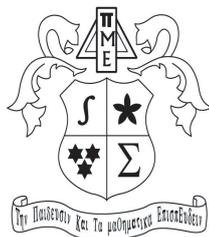




## MAA Student Chapters

The MAA Student Chapters program was launched in January 1989 to encourage students to continue study in the mathematical sciences, provide opportunities to meet with other students interested in mathematics at national meetings, and provide career information in the mathematical sciences. The primary criterion for membership in an MAA Student Chapter is “interest in the mathematical sciences.” Currently there are approximately 550 Student Chapters on college and university campuses nationwide.



## Pi Mu Epsilon

Pi Mu Epsilon is a national mathematics honor society with 355 chapters throughout the nation. Established in 1914, Pi Mu Epsilon is a non-secret organization whose purpose is the promotion of scholarly activity in mathematics among students in academic institutions. It seeks to do this by electing members on an honorary basis according to their proficiency in mathematics and by engaging in activities designed to provide for the mathematical and scholarly development of its members. Pi Mu Epsilon regularly engages students in scholarly activity through its *Journal* which has published student and faculty articles since 1949. Pi Mu Epsilon encourages scholarly activity in its chapters with grants to support mathematics contests and regional meetings established by the chapters and through its Lectureship program that funds Councillors to visit chapters. Since 1952, Pi Mu Epsilon has been holding its annual National Meeting with sessions for student papers in conjunction with the summer meetings of the Mathematical Association of America (MAA).

# Schedule of Student Activities

Unless otherwise noted, all events are at Lexington Center.

Please note that there are no MAA Sessions # 6, 10, 14, 18, or 22-26.

## Wednesday, August 3

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
2:00 pm - 4:00 pm	CUSAC Meeting	Hilton Hotel, Crimson Clover
4:30 pm - 5:30 pm	MAA/PME Student Reception	Thoroughbred 4
5:30 pm - 6:45 pm	Math Jeopardy	Thoroughbred 1

## Thursday, August 4

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
8:30 am - 11:30 am	PME Council Meeting	Hilton Hotel, Lilly of the Valley
8:30 am - 10:25 am	MAA Session #1	Thoroughbred 2
8:30 am - 10:25 am	MAA Session #2	Thoroughbred 3
8:30 am - 10:25 am	MAA Session #3	Thoroughbred 5
8:30 am - 10:25 am	MAA Session #4	Thoroughbred 6
8:30 am - 10:25 am	MAA Session #5	Thoroughbred 7
9:00 am - 5:00 pm	Student Hospitality Center	Exhibit Hall, Center Ballroom
1:00 pm - 1:50 pm	MAA Lecture for Students	Center Hall
2:00 pm - 3:55 pm	MAA Session #7	Thoroughbred 2
2:00 pm - 3:55 pm	MAA Session #8	Thoroughbred 3
2:00 pm - 3:55 pm	MAA Session #9	Thoroughbred 5
2:00 pm - 3:55 pm	PME Session #1	Thoroughbred 7
2:00 pm - 3:55 pm	PME Session #2	Thoroughbred 8
4:00 pm - 6:15 pm	MAA Session #11	Thoroughbred 2
4:00 pm - 6:15 pm	MAA Session #12	Thoroughbred 3
4:00 pm - 6:15 pm	MAA Session #13	Thoroughbred 5
4:00 pm - 6:15 pm	PME Session #3	Thoroughbred 7
4:00 pm - 6:15 pm	PME Session #4	Thoroughbred 8

## Friday, August 5

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
8:30 am - 10:45 am	MAA Session #15	Thoroughbred 2
8:30 am - 10:45 am	MAA Session #16	Thoroughbred 3
8:30 am - 10:45 am	MAA Session #17	Thoroughbred 5
8:30 am - 10:25 am	PME Session #5	Thoroughbred 7
8:30 am - 10:25 am	PME Session #6	Thoroughbred 8
9:00 am - 5:00 pm	Student Hospitality Center	Exhibit Hall, Center Ballroom
1:00 pm - 1:50 pm	MAA Student Activities Session: Polynomia Pasttimes	Thoroughbred 1
1:00 pm - 1:50 pm	MAA Student Activities Session: Unshuffling for the Imperfect Mathemagi- cian	Heritage Ballroom 3
2:00 pm - 3:55 pm	MAA Session #19	Thoroughbred 2
2:00 pm - 3:55 pm	MAA Session #20	Thoroughbred 3
2:00 pm - 3:55 pm	MAA Session #21	Thoroughbred 5
2:00 pm - 3:55 pm	PME Session #7	Thoroughbred 7
2:00 pm - 3:55 pm	PME Session #8	Thoroughbred 8
6:00 pm - 7:45 pm	PME Banquet and Awards Ceremony	Bluegrass Ballroom 2
8:00 pm - 8:50 pm	J. Sutherland Frame Lecture	Center Hall
9:00 pm - 10:00 pm	MAA Ice Cream Social and Awards	Bluegrass Pre-Function

## Saturday, August 6

<b>Time:</b>	<b>Event:</b>	<b>Location:</b>
9:00 am - 1:00 pm	Student Hospitality Center	Exhibit Hall, Center Ballroom
9:00 am - 10:00 am	MAA Modeling (MCM) Winners	Thoroughbred 1
1:00 pm - 2:15 pm	Student Problem Solving Competition	Thoroughbred 3

## **J. Sutherland Frame Lecture**

### **YOU CAN'T TOP THIS: MAKING THINGS BETTER WITH MATHEMATICS**

**Margaret H. Wright**

Courant Institute of Mathematical Sciences

Many problems in science, engineering, and medicine and life involve choosing the best way (or at least a better way) to do something. Mathematical optimization can often provide the answers we want; the speaker will describe when, why, and how this happens, along with a few examples.

*The J. Sutherland Frame Lecture is named in honor of the ninth President of Pi Mu Epsilon, who served from 1957 to 1966 and passed away on February 27, 1997. In 1952, Sud Frame initiated the student paper sessions at the annual Pi Mu Epsilon meeting, which is held at the Summer MathFests. He continually offered insight and inspiration to student mathematicians at these summer meetings.*

## MAA Lecture for Students

### MATH ICONS

**Roger Nelsen**

Lewis & Clark College

An icon (from the Greek for “image”) is defined as a picture that is universally recognized to be representative of something. The world is full of distinctive icons. Flags and shields represent countries, graphic designs represent commercial enterprises, and computer icons are essential tools for working with a variety of electronic devices from desktop computers to cell phones. What are the icons of mathematics? Numerals? Symbols? Equations? After many years of working with visual proofs, I believe that certain geometric diagrams play a crucial role in visualizing mathematical proofs. In this talk I’ll present several of these diagrams, which I call math icons, and explore the mathematics that lies within, and that can be created with them.

## MAA Undergraduate Student Activities Sessions

### POLYNOMIA PASTTIMES

**Dan Kalman**

American University

*Thoroughbred 1*

The Province of Polynomia has many interesting activities and diversions for the new visitor and old hand alike. In this session participants will explore some of the properties of Horner form, reverse polynomials, root power sums, and Lill's graphical method for finding roots of a polynomial.

### UNSHUFFLING FOR THE IMPERFECT MATHEMAGICIAN

**Doug Ensley**

Shippensburg University

*Heritage Ballroom 3*

In the world of card magic a perfect shuffle is one in which the deck is split exactly in half and the two halves combined by perfectly interlacing the cards. Magic tricks using this shuffle are not only very cool, but also extremely mathematical from the point of view of permutations. However, the perfect shuffle is difficult enough to do that few people can perform these tricks effectively. Our student activity will study a variation on the perfect shuffle that is easier to perform while preserving most of the mathematical properties. You'll be introduced to a little notation for permutations, learn a couple of magic tricks, and then set off to discover further generalizations of the ideas. Bring along a deck of cards and be ready to put practice into theory!

## MAA Student Speakers

Name	School	MAA Session
Stephanie Adamkiewicz	Augustana College	7
Brandon Alberts	Michigan State University	7
Enela Aliaj	Benedictine University	3
Matthew Allinder	DIMACS - Rutgers University	21
Ashley Dawne Anderson	Winona State University	20
Melanie M. Anderson	Winona State University	3
Holly Arrowood	Furman University	4
Michael Avallone	Augustana College	8
Rachel Elizabeth Bachman	Clarkson University	21
Crystal L. Bennett	North Carolina A&T State University	2
Nathan Yandala Bishop	St. Olaf College	19
Kelly Blakeman	Loyola Marymount University	19
Dubravka Bodiřoga	Hood College	1
Clark Bowman	University of Rochester	17
Natasha Brackett	Central Michigan University	16
Kelsea Brewer	Arkansas State University	2
Amelia Brown	Simpson College	17
Caitlin Bryant	University of North Carolina - Asheville	16
Jon Calhoun	Arkansas State University	8
Karleigh Cameron	Central Michigan University	20
Ray L. Chen	Danbury Math Academy	17
Michelle Chu	Emory University	21
Ping Ngai Chung	Massachusetts Institute of Technology	4
Michael Coates	Youngstown State University	7
Eugene D. Cody	University of Kansas	15
Mike Comer	Simpson College	17
Jason Cooke	Youngstown State University	11
Nathaniel Lawton Coursey	Kennesaw State University	15
Chaz Cox	University of Tennessee at Martin	5
Emily Davis	Brigham Young University	19
Jennifer Davis	University of North Carolina - Asheville	16
Amal DeAlwis	Southeastern Louisiana University	16
Neil J. Deboer	Michigan State University	4
Theodore Dokos	Ohio State University	9
Marianne Dubinsky	University of Mary Washington	15
Tim Dwyer	University of Florida	13
Trevor Edwards	Elon University	1
Charles Lloyd Elloie	Southern University at New Orleans	7
AnnaVictoria Ellsworth	Mount Holyoke College	4
Marie Nicole Ermete	Central Michigan University	16
Ashton Erwin	Arkansas State University	5
Miguel A. Fernandez Flores	Truman State University	16

## MAA Student Speakers (Continued)

Name	School	MAA Session
Christopher Robert Frye	University of Central Florida	12
Stephanie Ger	Boston College	2
Michael James Gustin	Central Michigan University	20
Justin Halverson	University of Wisconsin - Stout	1
Daniel R. Hast	University of Michigan	4
Timothy Hayes	Drexel University	21
Kevin He	University of California, Berkeley	17
Sam Roderick Helmich	Winona State University	15
Miranda Henderson	Benedictine University	2
Mike Henry	Simpson College	21
Jennifer Herdan	Winona State University	15
Chaim Hodges	Goshen College	17
John Holden	Central Michigan University	20
James Alban Christian Howe	Walla Walla University	5
Joshua D. Ide	Shippensburg University	5
Ibrahim Jadoon	Centre College	16
Yunhan Jing	High School Affiliate to Jilin University	17
Alexis Johnson	Grand Valley State University	1
Jackie Kastil	Augustana College	8
Lisa Marie Kaylor	Westminster College	2
Jennifer Kenkel	Washington University in St. Louis	4
Reema H. Khatri	Benedictine University	3
Jie Ling Liang	University of Central Florida	11
Missy Lucas	Rice University	9
Heather Malbon	Simpson College	21
Nathan Marculis	Grand Valley State University	17
Sami Eid Merhi	Western Connecticut State University	17
Yao Messan	North Carolina A&T State University	20
Juraj Milcak	University of Toronto	9
Rebecca Miller	University of Central Oklahoma	7
Annie Murphy	Clarkson University	20
Max Nguyen	Simpson College	21
Catherine Rose O'Doherty	University of Mary Washington	19
Pierce O'Donnell	Western Connecticut State University	17
SaraJane Parsons	Indiana University of Pennsylvania	17
James Derrick Peiskee	Tarleton State University	3
Van Pham	Southwestern University	13
Karli N. Powell	Central Michigan University	16
Eddie Price	Benedictine University	1

