The Crossing Map of Knots in the Cubic Lattice

Eric Klemchak Wilkes University
Advisor(s): Christian Laing, Wilkes University

Given a polygonal knot on the simple cubic lattice, we describe a method for computing the change in the number of knot crossings, known as the crossing map, as one changes the knot projection, as the average of weighted projected crossing numbers of the knot in a few directions. These directions are determined by the lattice geometry, the weights are determined by areas of regions on the unit 2-sphere, and the regions are formed by the crossing map to the polygonal knot. This work will describe several formulas, patterns that emerge, and algorithms that can be used to generate any crossing map for any polygonal knot embedded in a $n \times n \times n$ box. Particularly, as n increases, the complexity of the crossing map will also increase.

306. An Extension of Stanley Depth for Refinement-Ordered Posets

Ying Gao Newton North High School
Advisor(s): Sergei Bernstein, MIT

A property of posets concerning the partitionability of a poset into intervals was recently discovered to be analogous to the Stanley depths of certain modules. This property, which we call $ndepth$, provides a link between combinatorics and commutative algebra. Recently, the ndepth of the poset of the nonempty subsets of sets of size n ordered by inclusion was bounded. Also, ndepth was found for the poset formed as the product of chains of equal length. We study the properties of this Stanley-like depth in relation to another well-known class of posets - the posets consisting of non-empty partitions of sets ordered by refinement We used combinatorial and algebraic methods to obtain results showing that ndepth of refinement-ordered posets of the set-partitions of a set of size i is non-decreasing as i increases. We then showed linear upper and lower bounds for the ndepth of these refinement-ordered posets based on the value of i.

307. The Fundamental Groups of the Digital Line and Circles

Dane Lawhorne University of Mary Washington
Advisor(s): Randall Helmstutler, University of Mary Washington

The topological spaces known as the digital line ($\mathbb{D}$) and digital n-circles (C_n) are important objects of study in digital topology. From a homotopy theoretic viewpoint, the digital line and digital circles are discrete versions of the real line and S^1. In support of this statement, we first prove that $\pi_1(\mathbb{D})$ is trivial, as is $\pi_1(\mathbb{R})$, by constructing a series of homotopies from an arbitrary loop in $\mathbb{D}$ to the constant loop at the basepoint. Next, we construct for each C_n a covering map $p : \mathbb{D} \to C_n$ analogous to the standard periodic covering map $q : \mathbb{R} \to S^1$. We then use this fact, along with techniques from the standard proof that $\pi_1(S^1) \cong \mathbb{Z}$, to show $\pi_1(C_n) \cong \mathbb{Z}$. This last result is known, but previous proofs relied on the use of simplicial methods.

308. Determining the Structure of Length-k Steenrod Operations as $A(r)$-Modules

Benjamin Kraft Massachusetts Institute of Technology
Advisor(s): Mark Behrens, MIT

The Steenrod Algebra $\mathcal{A}$ is the algebra of stable natural endomorphisms of the $\mathbb{Z}/2$-cohomology functor; it is generated by elements Sq^{2^i}. Let $\mathcal{A}(k)$ be the subalgebra generated by the Sq^{2^i} for $i \leq k$. Consider the modules $L(k)$ spanned by sequences of Steenrod operations of length k. Welcher proved that $L(k)$ is a free module over $\mathcal{A}(k-1)$. We are interested in finding the structure of $L(k)$ as an $\mathcal{A}(r)$-module for any r. We conjecture that $L(k)$ is built as an $\mathcal{A}(r)$-module out of $\mathcal{A}(r)//\mathcal{A}(r-k)$, in the sense that it has an increasing filtration with quotients isomorphic to $\mathcal{A}(r)//\mathcal{A}(r-k)$, and present partial results towards that claim. In addition, we prove some interesting commutation relations in the Steenrod algebra relating to representations of Steenrod Algebra elements in Wood's Z-basis.

309. Boundary Slopes of Multiple, Non-isotopic Incompressible Surfaces

Ryan Manheimer The College of New Jersey
Advisor(s): Cynthia Curtis, The College of New Jersey

Over the past thirty years, incompressible surfaces have played an increasingly large role in our understanding of knots and the 3-manifolds resulting from Dehn surgery on knots. For two-bridge knots, the set of slopes of such surfaces is well-understood due to work of Hatcher and Thurston. However, note that two non-isotopic incompressible surfaces in the complement of a two-bridge knot can have the same boundary slope. We provide a criterion to identify boundary slopes of two bridge knots which are boundary slopes of multiple, non-isotopic incompressible surfaces. Furthermore, we identify an infinite family of knots for which no non-isotopic incompressible surfaces share a boundary slope.

310. Three-Variable Bracket Polynomial for Two-Bridge Knots

Matthew Overduin California State University, San Bernardino
Advisor(s): Rolland Trapp, California State University, San Bernardino

We derive recursive formulas for the three-variable bracket polynomial of a twist connected to a tangle and for a twist that connects to a tangle in two places. We use these formulas to derive the three-variable bracket polynomial for two-bridge knots. From this, we determine that the highest exponent of d is equal to the crossing number minus the twist number when the two-bridge knot has no twists with single crossings. We also state a theorem that allows one to determine the number of states corresponding to the maximal exponent of d for a two bridge knot not containing any twist with single crossings.

311. Ribbon Graphs of Oriented Links and Homfly Polynomial

Jordan Clark Morehouse College
Advisor(s): Neal Stoltzfus, Louisiana State University

The Homfly polynomial encodes information for oriented links. Together with the writhe, the Jones can be obtained from the Kauffman bracket, which in turn can be obtained via the rank polynomial of the all-A ribbon graph. The

writhe can be computed from the Seifert state. Using Peter Cromwell's canonical resolution of a link diagram an outline of we give an algorithm for computing the HOMFLY polynomial from its ribbon graph and the Seifert state

312. A Play on Topological Data Analyis in the NFL

Michael Paulson Southwest Minnesota State University
Advisor(s): Carl Olimb, Southwest Minnesota State University

Topological Data Analysis (TDA), a generalization of a Reeb graph analysis on a data set, provides tools that are effective in categorizing large data such as National Football League plays. We propose a novel approach using TDA to classify types of plays in the NFL and determine those classifications which yield maximal first down potential.

Index