## MAA Student Speakers (Continued)

Name	School	MAA Session
Jarrett Pung	Winona State University	9
Maria Christine Radcliffe	Benedictine University	3
Natalie Radziejewski	Augustana College	16
Alfredo Ramirez	University of Tennessee at Martin	5
Nathan Reeves	Walla Walla University	8
Megan Marie Reiner	Winona State University	9
Sarah Ritchey	Youngstown State University	19
Matthew Daniel Romney	Brigham Young University	9
Donald J. Sanfilippo	Penn State University	19
Gretchen E. Schillemat	Brigham Young University	15
Ariel Setniker	Western Oregon University	2
Niralee Shah	Williams College	4
Evan Shirley	Centre College	16
Sajan Shrestha	Benedict College	19
Stacy Siereveld	Central Michigan University	20
Adam Solomon	Texas A&M University	8
Yonghyun Song	Hamilton College	21
Amelia Stonesifer	St. Olaf College	1
Laquinta Shanti Stuart	The College of New Rochelle	7
Anthony Taylor	Central State University	8
Jennifer Eileen Thompson	University of Cambridge	5
Jes Toyne	Simpson College	17
Hong Lien Tran	Kennesaw State University	2
Christopher Paul Van Putten	Lewis & Clark College	1
John VanBuren	Augustana College	3
Luis Alfonso Sordo Vieira	Wayne State University	4
Jakob Weisblat	Cleveland State University	7
Leah Whitaker	Pepperdine University	8
Jaelyn Wilson	Robert Morris University	5
Chad Witbeck	Brigham Young University - Idaho	9
Jared Wolf	Arkansas State University - Jonesboro	12
Ada Yu	University of Puget Sound	17
Felix Yu	Texas A&M University	20
Sam Zeng	Danbury Math Academy	17

## Pi Mu Epsilon Speakers

<b>Name</b>	<b>School</b>	<b>Chapter</b>	<b>PME Session</b>
Matt Alexander	Youngstown State University	OH Xi	6
Joseph Bertino	Fordham University	NY Alpha Nu	1
Mark Blumstein	Elmhurst College	IL Iota	3
Hanqin Cai	St. Norbert College	WI Delta	4
Daniel Catello	Youngstown State University	OH Xi	5
Kayla Comeaux	Southwestern University	TX Pi	1
Erin Compaan	University of North Florida	FL Eta	4
Scott Constable	Ithaca College	NY Upsilon	5
Anthony DeCino	University of California, Los Angeles	CA Alpha	7
Michael Donatz	Oregon State University	OR Beta	2
Gary R. Engler	William Paterson University of New Jersey	NJ Lambda	6
Joseph Ferrara	University of North Florida	FL Eta	6
Stephen Fox	Fordham University	NY Alpha Nu	1
Richard Freedman	Wake Forest University	NC Lambda	8
Kelvin Guilbault	North Central College	IL Nu	6
Ryan Hallberg	St. Norbert College	WI Delta	4
Nicole Harp	St. Norbert College	WI Delta	7
Brian Harrison II	John Carroll University	OH Lambda	5
Katie Heaps	Duquesne University	PA Upsilon	1
Reynaldo Herrera	Grand Valley State University	MI Iota	2
Kady Hossner	Western Oregon University	OR Delta	6
Stephanie Jessie	Austin Peay State University	TN Epsilon	7
Heather Johnston	Western Oregon University	OR Delta	7
Sepideh Khavari	Youngstown State University	OH Xi	5
Heather Hisako Kitada	Lewis & Clark College	OR Eta	2
Josh Koslosky	Duquesne University	PA Upsilon	1
Kelsey Larson	College of Saint Benedict	MN Delta	3
Kirill Lazebnik	State University of New York at Geneseo	NY Alpha Delta	8
Rachel Levanger	University of North Florida	FL Eta	4
Gabriel Montes de Ocs	Lewis & Clark College	OR Eta	3
Katherine Moore	Kenyon College	OH Pi	8
Kevin Moss	Iowa State University	IA Alpha	2
William Olsen	University of North Florida	FL Eta	6
Drake Parker	Grand Valley State University	MI Iota	2
Luigi Patruno	Fordham University	NY Alpha Nu	3
Brian Pietsch	St. Norbert College	WI Delta	4
Robert Rand	Yeshiva University	NY Mu	3
Tara Sansom	Youngstown State University	OH Xi	5
Christopher Schafhauser	University of Wisconsin - Platteville	WI Eta	3
Robert Short	John Carroll University	OH Lambda	8

## **Pi Mu Epsilon Speakers (Continued)**

<b>Name</b>	<b>School</b>	<b>Chapter</b>	<b>PME Session</b>
Glenn Sidle	Duquesne University	PA Upsilon	1
Justin Simpson	University of North Florida	FL Eta	7
Bradley Slabe	Youngstown State University	OH Xi	5
Mario Sracic	Youngstown State University	OH Xi	8
Abigail Wendricks	St. Norbert College	WI Delta	7
James Winegar	Austin Peay State University	TN Epsilon	4
Sarah Witt	Roanoke College	VA Delta	2
Hongying Zhao	College of Saint Benedict	MN Delta	8

## MAA Session #1

Room: Thoroughbred 2

8:30A.M. – 10:25A.M.

8:30–8:45

***p*-adic Numbers and their Galois Groups**

Trevor Edwards

*Elon University*

This paper will explore the subject of *p*-adic numbers. It will aim to give the reader a better understanding of this subject but also explore new aspects of *p*-adic numbers. The end goal will be to come up with the Galois groups of cubic field extensions of  $\mathbb{Q}_3$ .

8:50–9:05

**Using Cayley Graphs to do Rewriting**

Eddie Price

*Benedictine University*

In mathematics, one method of defining a group is by a presentation. Every group has a presentation. A presentation is often the most compact way of describing the structure of the group. However there are also some difficulties that arise when working with groups in this form. One of the problems is called the word problem which is an algorithmic problem of deciding whether two words represent the same element. I am studying the word problem on group extensions. To solve this problem in a group extension,  $G$ , we will need to know how to solve the word problem in a normal subgroup,  $N$ , and a quotient group  $G/N$ . This information along with information about how these groups stick together to make  $G$  is enough to solve the word problem in this case. In this talk I will discuss some observations I have made using the Cayley Graphs for  $N$  and  $G/N$  to study the structure of the resulting group extension  $G$ .

9:10–9:25

**The Cubic Formula in Characteristic 3**

Dubravka Bodiřoga

*Hood College*

Cardano's formulas express solutions of a cubic equation  $x^3 + px^2 + qx + r = 0$  in terms of its coefficients  $p$ ,  $q$ , and  $r$  by extracting square root and cube roots. However, Cardano's method does not work in characteristic 3, where  $3 = 0$ , because the formulas require dividing by 3 and because cube roots behave badly in characteristic 3. The proper replacements for cube roots in characteristic 3 are solutions  $Y$  to "Artin-Schreier" equations  $Y^3 = Y + b$ . I will explain how to construct a characteristic-3 replacement for Cardano's formulas that solves cubics in terms of Artin-Schreier equations. I use the methods of Euler, Bézout, and Lagrange to construct and analyze the solutions with the help of computer algebra. The methods generalize and can be used to study solvable equations of degree  $p$  in characteristic  $p$ .

9:30–9:45

**Commutative Ideals of Upper Triangular Matrices Bijected to Well-Known Catalan Objects**

Christopher Paul Van Putten

*Lewis & Clark College*

We will examine the commutative ideals of the ring of upper triangular matrices. The Catalan sequence, defined by  $C_n = \frac{1}{n+1} \binom{2n}{n}$ , counts a variety of objects including the ideals of the ring of upper triangular matrices. The cardinality of the set of commutative ideals of the ring of  $(n-1) \times (n-1)$  upper triangular matrices is  $2^{(n-2)}$ . These commutative ideals of upper triangular matrices biject to other objects such as Dyck paths, binary trees, and polygon triangulations. We establish a bijection from the set of commutative ideals of upper triangular matrices to a set of Dyck paths using an involution developed by J.C. Lalanne, from the set of Dyck paths to a set of binary trees, and from the set of binary trees to a set of polygon triangulations.

9:50–10:05

**Dead End Depth in Thompson's Group  $F$** 

Justin Halverson

*University of Wisconsin - Stout*

Thompson's groups  $F$  and  $T$  were introduced by Richard Thompson in the 1960's and have since found applications in many areas of mathematics including algebra, logic and topology. These groups can be realized as groups of piecewise linear homeomorphisms of the unit interval. In this talk, we will focus on metric properties of  $F$  equipped with the word metric with respect to its standard consecutive generating sets. The property we focus on is the dead end depth of  $F$ , which is the minimal integer  $N$  such that for any group element,  $g$ , there is guaranteed to exist a path of length at most  $N + 1$  in the Cayley graph of  $F$  leading from  $g$  to a point farther from the identity than  $g$  is. We will discuss improved bounds on the dead end depth of  $F$  with respect to the consecutive generating sets.

10:10–10:25

**Word Length in Alternate Presentations of Thompson's Group  $F$** 

Alexis Johnson and Amelia Stonesifer

*Grand Valley State University and St. Olaf College*

Thompson's groups  $F$  and  $T$  were introduced by Richard Thompson in the 1960's and have since found applications in many areas of mathematics including algebra, logic and topology. These groups can be realized as groups of piecewise linear homeomorphisms of the unit interval. In this talk, we focus on the word metric for  $F$  with respect to various finite generating sets. By viewing  $F$  as a diagram group, we are able to generalize techniques from the known algorithms for calculating word length with respect to the so-called consecutive generating sets to other non-standard generating sets. The generating sets we focus on are more symmetric than the standard generating sets in the sense that they contain elements whose corresponding piecewise linear homeomorphisms are the identity on long segments of the form  $[0, b]$  as well as elements whose homeomorphisms are the identity on long segments of the form  $[a, 1]$ .

## MAA Session #2

Room: Thoroughbred 3

8:30A.M. – 10:25A.M.

8:30–8:45

**Effects of Temperature on Sickle Cell CO – mediated Polymer Melting**

Crystal L. Bennett

*North Carolina A&T State University*

Sickle cell is a homozygous recessive genetic disorder. Sickle cell affects the protein hemoglobin (HbA) inside the red blood cell that allows oxygen to be circulated throughout the body. CO induced polymer melting enables the sickle cell hemoglobin (HbS) to revert to a close to normal hemoglobin state by CO molecules binding to a hemoglobin molecule. First, analysis of a mathematical model to assess the effects of CO-mediated polymer melting was implemented. Next, replication of the results using multiple numerical schemes using Matlab and Excel programming languages were executed to measure accuracy of each method. Comparison of different programs helped to determine the validity of the existing model. Next, using SimBiology (a systems biology markup language), verification of the numerical results from the mathematical model were obtained including a single graph of the system with various CO concentrations used. Then, a graphical user interface was created that allows users to manipulate the dynamical systems parameters (time, CO concentration, initial conditions, melting and binding rates, etc.) and retrieve the corresponding results. The final phase of this experiment will be the exploration of how temperature variation can be incorporated to improve the results of the dynamical system.

8:50–9:05

**Domination and Independence on the Triangular Honeycomb Chessboard**

Hong Lien Tran

*Kennesaw State University*

Puzzles on the chessboard have long been studied by mathematicians. “Across the Board: The Mathematics of Chessboard Problems” by John Watkins is an indispensable collection of mathematically themed chessboard problems. We do not restrict ourselves to the standard  $8 \times 8$  chessboard. Generalizations are quickly made to the square board of sides other than  $n = 8$ ,  $m \times n$  rectangular boards and other variant surfaces. Chessboard problems are most frequently set in the context of Graph Theory. Two classic problems in Graph Theory that appear again and again are those of dominating sets of minimum cardinality and independent sets of maximum cardinality. For chessboards the question of a minimum dominating set transforms into how to threaten or occupy every square on the board with the fewest pieces. It is possible to dominate the  $8 \times 8$  board with 5 queens but impossible to dominate the same board with only 4 queens. Maximum independent sets become the problem of how to place the maximum number of non-attacking pieces. It is possible to place 8 non-attacking queens on the  $8 \times 8$  board but it is impossible to place 9 non-attacking queens on the same board. Our project explores these two combinatorial problems on the variant triangular honeycomb chessboard.

9:10–9:25

**Straightline Hanoi**

Lisa Marie Kaylor

*Westminster College*

We investigate a variation of the Tower of Hanoi Puzzle called Straightline Hanoi in which disks may only be moved to or from the first peg. For  $n$  disks and for varying numbers of pegs we devise an algorithm for playing the game which we conjecture gives the minimal number of moves needed to solve it. We will present formulas for the number of moves needed to complete the puzzle when using our algorithm.

9:30–9:45

**Comparison of Boolean and Continuous Dynamics of Three-gene Regulatory Networks**

Miranda Henderson

*Benedictine University*

Recent work of Gehrmann and Drossel (2010) has shown significant differences between Boolean and continuous dynamics in two gene regulatory networks. Here we investigate three gene regulatory networks and compare the Boolean and continuous dynamics of these networks. In particular, we highlight conditions in which these two types of dynamical behavior differ.

9:50–10:05

**Inverse Modeling of Dynamical Systems**

Ariel Setniker and Stephanie Ger

*Western Oregon University and Boston College*

Conditioning likelihoods are typically much simpler to model than the full joint distribution which may be difficult or impossible to find analytically. Conditioning has the potential to improve the identifiability of the estimation problem. We will quantify various features of dynamical systems, for example, frequency, peak amplitude, inter-peak intervals, phase synchrony, etc. Parameter estimates are often obtained as the minimizers of a loss function which measures departure between the model prediction and the data at each of  $N$  points. Estimators with improved prediction bias are obtainable by adding conditional penalties to the loss function. We define a cumulative power penalty and compare its performance to the derivative matching penalty (Ramsay et al, 2007), as well as an optimal weighted average of different methods. The penalized losses will be included in a Bayesian Markov-Chain Monte Carlo (MCMC) estimation scheme. We provide case studies of stochastic switching, bistable oscillation (Terman and Wang, 1995), and predator-prey relationships. We will suggest applications of these methods to wildlife population management, neuroscience, and cryptography.

10:10–10:25

**Computational Algorithms for Constructing Nonregular Robust Parameter Designs**

Kelsea Brewer

*Arkansas State University*

The general robust parameter design (RPD) problem involves finding the settings of the control factors that minimize process variation due to changes in the (uncontrollable) noise factors. A computationally efficient algorithm has been developed for the construction of optimal non-regular RPDs, and the effectiveness of the algorithm is studied.

## MAA Session #3

Room: Thoroughbred 5

8:30A.M. – 10:25A.M.

8:30–8:45

**Exploration of the SIR Model**

John VanBuren

*Augustana College*

We will introduce the well-known SIR model for the spread of an infection in a population and extend the model in several ways. We will add new uninfected people at a constant rate. We will introduce a second disease, which can only infect people who have already been infected by the first disease. By varying the parameters in the model, we will minimize the maximum number of infected people at one time and look at the relationships between the parameters.

8:50–9:05

**Understanding Biological Models via Difference Equations**

Melanie M. Anderson

*Winona State University*

Difference equations are those equations that look at relationships between varying quantities and how these quantities change over an interval such as: time, space, etc. Epidemic Models are those that explain the spread of disease in a population. The goal of the application of these models is to eradicate potentially harmful diseases. Two examples of these models include: SIR and SIS models. SIR models are applied to diseases that result in the victim becoming immune to the disease after infection. SIS models are applied to diseases that do not result in a temporary recovery or no recovery of the victim after infection. This presentation will explore the applications of these epidemic models, what equations they are evaluated with, and actual calculations using the above models.

9:10–9:25

**A Node-weighted Model for the Spread of a Non-indigenous Species**

Maria Christine Radcliffe

*Benedictine University*

We present and analyze a model for the spread of a non-indigenous species (NIS) through a network. The model is a modified version of an SI epidemic model on a network in which the transmissibility rate from one node to another depends on the density of the NIS at the node at which the NIS is already present. This work is part of a larger project devoted to the study of the spread of the urban weed *Ailanthus altissima* (tree of heaven) throughout the United States.

9:30–9:45

**An Edge-weighted Model for the Spread of a Non-indigenous Species**

Enela Aliaj

*Benedictine University*

We present and analyze a model for the spread of a non-indigenous species (NIS) through a network. The model is a modified version of an SI epidemic model on a network in which the transmissibility rate from one node to another depends on a weight assigned to the edge connecting a pair of nodes. This work is part of a larger project devoted to the study of the spread of the urban weed *Ailanthus altissima* (tree of heaven) throughout the United States. In the *Ailanthus altissima* model, the weights are proportional to the number of roads and railroad lines between distinct locations.

9:50–10:05

**Optimal Control of the Spread of Cholera**

James Derrick Peiskee

*Tarleton State University*

Effective treatment and prevention strategies as they relate to the incidence of cholera outbreaks and the spread of the disease are of significant interest to the global health community. We present an SIR-type model which describes the transmission dynamics of the disease within the human population, as well as the growth of the two-class (hyper- and less infectious) bacterial concentrations. Treatment and prevention controls are analyzed using optimal control theory to suggest optimal control protocols.

10:10–10:25

**An Epidemic Model with a Multi-Stage Vaccine**

Reema H. Khatri

*Benedictine University*

Creating a mathematical model for an epidemic is necessary to describe the transmission of disease through individuals in a population. In order to prevent the possibilities of certain diseases, individuals would need to go through multi-stage vaccines to be fully immune. This creates a strain on resources as well as the individuals receiving the vaccines, who need to receive all of the vaccines to be fully immune. Thus, the goal of my project is to set up and analyze an epidemic model for multi-stage vaccines.

## MAA Session #4

Room: Thoroughbred 6

8:30A.M. – 10:25A.M.

8:30–8:45

**Exploring Periodic Orbits of The Outer Billiard Maps**

Neil J. Deboer and Daniel R. Hast

*Michigan State University and University of Michigan with Grand Valley State University*

We analyze the orbit structures and the geometric properties of the outer (dual) billiard map in the Euclidean and the hyperbolic planes. In particular, we concentrate on orbits of the map with periodicity divisible by the number of the vertices of its polygonal table. In the hyperbolic plane, we use Klein, Poincare and Upper Half Plane models to explore the orbits.

8:50–9:05

**Perimeter-minimizing Planar Tilings by Pentagons**

Niralee Shah

*Williams College*

Hales proved that regular hexagons provide the least-perimeter way to partition the plane into unit areas. What are the best pentagons?

9:10–9:25

**Isoperimetry in Surfaces with Density**

Luis Alfonso Sordo Vieira

*Wayne State University*

We study the isoperimetric problem in some simple surfaces with density. The isoperimetric problem seeks to enclose prescribed area with minimum perimeter. In the Euclidean plane, the answer is the circle.

9:30–9:45

**Tilings with Nonconvex Pentagons**

Ping Ngai Chung

*Massachusetts Institute of Technology*

We study tilings of the plane by unit-area pentagons. Could tilings with mixtures of nonconvex and regular pentagons have less perimeter than that with convex irregular pentagons?

9:50–10:05

**Equal Circle Packing on Tori**

AnnaVictoria Ellsworth and Jennifer Kenkel

*Mount Holyoke College and Washington University in St. Louis*

The study of maximally dense packings of disjoint equal circles in different types of containers is a problem in Discrete Geometry that has developed over the past forty years. The optimal densities and arrangements are known for packings of small numbers of equal circles into hard boundary containers, including squares, equilateral triangles and circles. In this presentation, we will explore packings of small numbers of equal circles into a boundaryless container called a flat torus. Using numerous figures we will introduce all the basic concepts (including the notion of a flat torus, an optimal packing and the graph of a packing), demonstrate many maximally dense arrangements, and outline the proofs of their optimality. This research was conducted as part of the 2011 REU program at Grand Valley State University.

10:10–10:25

**The Isoperimetric Inequality on Constant Gaussian Curvature Surfaces**

Holly Arrowood

*Furman University*

A new proof of the isoperimetric inequality on spheres and hyperbolic planes. We use a new method of optimization proof called metacalibration, in which competitors are compared directly to the proposed minimizer via vector fields and the divergence theorem. This approach paves the way to solve open problems such as multiple bubbles and isoperimetric problems with boundary on constant Gaussian curvature surfaces.

## MAA Session #5

Room: Thoroughbred 7

8:30A.M. – 10:25A.M.

8:30–8:45

**Improving on the Range Rule of Thumb**

Alfredo Ramirez and Chaz Cox

*University of Tennessee at Martin*

In manufacturing it is useful to have a quick estimate of the standard deviation. This is often done with the range rule of thumb:  $\sigma \approx \frac{\text{sample range}}{4}$ . This rule works well when the data comes from a normal distribution and the sample size is around 30, but fails miserably for other distributions and sample sizes. Through the use of Monte Carlo simulations we suggest new rules of thumb for the normal distribution ( $\sigma \approx \frac{\text{range}}{(3\sqrt{\ln n - 1.5})}$ ), uniform distribution ( $\sigma \approx \frac{n+1}{n-1} \cdot \frac{\text{range}}{\sqrt{12}}$ ) and exponential distribution ( $\sigma \approx \frac{\text{range}}{\ln(n)+4/9}$ ) which are dependent on sample size. We then seek to verify these empirical results theoretically.

8:50–9:05

**Weighted Least-Squares Estimates for the Autoregressive Parameter**

Ashton Erwin

*Arkansas State University*

In time series analysis, we often wish to estimate the unknown parameter of a first-order autoregressive process. Weighted least-squares estimators yield good estimates for this parameter. In this talk, we will explore the various properties of these estimators such as bias and mean squared error. We will then use an asymptotic convergence result to construct confidence intervals for this autoregressive parameter centered around these different estimators, and then look at the coverage capability of the resulting intervals.

9:10–9:25

**An Extension of the Google PageRank Algorithm to College Football Rankings**

Joshua D. Ide

*Shippensburg University*

Have you ever wondered why it is that certain websites are listed before others when you perform a search using Google? There is a very specific reason behind this, and it heavily involves mathematical tools of varying difficulties. More specifically, Google's PageRank algorithm utilizes the stochastic matrices found in Markov chains to assign various probabilities to specific websites based on the notion that any given internet user will navigate to that specific page, with that probability. One then computes the steady state vector of the resulting matrix, ultimately providing a ranking of the web pages. Oddly enough, this same process can be applied to rank college football teams, subsequently providing viable predictions for the results of BCS bowl games.

9:30–9:45

**Optimal Couplings for Card Shuffling**

Jennifer Eileen Thompson

*University of Cambridge*

Finding a coupling between two instances of a Markov Chain is a useful way to bound the mixing time for that chain. We take the first chain in an arbitrary state, and the second in equilibrium, and control the transitions so that each chain when viewed marginally obeys the correct transition probabilities, and we try minimize the expected time until the two chains reach the same state. For an optimal coupling method, this time is a measure of the mixing time. How can we find optimal couplings for random card shuffling? We look at the simple card shuffling methods of ‘top card to random placement’, and ‘random card to random placement’, providing a coupling for the former and an optimal coupling for the latter. We find the optimal coupling by measuring the distance between two decks of cards with the ‘swapping metric’, and reduce this distance as the cards are shuffled.

9:50–10:05

**The Mathematics of Juggling: An Introduction**

James Alban Christian Howe

*Walla Walla University*

Mathematical diagrams and sequences allow jugglers to communicate, create, and learn juggling patterns in an effective and efficient way. The theorems that accompany these sequences provide a method for determining whether a given pattern can be juggled. This survey paper explores the definitions, theorems, and diagrams that form the foundation of the mathematics of juggling. The presentation will include demonstrations of juggling patterns that were created and learned with the help of sequences.

10:10–10:25

**Neo-Riemannian Theory and John Williams’ Music**

Jaclyn Wilson

*Robert Morris University*

The neo-Riemannian theory and its operations are used to analyze chord progressions. This is done by using the three mathematical operations: the parallel transformation ( $P$ ), leading transformation ( $L$ ), and reflective transformation ( $R$ ) and combinations of the three. Thus, the neo-Riemannian method presents the geometry that emerges within music. In this talk I will present an introduction to the neo-Riemannian basic operations as they relate to music theory. These operations will then be applied to The Star Wars (Main Theme) and The Imperial March (Darth Vaders’ Theme) by John Williams. A visual representation of the chord progressions will also be shown including a short video with a tonnetz, a network of tonal space lying on the torus.

## MAA Session #7

Room: Thoroughbred 2

2:00P.M. – 3:55P.M.

2:00–2:15

**Roots of Generalized Fibonacci-Type Polynomials**

Rebecca Miller and Laquinta Shanti Stuart

*University of Central Oklahoma and The College of New Rochelle*

Consider a Fibonacci-type Polynomial Sequence given by the recurrence  $F_0(x) = a$ ,  $F_1(x) = x + a$ ,  $F_n(x) = x^2 F_{n-1}(x) + F_{n-2}$ , where  $n \geq 2$  and  $a \in \mathbb{Z}^-$ . Let  $g_n$  be the maximal real root of  $F_n(x)$ . In this talk we will give asymptotic results for  $g_n$  numerical as well as analytic results will be presented. We will also give numerical results for the maximal real root of the derivatives and anti-derivatives of  $F_n(x)$ .

2:20–2:35

**On the Properties of a Quasi-Fibonacci Polynomial Sequence**

Brandon Alberts

*Michigan State University*

Properties of several Fibonacci-type polynomial sequences have been explored, especially the asymptotic behavior of the maximum roots. In this talk we will be exploring the properties of a Quasi-Fibonacci polynomial sequence defined by the recursion  $F_1(x) = -1$ ,  $F_2(x) = x - 1$ , and  $F_n(x) = F_{n-1}(x) + x^k F_{n-2}(x)$ , for  $k$  being a positive integer. Analytic as well as numerical results will be presented.

2:40–2:55

**The Lucas-Lehmer Test for Mersenne Primes**

Charles Lloyd Elloie

*Southern University at New Orleans*

In this project I will derive the test for whether a number in the form  $M = 2^p - 1$  is prime or composite when  $p$  is congruent to  $3 \pmod{4}$ . More precisely, this will allow me check whether or not a number in this form is a Mersenne prime or not, when  $p$  is congruent to 3 modulus 4.

3:00–3:15

**Anomalous Cancellation**

Michael Coates

*Youngstown State University*

The mathematical curiosity that allows one to perform the correct division  $\frac{64}{16} = \frac{4}{1}$  by incorrectly “canceling the sixes” in the numerator and the denominator is known to many. In this talk, we examine whether other examples of this phenomenon can be found in various number bases, provide generalized methods for doing so, and explain the significance of this result.

3:20–3:35

**Examining Efficient Factorization Methods**

Jakob Weisblat

*Cleveland State University*

In this talk, We will first examine the residues of perfect squares,  $\pmod{10}$  and  $\pmod{100}$ . We will then look at factoring large numbers efficiently using these residues, with Gauss' and Fermat's methods of factorization.

3:40–3:55

**Multiplicative Function**

Stephanie Adamkiewicz

*Augustana College*

We will define multiplicative and completely multiplicative functions. Multiplicative functions have the property that  $f(m) \cdot f(n) = f(mn)$  whenever  $m, n$  are relatively prime. Completely multiplicative functions, on the other hand, have the property that  $f(m) \cdot f(n) = f(mn)$  for all  $m, n$ . Furthermore, we will discuss some elementary results. Also, we will give several examples of infinite families of multiplicative and completely multiplicative functions.

## MAA Session #8

Room: Thoroughbred 3

2:00P.M. – 3:55P.M.

2:00–2:15

**Modeling Optimal Area of Cooling Zones Following a Marathon**

Adam Solomon

*Texas A&M*

The study aims to resolve overcrowding issues in marathons by determining the optimized area needed for each of two cooling zones to manage racer population fluctuations. We developed a modeling approach by transforming data from previous races into an appropriate distribution of finish times, in order to better organize future events. Specifically, three different models were constructed. The first two models were rejected due to unrealistic assumptions and outcome predictions. The final model was the most accurate, in which we used a constant traffic flow that is independent of the number of people in the zone. In this case, when the population is large, there is still a consistent outflow. This model yields populations of each cooling zone that are reasonable and less sensitive to delay terms. Using the curve fitting and parameter estimation we obtained feasible explanations of actual occurrences in a post-marathon expo. By closely analyzing the model, we obtained the percentage of allotted area for both of the cooling zones with the goal of providing the most efficient allocation of space.

2:20–2:35

**Seat Value in the Rose Bowl: Comparing Distance and Price**

Leah Whitaker

*Pepperdine University*

In the Rose Bowl Stadium, the ‘best’ seats are also the most expensive. In this preliminary version of our analysis, we find a measure of value for each seat location which is a function of distance to the field and price (subsequent analysis will include other factors such as viewing angles). Not surprisingly, we found that the measures for certain ‘less desirable’ seats are better than those for ‘more desirable’ seats.

2:40–2:55

**Modeling Motion in MATLAB: Predicting Wrist Placement in the Gait Cycle**

Anthony Taylor

*Central State University*

A MATLAB program called MOTION was created using algebraic techniques for solving inverse problems. Through these algorithms, the MATLAB program was structured to import the lengths of the upper arm and forearm from a text file and output the shoulder and elbow joint angles to place the wrist at a particular position and orientation in space. This paper, then, will explain how MOTION was designed and discuss the results of applying this program to Vicon Motion capture data collected from 15 human subjects.

3:00–3:15

**A Ranking and Prediction System for College Football**

Nathan Reeves

*Walla Walla University*

Ranking systems for college football have been much maligned of late. By taking into account current ranking systems, as well as readily-available statistics, a new, more accurate ranking and accompanying bowl game prediction system can be created. Using Spearman's Rank Correlation Coefficient, computer optimization techniques, and some simple logic, an algorithm has been found that can predict bowl game outcomes with a high degree of success.

3:20–3:35

**Life and Death: The Actuarial Model of Life Tables**

Michael Avallone and Jackie Kastil

*Augustana College*

Life tables show the number of survivors of a population as a function of time. We will show how to compute mortality rates and life expectancies. We will demonstrate these methods using examples, including one about the Chicago Bulls.

3:40–3:55

**Dynamic Contact of Viscoelastic Rods**

Jon Calhoun

*Arkansas State University*

We consider a viscoelastic rod that falls down and contacts a stationary rigid surface and then bounces off after its contact. This dynamic contact problem satisfies Signorini's contact conditions which can be interpreted in terms of complementary conditions (CCs). The motion of the viscoelastic rod is formulated by a third order partial differential equation. We use time discretization in the construction of the numerical formulation, where the midpoint rule is applied in the rod's elasticity and viscosity and the implicit Euler method is applied in CCs. Using finite element method over the special domain, we can set up a linear system at each time step and compute numerical solutions satisfying CCs. Numerical evidence of energy balance is observed in the viscoelastic model.

## MAA Session #9

Room: Thoroughbred 5

2:00P.M. – 3:55P.M.

2:00–2:15

**Convex Combinations of Minimal Graphs**

Missy Lucas and Chad Witbeck

*Rice University and Brigham Young University - Idaho*

A minimal surface in  $\mathbb{R}^3$  is a surface whose mean (or average) curvature vanishes at every point on the surface; bending in one direction is “canceled out by opposite bending in the perpendicular direction. Complex-valued, planar harmonic mappings of the form  $f = h + \bar{g}$  can be used along with the Weierstrass representation to provide a formula to parameterize minimal graphs in  $\mathbb{R}^3$ . We explore conditions under which known minimal graphs can be combined to form new minimal graphs and continuous transformations between them.

2:20–2:35

**Planar Harmonic Mappings and Symmetric Minimal Surfaces**

Matthew Daniel Romney

*Brigham Young University*

A minimal surface in  $\mathbb{R}^3$  is a regular surface for which the mean curvature equals zero at every point. In the theory of minimal surfaces, the Weierstrass representation provides a formula using complex analysis for the local representation of a minimal surface. One new approach to investigating minimal surfaces is to use results about planar harmonic mappings,  $f = h + \bar{g}$ , where  $h$  and  $g$  are complex analytic functions. In particular, we can reformulate the classical Weierstrass representation using the harmonic univalent map  $f = h + \bar{g}$ . We will discuss how this can be used to generate minimal surfaces of higher rotational symmetry by a transformation of the corresponding harmonic map.

2:40–2:55

**Modeling Fluid Interactions Around an Airfoil**

Megan Marie Reiner

*Winona State University*

The design of an airfoil can range from simple raindrop like shapes to complex curved figures. Through the art and science of Conformal Mapping, one can take a simple shape, such as a circle, and transform it into many complex shapes that can be useful in solving problems. One such example is the shape of an airplane wing and the way fluid flows around it to produce lift. A cross-section shape, known as an airfoil, will model the airplane wing. To create the shape of the airfoil, a transformation will be made to convert a cylinder to an airfoil. This presentation will focus on a few of the techniques and theories required to map the airflow around a cylinder. It will also indulge in the mathematics of the Joukowski Transformation to make a connection from the cylinder to an airfoil.

3:00–3:15

**An Introduction to the Development of Multidimensional Complex Analysis**

Jarrett Pung

*Winona State University*

As one begins studying mathematics, the starting point is always single variable cases. Calculus eventually develops into multivariate calculus. The usefulness of integration and derivatives carries over to the higher dimensional plane. Integrals become double integrals and derivatives become partial derivatives. After studying the complex plane, one will undoubtedly begin to wonder about the mathematical phenomena of multidimensional complex analysis. Just like in real analysis, many key theorems carry over to a multidimensional context. One can use Cauchy's Integral Formula to obtain a double integral formula for a bidisc in the Complex Euclidean space. Similarly, derivatives become partial derivatives. We can continue to use tools of complex analysis to further understand this multidimensional context. Important factors in complex analysis such as zeros, singularities and convergences all have a new and more in-depth meaning than in the single variable cases. Many mathematical discoveries have allowed the study of multidimensional complex analysis to be understood and utilized. By the end of this presentation one will be able to comfortably navigate the Complex Euclidean space without feeling overwhelmed.

3:20–3:35

**Generalization's of Golomb's "Strange Recursion"**

Juraj Milcak

*University of Toronto*

A recursion  $g$  is called nested if an argument of a recursive call of  $g$  refers to another invocation of  $g$ . An example of a nested recursion is Golomb's "strange recursion"  $g(g(n) + kn) = 2g(n) + kn$ , where  $k$  is some fixed natural number. This recursion has, as one of its solutions, the function  $\lfloor r^{(k)}n \rfloor$ , where  $r^{(k)}$  is the positive root of the quadratic equation  $f(x) = x^2 + (k - 2)x - k$ . This motivates us to ask whether similar behaviour occurs for generalizations of Golomb's recursion and other irrational floor functions. We show that indeed for any algebraic number  $r$ , there exists a "strange recursion" such that  $\lfloor rn \rfloor$  is a formal solution.

3:40–3:55

**Inversion Wilf Equivalence in Permutations**

Theodore Dokos

*Ohio State University*

For a given permutation  $\pi$  we use  $Av_n(\pi)$  for the set of permutations on  $\{1, \dots, n\}$  that *avoid*  $\pi$ , that is, they do not contain a subsequence where the elements are in the same relative size as  $\pi$ . For example, 14325 contains 132, by the subsequence 142, but avoids 4321. We call two permutations  $\pi, \pi'$  *Wilf equivalent* if  $|Av_n(\pi)| = |Av_n(\pi')|$  for all  $n$ . Surprisingly, if  $\pi, \pi' \in S_3$ , then they are Wilf equivalent. A function from a set of permutations of  $\{1, \dots, n\}$  to the natural numbers is called a *permutation statistic*. As an example, the number of inversions  $\text{inv } \pi$  of a permutation forms a statistic, where an *inversion* is a pair of out of order elements. Related to the inversion number is the *inversion polynomial* on a set of permutations:  $I(A, q) = \sum_{\pi \in A} q^{\text{inv } \pi}$ . From these polynomials we define *inversion Wilf equivalence*: we say  $\pi, \pi'$  are inversion Wilf equivalent if  $I(Av_n(\pi), q) = I(Av_n(\pi'), q)$  for all  $n$ . In this presentation we will investigate when two permutations are Wilf equivalent.

**PME Session #1****Room: Thoroughbred 7**  
2:00–2:15**2:00P.M. – 3:55P.M.****Variational Image Denoising and Decomposition Using Duality**

Katie Heaps

*Duquesne University*

Minimizing the total variation of an image coupled with an appropriate fidelity term has been used in the image processing community for decades as a way to denoise images while preserving important information such as edges and smooth regions. Unfortunately, it also results in staircasing, replacing smooth transitions in color with disconnected flat regions. In this talk, we examine several methods based on the total variation functional for reducing staircasing, including one that uses edge detection to control smoothing and a scheme that minimizes second differences (instead of first) to better preserve ramps. These functionals can be minimized using the dual problem, which allows for a more accurate solution. We will also look at how these methods can be extended to decompose images into their cartoon and texture components. We will observe the effects of these algorithms on a variety of images.

2:20–2:35

**Image Fusion Using Gaussian Mixture Models**

Josh Koslosky

*Duquesne University*

Recent research in image processing has shown that sparse and redundant dictionary representations of image patches can be used to perform various techniques such as denoising, deblurring and inpainting of images. Yu and Sapiro showed that a related, but more stable, solution can be found by estimating the patches using Gaussian Mixture Models (GMMs). In this talk we discuss how these GMM models can be used for fusing images of the same field of view but with different degradation operators applied.

2:40–2:55

**Image Representation Using Bases and Redundant Dictionaries**

Glenn Sidle

*Duquesne University*

In order to better process and analyze signals, such as images, it is oftentimes useful to represent them in different domains other than the “usual” spatial domain. In this presentation we will survey several different image representations, including the standard Fourier and wavelet domains as well as the newer patch based methods using sparse and redundant dictionaries. We will discuss the benefits of these different representations, as well as their connection to modeling human vision. We will also present several examples of processing images in using these different representations.

3:00–3:15

**Solving Systems of Polynomial Equations Using Gradient Descent**

Joseph Bertino

*Fordham University*

Problems of optimization can become very large, in which case computation by hand is slow and prone to error. Fortunately, large optimization problems can be expressed as systems of polynomial equations and, assuming suitable initial conditions, approximated using a variety of computer algorithms. This thesis explores the gradient descent algorithm (Augustin Cauchy), other conjugate gradient methods (Polak-Ribiere), as well as Darwinian and Evolutionary methods in finding approximate solutions to systems of polynomial equations. ‘Darwinian’ and ‘Evolutionary’ imply, respectively, the destruction and creation of candidate solutions to potentially improve the chances of convergence and avoid settling at local minima. The objective is to compare the performance of these various methods, based on rate of convergence and running times, while discussing their uses for different examples of optimization problems.

3:20–3:35

**Multiplying Univariate Polynomials Via Fast Fourier Transforms Over Finite Fields**

Stephen Fox

*Fordham University*

The Fast Fourier Transform (FFT) has been known and used for several decades to manipulate polynomials with real coefficients. Although the coefficients are usually real in important applications, one needs to use complex numbers and their roots of unity. Since the fact that the FFT relies on roots of unity in  $\mathbb{C}$  is never used in the algorithm, we can implement the FFT over a finite field using the same mathematical properties of fields needed for the FFT. We have implemented the Fast Fourier Transform over  $\mathbb{Z} \bmod p$ . Using the Chinese Remainder Theorem, this gives rise to an  $O(n \log n)$  algorithm for multiplication of univariate polynomials with integer coefficients, integers of unlimited number of digits as is common in computer algebra systems. Results suggest that the FFT over finite fields can provide a significant speedup in polynomial multiplication time over  $\mathbb{Z}$ .

3:40–3:55

**Statistical Analysis of an Infrared Thermography Inspection of Reinforced Carbon-Carbon**

Kayla Comeaux

*Southwestern University*

Each piece of flight hardware being used on the shuttle must be analyzed and pass NASA requirements before the shuttle is ready for launch. One tool used to detect cracks that lie within flight hardware is Infrared Flash Thermography. This is a non-destructive testing technique which uses an intense flash of light to heat up the surface of a material after which an Infrared camera is used to record the cooling of the material. Since cracks within the material obstruct the natural heat flow through the material, they are visible when viewing the data from the Infrared camera. We used Ecotherm, a software program, to collect data pertaining to the delaminations and analyzed the data using Ecotherm and University of Dayton Log Logistic Probability of Detection (POD) Software. The goal was to reproduce the statistical analysis produced by the University of Dayton software, by using scatter plots, log transforms, and residuals to test the assumption of normality for the residuals.

## PME Session #2

Room: Thoroughbred 8

2:00P.M. – 3:55P.M.

2:00–2:15

**Enumerative Graph Theory: Perfect Matchings on Grid Graphs and Domino Tilings**

Reynaldo Herrera

*Grand Valley State University*

In 1961, mathematician P.W. Kasteleyn calculated the number of diatomic molecule configurations on the quadratic lattice. This led to the discovery of the following formula,

$$4^{mn} \prod_{k=1}^m \prod_{l=1}^n \left( \cos^2 \left( \frac{k\pi}{m+1} \right) + \cos^2 \left( \frac{l\pi}{n+1} \right) \right).$$

These “dimer” configurations are actually perfect matchings on  $2n \times 2m$  grid graphs. We will discuss the connection between grid graphs, Domino tilings and Kasteleyn’s results.

2:20–2:35

**Archimedes’ Heuristics: Actual Infinity in the Method of Mechanical Theorems, Prop. 14**

Drake Parker

*Grand Valley State University*

Study over the past decade by the Archimedes Palimpsest Project of the Method of Mechanical Theorems, recovered in 1998, has produced evidence that the ancient Greek mathematician may have made informal use of actual infinity in his method of discovering geometric theorems. This runs contrary to the Greek tradition of rigorous proof which allows only for the use of potential infinity. We will examine the relevant argument, Proposition 14, compare it with other, more or less traditional, Greek infinitary arguments (including Euclid’s classic proof in Elements XII.2), and consider the questions arising about the Greek attitude toward infinity. Greek geometry benefited from illustration and plenty will be provided.

2:40–2:55

**Domino Tilings and the Fibonacci Sequence**

Kevin Moss

*Iowa State University*

How many ways can a  $2 \times N$  grid be tiled by dominoes? The solution, involving the Fibonacci sequence, links the fields of recurrent sequences and tiling problems. In this talk, I will discuss some other tiling problems and how they can be used to prove combinatorial identities.

3:00–3:15

**Using Young Tableaux to Count Commutative Ideals**

Heather Hisako Kitada

*Lewis & Clark College*

In this talk, we will demonstrate a counting argument for the number of ideals in the ring of upper triangular matrices by right justified Young tableaux. It is known that the  $n$ -th Catalan number is the number of ideals in the ring of  $(n - 1) \times (n - 1)$  upper triangular matrices. In addition, we will focus on the subset of ideals that are commutative. It turns out that there are  $2^{n-2}$  commutative ideals in the ring of  $(n - 1) \times (n - 1)$  upper triangular matrices. We will show through a partition of Young's lattice how one can count these commutative ideals using binomial coefficients, which are manifested in Pascal's triangle. Furthermore, we hope to illuminate how commutative ideals translate to other Catalan objects, including Dyck paths, 312-avoiding permutations and ordered plane trees.

3:20–3:35

**On the Relationship Between John Conway's Tangles and Integer Partitions**

Michael Donatz

*Oregon State University*

Both knot theory and the theory of integer partitions are quickly growing fields, that have much to be explored. In this talk we explore ways to understand partitions in terms of knots, and vice versa, hopefully creating a bridge between the two subjects.

3:40–3:55

**Using Graph Theory to Analyze Social Networks**

Sarah Witt

*Roanoke College*

Graph Theory has been used to provide insight into the structure of networks. In recent years, social networks have risen in importance especially including Facebook and Twitter. In this talk we will use various graph measures to investigate properties of social networks.

## MAA Session #11

Room: Thoroughbred 2

4:00P.M. – 6:15P.M.

4:00–4:15

**Exploring the Existence of Multiple Private Keys in RSA**

Jason Cooke

*Youngstown State University*

The RSA algorithm for public key encryption, which relies on the difficulty of factoring large numbers, is discussed. We are primarily concerned with the determination of the decryption key. Several keys are possible, but we will focus on the smallest one. Multiple keys are found to exist and this phenomenon is demonstrated using software.

4:20–4:35

**Congruences for The Number of Smallest Parts in The Partitions of  $n$** 

Jie Ling Liang

*University of Central Florida*

This paper is devoted to the study of the congruence properties of the total number of appearances of the smallest parts in all of the partitions of  $n$  denoted by  $spt(n)$ . In particular, we examined the problem of Ramanujan type congruences for  $spt(n)$  modulo powers of 2 and 3.

Abstracts for the remainder of this session were received too late to appear in print.

Please refer to the MAA Student web page at:

<http://www.maa.org/mathfest/students.html>

for presenters' names, talk titles, and abstracts.

## MAA Session #12

Room: Thoroughbred 3

4:00P.M. – 6:15P.M.

4:00–4:15

**Dynamic Contact of Particles with Adhesion**

Jared Wolf

*Arkansas State University - Jonesboro*

Our differential equations describe a physical situation where a particle drops down onto an adhesive stationary rigid foundation and bounces off after its impact. When the Signorini contact conditions with adhesion are imposed on the particle, the dynamic motion of the particle is expressed by the ordinary differential equation  $u_{tt} = f(t) + N_C(t) - \kappa (u - \varphi) \beta^2$  where the contact forces  $N_C(t)$  satisfy the complementarity conditions  $0 \leq u(t) - \varphi \perp N_C(t) \geq 0$ . Also the evolution of adhesion is formulated by the first order ordinary differential equation  $\beta_t = -\frac{\kappa}{a} (u - \varphi)^2 \beta$ , where the positive constant  $a$  is called the adhesion rate and the positive constant  $\kappa$  is the bonding coefficient. We use time discretization to set up numerical formulations and investigate convergence of numerical trajectories. In addition, the energy balance will be justified both theoretically and numerically.

4:20–4:35

**Spherical Harmonics and Legendre Polynomials in  $\mathbb{R}^p$** 

Christopher Robert Frye

*University of Central Florida*

This paper presents a review of the hyper-spherical harmonics and the related Legendre polynomials that arise in spherically symmetric problems in  $p$  dimensions, since it appears the related literature is scant. One of the few books that covers this subject, *The Functions of Mathematical Physics* by Harry Hochstadt, is now out of print. This paper makes the relevant material covered in Hochstadt's book accessible to intermediate undergraduates, as well as providing additional insight and several alternative proofs. The first chapter is brief and acts as an introduction, giving the reader motivation for studying this subject and intuition for working in  $\mathbb{R}^p$ . The second chapter provides the necessary background material used to prove the main results that come in the final chapter, in which the hyper-spherical harmonics are introduced as eigenfunctions of the Laplace operator on  $S^{p-1}$  and the corresponding Legendre polynomials as special cases of these spherical harmonics. Results include the orthogonality of the hyper-spherical harmonics and the completeness of this set of functions on  $S^{p-1}$ . Also shown are the  $p$ -dimensional analog of the addition theorem as well as analogs of many other properties that the ordinary Legendre polynomials possess in three dimensions as a set of orthogonal polynomials.

Abstracts for the remainder of this session were received too late to appear in print.

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## MAA Session #13

Room: Thoroughbred 5

4:00P.M. – 6:15P.M.

4:00–4:15

**Permutation Statistics and  $q$ -analogues of the Catalan numbers**

Tim Dwyer

*University of Florida*

We call  $\pi \in S_k$ , where  $S_k$  is the symmetric group, a pattern in  $\sigma \in S_n$  if  $\sigma$  has a subsequence which is in the same relative order as  $\pi$ , we might also say that  $\sigma$  contains  $\pi$ . For instance the permutation 3124 contains 213 as a pattern, since the terms 324 appear in the same relative order, but does not contain the pattern 321. A natural consideration then would be, for any  $\pi \in S_k$ , what  $\sigma \in S_n$  do, or do not, contain  $\pi$ . With this in mind we define  $Av_n(\pi) = \{\sigma : \sigma \text{ does not contain } \pi\}$ . An astonishing result in the field of patterns is that for all  $\pi \in S_3$  we actually have  $|Av_n(\pi)| = C_n$  where the  $C_n$  are the famous Catalan numbers. A function  $st : S_n \rightarrow \mathbb{N}$  is called a statistic or a permutation statistic and we consider generating functions of the form  $f(\pi; q) = \sum_{\sigma \in Av_n(\pi)} q^{st(\sigma)}$ . We will investigate the properties of the above generating function when  $st(\sigma)$  is the inversion number of  $\sigma$  (the number of entries of  $\sigma$  that appear out of their natural order), as a  $q$ -analogue of the Catalan numbers and relate them to various other  $q$ -Catalan numbers which have appeared in the literature.

4:20–4:35

**Selfish Sets and the Fibonacci Sequence**

Van Pham

*Southwestern University*

The Fibonacci sequence is seen in many mathematical problems, areas of art, and in nature. This talk will explore the Fibonacci sequence as it relates to Pascal's triangle and how that can be used to solve a set theory problem from the 1996 Putnam Exam.

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## PME Session #3

Room: Thoroughbred 7

4:00P.M. – 6:15P.M.

4:00–4:15

**The Gibbs Phenomenon on Orthogonal Bases of Polynomials**

Mark Blumstein

*Elmhurst College*

When using an infinite series of orthogonal polynomials to represent a function with a jump in it, the Gibbs overshoot is affected by the value of the weight function used in the inner product. When the value of the weight function is 0 in an  $x$ -neighborhood of the jump, the Gibbs overshoot will be reduced. Any weight function value besides 0 at the jump, will produce the Gibbs overshoot.

4:20–4:35

**The Convex Hull and Its Applications**

Luigi Patruno

*Fordham University*

Convex sets in vector spaces possess certain algebraic properties. A plane (or hyper-plane) divides a space (or hyper-space) into two parts called half-spaces. Every convex set can be characterized as an intersection of half-spaces. Every set of points  $S$  in any vector space is contained in a convex set called the convex hull of  $S$ . The convex hull of  $S$  can be defined as the smallest convex set containing  $S$ , or equivalently, as the intersection of all convex sets that contain  $S$ . Every point in the convex hull can be written as a linear combination of points in  $S$ , with coefficients between 0 and 1, and all totaling 1. Our discussion will consider the convexity of certain equivalence classes of  $n \times n$  positive definite matrices, viewed as points in  $\mathbb{R}^{n^2}$ . We then discuss real applications of the convex hull.

4:40–4:55

**Orbifold Graphs from the Perspective of Spectral Geometry**

Gabriel Montes de Ocs

*Lewis & Clark College*

Spectral geometers ask, “Can you hear the shape of a manifold?” By analogy, spectral graph theorists ask, “Can you hear the shape of a graph?” Inspired by recent advances in spectral geometry of singular spaces known as orbifolds, this project presentation examines the spectral properties of orbifold graphs.

5:00–5:15

**Alpha-Regular Stick Unknots**

Christopher Schafhauser

*University of Wisconsin - Platteville*

A stick knot is a closed loop in  $\mathbb{R}^3$  formed by straight line segments. Such a knot is said to be  $\alpha$ -regular if each segment has length 1 and the angle between any two adjacent sticks is  $\alpha$ . We show there is an  $\alpha$ -regular stick knot with  $n$  sticks and a trivially knot type when  $n \geq 4$  is even and  $0 < \alpha \leq \frac{n-2}{n}\pi$  and when  $n \geq 7$  is odd and  $\frac{\pi}{3} \leq \alpha < \frac{n-2}{n}\pi$ .

5:20–5:35

**Ordered Directness and Efficiently Computing Closures**

Robert Rand

*Yeshiva University*

We analyze three bases of closure systems: The Canonical Basis, Direct Optimal Basis and  $D$ -Basis. We compare the efficiency of each basis in terms of its length and the time necessary to extract all implications. We discuss the concept of “Ordered Directness” and show where it is possible and impossible.

5:40–5:55

**Art Meets Algebra: Exploring Colored Graphs of Groups**

Kelsey Larson

*College of Saint Benedict*

The concept of distance in the reals generalizes to all groups using edge colored complete graphs. We investigate the group of color permuting bijections of the graph of a group. The automorphisms of a group can provide key insights into the group of these color permuting bijections.

## PME Session #4

Room: Thoroughbred 8

4:00P.M. – 6:15P.M.

4:00–4:15

**Some Coefficient Bounds for a New Subclass of Univalent Functions**

James Winegar

*Austin Peay State University*

We investigate coefficient bounds for a new subclass of univalent functions. These functions are analytic, and as such have a Taylor series expansion about every point in the unit disk in  $\mathbb{C}$  of the form  $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ , normalized such that  $f(0) = 0$  and  $f'(0) = 1$ . Our results will generalize known results for several subclasses of functions univalent in the unit disk.

4:20–4:35

**Mathematics Involved in the Game “Shut Box”**

Ryan Hallberg

*St. Norbert College*

In this talk, we will discuss the game “Shut Box”. The game involves two dice and numbered boxes. The dice are rolled simultaneously, and the player must choose a combination of boxes to shut based on the sum of the dice. The rules of the game are simple, but the math involved is challenging due to the randomness of dice rolls and player’s choice. To solve these problems we reduce the game to a smaller variant and propose a fixed strategy. The primary math used are probability trees and Markov chains.

4:40–4:55

**Parameterizing the Koch Curve**

Brian Pietsch

*St. Norbert College*

One of the most famous fractals is the self-similar Koch curve. It is known for having infinite length, and it is generated by infinite iterations of four affine transformations. We use these transformations to create a continuous parameterization of the Koch curve.

5:00–5:15

**Fractals and Their Dimensions**

Hanqin Cai

*St. Norbert College*

Fractals such as the Koch curve and Sierpinski Triangle are self-similar geometric shapes. During the summer of 2011, I participated in a research program at St. Norbert College focusing on fractals. We will begin by introducing fractal dimension which describes how densely the shape occupies space, and use it to compare fractals. Then, I will present some of my findings.

5:20–5:35

**Imagining the Banach-Tarski Paradox**

Rachel Levanger

*University of North Florida*

The Banach-Tarski Paradox says that it is possible to take a solid ball in three-dimensional space, divide it into a finite number of non-overlapping subsets, and then recombine those sets to create two balls with the original size and volume of the first, thus duplicating the sphere. In this presentation, we will examine the precise formulation of duplicative paradoxes, and then follow the development of the Banach-Tarski Paradox from the hollow sphere to the solid ball. Through the use of Mathematica throughout the presentation, we will provide a visual interpretation of how such a decomposition is possible, aiding in developing an intuition for the sets involved.

5:40–5:55

**Differential Forms and the Generalized Stokes' Theorem**

Erin Compaan

*University of North Florida*

In this talk, the theory of differential forms will be used to expose a unity among several fundamental theorems of calculus. Basic properties of differential forms are discussed, and then used to state the Generalized Stokes' Theorem. This broad theorem reveals a symmetry between a number of familiar, but apparently dissimilar, theorems of two- and three-dimensional analysis.

## MAA Session #15

Room: Thoroughbred 2

8:30A.M. – 10:45A.M.

8:30–8:45

**Minimum Rank of Certain Graph Classes**

Gretchen E. Schillemat

*Brigham Young University*

Given a symmetric or Hermitian matrix  $A = [a_{ij}]$ , we associated a simple undirected graph  $G(A)$  with vertex set  $V = \{1, \dots, n\}$  and edge set  $E = \{ij : a_{ij} \neq 0, 1 \leq i < j \leq n\}$ . The diagonal entries of  $A$  have no effect on the structure of  $G(A)$ . The minimum rank of a graph is the smallest possible rank of any real symmetric matrix associated to the given graph. The real (complex) minimum semidefinite rank of a graph is the minimum rank among symmetric (Hermitian) positive semi-definite matrices associated to the given graph. In this talk, we will present results on the minimum rank and minimum semidefinite rank of certain classes of circulant graphs. In particular, we will provide bounds for these graph parameters arising from the zero forcing number or OS number of a graph.

8:50–9:05

**A Geometric Approach to Eigenvectors and Eigenvalues in the Octonions**

Jenifer Herdan

*Winona State University*

In linear algebra, we learn that the linear transformations of matrices can be understood geometrically. They can do rotations, reflections, and stretch/shrink. In this talk, I'll discuss the geometry of the transformations of a class of  $3 \times 3$  matrices over the octonions, the largest of the normed division algebras. I'll also show how the eigenvalues and eigenvectors of the class behave geometrically, and discuss some interesting consequences of these results, including how such transformations violate the normal properties of linear transformations on vector spaces.

9:10–9:25

**Laplace Transforms and Exponential Matrices**

Marianne Dubinsky

*University of Mary Washington*

The exponential function plays a major role in Calculus and differential equations. More generally, matrix exponentials have wide applications - they are easily computed for certain square matrices. For others, the Laplace transform plays a key role. We will discuss how this is done, and what benefits ensue from knowing the exponential of a given matrix.

9:30–9:45

**Distance to Uncontrollability with Hermitian Matrices**

Eugene D. Cody

*University of Kansas*

Controllability is a concept that plays a fundamental role in systems and control. If a system,  $(A, b)$ , where  $A$  is a square matrix and  $b$  is a column vector, is controllable, how large a perturbation is necessary so that the resulting system is uncontrollable? This can algebraically be expressed by the distance to uncontrollability, which is a minimization problem over the complex field. We consider the distance problem with a special case when the matrix  $A$  is Hermitian. In this case, the system  $(A, b)$  is reduced to the pair  $(\Lambda, z)$ , where  $\Lambda$  is a real diagonal matrix. By using the real diagonal matrix structure we prove that when  $A$  is Hermitian, the search field for the minimization problem of the distance to uncontrollability is reduced to the real field. We observe the behavior of the secular equation and study the relationship between  $\Lambda$  and  $z$  to determine the minimizer using a combination of two methods.

9:50–10:05

**Mutually Orthogonal Sudoku Latin Squares**

Nathaniel Lawton Coursey

*Kennesaw State University*

A Latin square of order  $n$  is an  $n \times n$  matrix where each row and column is a permutation of the integers  $1, 2, \dots, n$ . Two Latin squares  $A$  and  $B$ , both of order  $n$ , are orthogonal if all  $n^2$  ordered pairs formed by juxtaposing the two matrices are unique. It is well known that there exists a pair of orthogonal Latin squares of order  $n$  for every positive integer  $n \neq 2, 6$ . A family of mutually orthogonal Latin squares (MOLS) of order  $n$  is a collection of Latin squares of order  $n$  such that each Latin square in the collection is orthogonal to every other Latin square in the collection. It is relatively easy to show that the maximum size of a collection of MOLS of order  $n$  is  $n - 1$ . A gerechte design is an  $n \times n$  matrix where the matrix is partitioned in  $n$  regions  $S_1, S_2, \dots, S_n$  where each row, column and region is a permutation of the integers  $1, 2, \dots, n$ . The popular puzzle Sudoku is an example of a gerechte design. Results about mutually orthogonal Sudoku Latin squares of order  $n = k^2$  are beginning to appear in journals. This talk discusses the adjustments that must be made when  $n$  is not a perfect square and the size of critical sets (clues) of mutually orthogonal Sudoku Latin squares.

10:10–10:25

**Into the Wild 8-dimensional Yonder: Eigenvectors and Eigenvalues in the Octonions**

Sam Roderick Helmich

*Winona State University*

Eigenvectors and eigenvalues in the Reals ( $\mathbb{R}$ ) and the Complexes ( $\mathbb{C}$ ) are very well known, their properties are well documented and their derivation and use has become little more than trivial. However, when we move up into higher normed division algebras (like the Quaternions ( $\mathbb{H}$ ) and the Octonions ( $\mathbb{O}$ )), methods that were consistent across the  $\mathbb{R}$  and  $\mathbb{C}$  do not always generalize to  $\mathbb{H}$  and  $\mathbb{O}$  cases. In this talk, we will investigate how to start looking into these new cases, and offer a method to proceed in further research.

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## MAA Session #16

Room: Thoroughbred 3

8:30A.M. – 10:45A.M.

8:30–8:45

**The Role of Dynamic Representations in Development of Algebraic Concepts**

Marie Nicole Ermete, Natasha Brackett, and Karli N. Powell

*Central Michigan University*

One of the goals of mathematics education is to allow students to understand mathematical ideas from various perspectives. The ability to move among representations allows the student to become better problem-solvers. Based on the symbolic mediation model proposed by Kaput, Blanton, and Moreno (2008), this talk lays out a theoretical framework for the use of dynamically-connected representations to help students develop algebraic concepts. In addition, research on the use of data collection devices suggests that real-time connection of representations is essential for students to make linkages among various aspects of the concepts (Lapp, 2000). This talk discusses the use of technology that dynamically links multiple representations as a tool so that the learner can make connections of algebraic concepts. This particular study is situated in a College Algebra class where students had access to both a computer algebra system (CAS) and dynamically linked representations. Results regarding both student learning and teacher change from two semesters (one using a basic graphing calculator and the other using CAS) will be shared. Directions for future research will also be discussed.

8:50–9:05

**Geometric Problems Associated with Families of Cubic Polynomials**

Amal DeAlwis

*Southeastern Louisiana University*

In this paper, we first investigated a special property of tangent lines associated with cubic polynomials that have distinct, real zeros. We were able to generalize this property to any polynomial with distinct, real zeros. We also considered a certain family of cubics with two fixed zeros and one variable zero, and explored the loci of special points of triangles associated with the family. Special points include the centroid, circumcenter, and orthocenter of the triangle. Some fascinating connections were observed between the original family of the cubics and the loci of these special points of the triangles. For example, we were able to prove that the locus of the centroid of a certain triangle associated with the family of cubics is another cubic whose zeros are in arithmetic progression. Motivated by this, in the latter section of the paper, we obtained several results of polynomials whose zeros are in arithmetic progression. Throughout the paper, we used the computer algebra system, Mathematica to form conjectures and facilitate calculations. Mathematica was also used to create various animations to explore and illustrate many of the results.

9:10–9:25

**Determining the Ideal Class Size**

Caitlin Bryant and Jennifer Davis

*University of North Carolina - Asheville*

After the first 40 days of school in the state of North Carolina, the current class size for K-3 is mandated to be no larger than 24 students. For grades 4-12, however, there is no maximum enrollment. It is also becoming increasingly common for students at different levels and abilities, both disabled and non-disabled, to be integrated into the same classroom. In this study, we surveyed teachers and teaching licensure students across the state to investigate where improvements can be made when determining class size, and how to appropriately accommodate the many different types of students. These results are used to develop a model for determining the ideal classroom size, taking into account the different needs of many of our students, which can provide a framework for proper allocation of resources in North Carolina schools.

9:30–9:45

**Geometry of Surfaces with Densities**

Miguel A. Fernandez Flores

*Truman State University*

Perelman's stunning proof of the million-dollar Poincaré Conjecture needed to consider not just manifolds but "manifolds with density" (like the density in physics you integrate to compute mass). We explore some basic geometry of such spaces.

9:50–10:05

**Using Topology to Protect a Forest**

Evan Shirley and Ibrahim Jadoon

*Centre College*

To safeguard a forest against fire, we can enclose the area within a sensor fence and spread a large number of cheap sensors inside. Each sensor senses fire within its own vicinity and broadcasts signals to a central facility. How do we guarantee that no spot in the forest evades detection? We show that the answer is topological and elementary.

10:10–10:25

**Cutting Topological Models**

Natalie Radziejewski

*Augustana College*

Taking a cue from Stephen Barr's Experiments in Topology, we will construct paper models of cylinders and Möbius strips. We will tape the figures together in various combinations and cut them down the middle. Paper, tape, and scissors will be provided for a hands-on and interactive experience.

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## MAA Session #17

Room: Thoroughbred 5

8:30A.M. – 10:45A.M.

8:30–8:45

**Modeling the DNA Unknotting Action of Type II Topoisomerases**

Kevin He

*University of California, Berkeley*

In nature, circular DNA molecules often become knotted or linked. Type II topoisomerases are enzymes that remove these undesirable topological forms by passing one segment of double-stranded DNA through another (“strand-passage”). The enzymes are believed to preferentially target certain 3-dimensional features on the DNA molecule to increase the efficiency of unknotting. In our study, we model circular DNA as a self-avoiding polygon in the simple cubic lattice and use Monte Carlo methods to sample polygons of each knot type. We simulate strand-passage on our polygons and compare the efficiency of targeting different local topological features. We find that preferential targeting of clasped configurations in our model leads to both a significant increase in one-step unknotting probabilities and a decrease in the proportion of knots at the steady-state. Our study produces one-step unknotting probabilities highly comparable to results of other groups working on modeling DNA unknotting (such as Chan et al. 2006). At the same time, our knot-centric rather than juxtaposition-centric approach to modeling allows investigation into more knot types and unbiased sampling of initial knot populations.

8:50–9:05

**Coloring Knots and Tangles**

Amelia Brown, Mike Comer, and Jes Toyne

*Simpson College*

A knot or tangle is tricolorable if it has a diagram whose arcs can be labeled with three colors such that the colors at any given crossing are all the same or all different. This colorability invariant was introduced by Fox in the 1950s and he generalized the invariant to use any number of colors. We will discuss a technique for determining the colorability of knots and tangles that uses matrices. We will also explain how this technique leads to two other invariants, one using a determinant and the other a submatrix.

9:10–9:25

**Can Iterative Voting Solve the Separability Problem in Referendum Elections?**

Clark Bowman and Ada Yu

*University of Rochester and University of Puget Sound*

In referendum elections, voters are often required to register simultaneous votes on multiple proposals. The separability problem occurs when a voter’s preferred outcome on one proposal depends on the known or predicted outcomes of other proposals. This type of interdependence can lead to unsatisfactory or even paradoxical election outcomes, such as a winning outcome that is the last choice of every voter. Here we will propose an iterative voting scheme that allows voters to revise their voting strategies based on the outcomes of previous iterations. Using computer simulation and tools from game theory, we will investigate the potential of this approach to solve the separability problem. This research was conducted as part of the 2011 REU program at Grand Valley State University.

9:30–9:45

**Fair Division with Budget Constraints**

Chaim Hodges

*Goshen College*

Divisible goods are to be allocated fairly among two or more players. For example, Antonio and Beth are to share the gift of a chocolate bar. Different players value the goods differently. Perhaps Antonio thinks the chocolate bar is worth \$3.00 while Beth thinks it is only worth \$1.90. Players are willing to exchange money to obtain a better allocation: Antonio taking the whole chocolate bar and giving Beth \$1.20 is more valuable to both than simply splitting the chocolate bar in half. Nonetheless, players have budget constraints: Antonio may be unwilling to spend more than \$0.50. In this situation, what is a “fair” allocation? We investigate allocations that are efficient (there is no way to simultaneously improve the allocation for everyone), envy-free (no one would want to trade portions with another), or satisfy other fairness properties.

9:50–10:05

**Wavelet-Based Quantum Computing Algorithm for Face Recognition**

Sami Eid Merhi, Pierce O’Donnell, and Yunhan Jing

*Western Connecticut St. Univ., Western Connecticut St. Univ., and High School Affiliate of Jilin Univ.*

As security needs grow, the capability of positively recognizing individuals becomes a necessity. In an attempt to solve identity issues, researchers have devised numerous algorithms. Among these, we have seen: signatures, fingerprints, retinal scans, voiceprint, and many others. However, face recognition is arguably the most effective mode of identification. Biomedical research has shown that human beings often recognize one another through facial characteristics. In as early as the late nineteenth century, researchers have tried to identify dominant facial features through automatic methods of classification. Popular recognition algorithms include Principal Component Analysis with eigenface, Linear Discriminate Analysis, Elastic Bunch Graph Matching fisherface, the Hidden Markov model, and the neuronal motivated dynamic link matching. In facial recognition, disc space is often an issue, especially when dealing with a large population. Wavelets will not only represent faces as primary and secondary features, but also compress data dramatically and thus save memory. In this paper, we try to identify the most efficient of the current wavelet transforms in order to satisfy both of the previous requirements. When using traditional methods in face recognition, success rates decrease rapidly as the orientation of the face or the image size change. This can be overcome by combining Hidden Markov Models with Wavelet Analysis. In addition, Quantum Computing can be employed to achieve better face detection when combined with Wavelet transforms, as well as achieving higher compression ratios.

10:10–10:25

**Smartphone Sensors and Wavelets**

Nathan Marculis and SaraJane Parsons

*Grand Valley State University and Indiana University of Pennsylvania*

Data logs from accelerometers in iPads and smartphones are a source of noisy, spiky data. This data can be collected while driving on bumpy roads or from other dynamic activities. This talk will discuss analyses of this data using wavelets and other mathematical tools. This research was conducted as part of the 2011 REU program at Grand Valley State University, with Prof. Edward Aboufadel.

10:30–10:45

***M*-band Wavelet Based Image Compression with Quantum Computing**

Sam Zeng, Ray L. Chen, and Yunhan Jing

*Danbury Math Academy, Danbury Math Academy, and High School Affiliate of Jilin University*

In recent years, we have observed a massive expansion in wavelet analysis to fit our demanding need for faster and more accurate signal processing. Different types of wavelets have been developed to solve problems in signal and image analysis, medical diagnostics, statistical analysis, pattern recognition, and many other problems. In the age informatics, compressions are essential to the rapid transfer of information. In this research, we explore data compression by focusing on image compression. We utilize *M*-band wavelet transforms to compact the image and preserve the energy. However, conventional information theory fails to properly describe how information can be represented and transformed in corresponding physical systems. To solve this problem, we can apply ideas from quantum mechanics to create a quantum algorithm that will allow for faster and more efficient data compression. We perform a wavelet transform of the signal and then set all values less than a predetermined threshold equal to zero. We then apply quantum arithmetic coding to the transformed data. To decompress, we decode the values and then apply the inverse wavelet transform.

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## PME Session #5

Room: Thoroughbred 7

8:30A.M. – 10:25A.M.

8:30–8:45

**The Impact of Climate Change on the Power Grid in Rochester, New York**

Scott Constable

*Ithaca College*

It is well documented that power consumption in the US increases drastically when summer temperatures spike. As more and more people resort to air conditioning to thwart the heat, the strain on the power grid can become overwhelming, occasionally leading to blackouts. Already, this is a serious issue in some areas of the US. In New York State, however, massive blackouts resulting from excessively hot weather are much less frequent, and are generally on a smaller scale. Yet as average temperatures climb at an unprecedented rate, how will this affect the likelihood of blackouts in New York State in both the near and distant future? This study employs an extensive weather simulation model, NYISO power consumption data, and concepts of extreme value theory to show that indeed, measures must be taken to mitigate the impact of climate change.

8:50–9:05

**From Primes to Polygons**

Brian Harrison II

*John Carroll University*

While I was trying to discover my own proof that there are infinitely many primes, I noticed a pattern in the multiplication table. This pattern is that if a rhombus with vertical and horizontal diagonals is overlaid on the table, the sum of the values along one diagonal is equal to the sum of those along the other diagonal. I then discovered that this pattern generalizes to a property algebraically relating the center of mass of certain polygons in the plane to the coordinates of that polygons vertices. I will present this property and show it is true for any regular polygon in the Cartesian plane.

9:10–9:25

**Using Eigenvalues and Eigenvectors to Produce Accurate Measurements**

Daniel Catello

*Youngstown State University*

The Analytic Hierarchy Process (AHP), developed by Thomas Saaty, has proven to be a reliable method for making multicriteria decisions. By calculating the principal eigenvalue and its corresponding eigenvector of sets of comparison matrices, the AHP produces a vector, whose components help the decision maker choose the “best” course of action. Using these concepts, it is possible to produce accurate estimates of areas, volumes, and other parameters of physical objects. In this interactive presentation, the method will be introduced and the audience will also produce some strikingly accurate measurements by using a simple seven point comparison scale.

9:30–9:45

**Theoretical Framework and Analysis of Time-to-Peak Response in Biological Systems**

Sepideh Khavari

*Youngstown State University*

Living organisms are constantly exchanging information with their environment. The process of sending and receiving signals in response to environmental changes such as fluctuations in concentration of hormones, neurotransmitters, growth factors, and drugs is a part of a complex network of cellular communication. We present a theoretical framework that first examines a simple reaction,  $A \xrightarrow{f(t)} B$  with a regulating function  $f(t)$ , to establish various properties of the response variable  $B(t)$ . The qualitative properties of  $B(t)$  include its non-negativity, existence for all time, and the presence of a unique maximum. We use ordinary differential equation and calculus-based proofs to establish these and other properties. Furthermore, we extend this theoretical framework to incorporate more complex signaling networks including feedback and degradation. This work is based on the recent publication by Theis et al., *Bull Math Biol* (2011) 73: 978-1003 which we extend to include nonlinear features.

9:50–10:05

**Differential Equation Model of the Metabolism of Toxic Metals in Bacteria**

Tara Sansom

*Youngstown State University*

A strain of bacteria, *S. Maltophilia* O2, reduces toxic selenite to nontoxic selenium. A coupled system of differential equations models the cell growth and selenite reduction within the cells. Experimental results show that the growth rate and carrying capacity depend on selenite concentration and are accounted for in the model. This compartment model not only models the bacterial growth but also predicts the concentration of the various metabolic constituents throughout the growth cycle.

10:10–10:25

**Mathematical Model of Acetone-Butanol-Ethanol Fermentation by *Clostridium beijerinckii***

Bradley Slabe

*Youngstown State University*

Using Michaelis-Menten relations and the metabolic pathways for glucose and xylose, we model the Acetone-Butanol-Ethanol (ABE) fermentation by *C. beijerinckii* with a system of differential equations. Enzyme kinetics including competition, inhibition and final product states are examined. Comparisons are made between the butanol production obtained from various carbon sources. Experimental data and results of the mathematical model are also analyzed and compared.

## PME Session #6

Room: Thoroughbred 8

8:30A.M. – 10:25A.M.

8:30–8:45

**Proof and Application of Leibniz's Formula**

Matt Alexander

*Youngstown State University*

We will prove and use Leibniz's Formula to then prove the following identity:

$$|GA| \cdot |GB| \cdot |GC| (|GA| \sin \alpha + |GB| \sin \beta + |GC| \sin \gamma) = 2S (R^2 - |GO|^2),$$

given in problem 11550 from the American Mathematical Monthly. Where  $G$  is a point inside triangle  $ABC$ .  $\alpha, \beta, \gamma$  are the radian measures of angles  $BGC, CGA, AGB$ , and  $O, R, S$  are the triangle's circumcenter, circumradius, and area, respectively.

8:50–9:05

**Fuzzy Sets and Fuzzy Groups with Partial Order**

Kelvin Guilbault

*North Central College*

Traditionally, fuzzy sets grade the membership of their elements using the unit interval, imposing total order on elements' membership in the set. In this talk, I discuss how we can use lattices to create partially ordered fuzzy sets, and explore how these sets can be used to create fuzzy groups.

9:10–9:25

**Exterior Algebra and the Maxwell-Boltzmann Equations**

Joseph Ferrara

*University of North Florida*

Starting with the properties and definitions that form exterior algebra we lay the foundation to solve and prove some abstract problems. We then realize the application exterior algebra has to physical problems, and apply these methods to the Maxwell-Boltzmann equations.

9:30–9:45

**An Exploration of the Group Structure on Elliptic Curves**

William Olsen

*University of North Florida*

The study of elliptic curves has yielded numerous advances in mathematical fields such as algebraic geometry, number theory, cryptography, and complex analysis. One main cause of this diversity is the unique group structure that can be defined on an elliptic curve. In this presentation the group structure of elliptic curves will be defined, a tour of examples that showcase the uniqueness of the group structure will be provided, and relevant theorems and conjectures will be exposed. Theorems and conjectures of particular interest will be Mordell's theorem on the finitely generated group structure of elliptic curves, Mazurs torsion subgroup theorem, and the Birch Swinnerton-Dyer Conjecture. The purpose of the exposition is to reveal the interesting and peculiar fundamentals of elliptic curve theory.

9:50–10:05

**Cayley-Sudoku Tables and Loop Theory**

Kady Hossner

*Western Oregon University*

A Cayley-Sudoku Table is a Cayley Table that also has smaller subrectangles that contain every element of the group exactly once. One restriction needed to form a “Cayley-Sudoku Table” leads to loop theory and a 1939 theorem of R. Baer.

10:10–10:25

**Non-Commutative Algebras Generated by  $n$ -Vertex Paths**

Gary R. Engler

*William Paterson University of New Jersey*

The talk will include an introduction to the Diamond Lemma for Ring Theory. The method will be applied to finding the Hilbert Series for an algebra generated by the  $n$ -vertex path.

## MAA Session #19

Room: Thoroughbred 2

2:00P.M. – 3:55P.M.

2:00–2:15

**Ratio Test and Its Generalizations**

Donald J. Sanfilippo

*Penn State University*

Given an infinite series  $S_{a_n}$ , the Ratio Test, to test the convergence and divergence of the series, is as follows: Consider  $\lim \left\| \frac{a_n}{a_{n+1}} \right\| = L$  as  $n$  approaches infinity. If  $L < 1$ , then the series  $S_{a_n}$  converges absolutely; if  $L > 1$ , then the series  $S_{a_n}$  diverges; and if  $L = 1$ , then the test is inconclusive.

In this talk we will provide various generalizations of ratio test; one is using the concepts of  $\lim \sup$  and  $\lim \inf$ , and another using Raabe's Test when ratio test fails. We will further discuss the situation when Raabe's Test fails and provide a generalized version of Raabe's Test. Finally, we will try to provide various examples of infinite series explaining different situations under which these test applies.

2:20–2:35

**Sizing Up The Rationals**

Nathan Yandala Bishop

*St. Olaf College*

It's generally accepted that Lebesgue measure is the best way to measure subsets of  $\mathbb{R}$  because, when applied to any real interval, it agrees with an interval's length. But what if we wanted to measure subsets of  $\mathbb{Q}$ ? Lebesgue measure proves to be unhelpful, so instead, we use density to study subsets of  $\mathbb{Q}$  and find that an interval's density is represented by its length.

2:40–2:55

**Applications of the Laplace Transform**

Catherine Rose O'Doherty

*University of Mary Washington*

We explore applications of the Laplace transform in analysis, probability, and differential equations. We note how the construction and properties of some important functions flow naturally from determining the transforms of certain functions.

3:00–3:15

**Adding Harmony to the Harmonic Series**

Sarah Ritchey

*Youngstown State University*

It is well known that the harmonic series diverges. The question as how this series may be "thinned" so that the resulting series converges has been the subject of continuing research. For example, in 1914, Kempner showed that the series determined by removing every term from the harmonic series whose denominators contain the digit 9 is a convergent series, while others have shown that the series that results from removing all prime denominators remains divergent. This presentation will analyze the series named above, explore other series that can be created by the thinning process, and introduce a startling consequence of these results.

3:20–3:35

**On Legendre Multiplier Sequences**

Kelly Blakeman and Emily Davis

*Loyola Marymount University*

Let  $B = b_k(x)$  be a simple set of polynomials. A sequence of real numbers  $G = g_k$  is called a  $B$ -multiplier sequence if given any polynomial  $p(x) = a_n b_n(x) + \cdots + a_1 b_1(x) + a_0 b_0$  with only real zeros, the polynomial  $G[p(x)] = a_n g_n b_n(x) + \cdots + a_1 g_1 b_1(x) + a_0 g_0 b_0$  also has only real zeros. Such sequences have been completely characterized in case  $B$  is the standard basis (Polya-Schur, 1914), or the set of Hermite polynomials (Piotrowski, 2007). Good progress has been made recently in the case when  $B$  is the set of generalized Laguerre polynomials (Forgacs and Piotrowski, 2011) although the characterization of generalized Laguerre multiplier sequences is still incomplete. In this talk we describe multiplier sequences for the set of Legendre polynomials, and discuss how Legendre multiplier sequences are related to the classical, Hermite and Laguerre multiplier sequences.

3:40–3:55

**Optimization of Fuel Use**

Sajan Shrestha

*Benedict College*

In present world of Scarcity, everything should be used in an optimal way that is utilizing minimum resources to get best possible benefit. Among the bunch of scarce resources, the resource that has caught the most attention of the whole World is the fuel resource. In this study I was looking for how to optimize the fuel use. Application of Derivative was used as a method.

## MAA Session #20

Room: Thoroughbred 3

2:00P.M. – 3:55P.M.

2:00–2:15

**Exploration of Cellular Membrane Potential via Mathematical Formulations**

Ashley Dawne Anderson

*Winona State University*

Electrically active cells play a vital role in maintaining life. Within a living organism they are able to transmit a signal to a certain destination to create a biological reaction, such as transmitting a neurological signal or creating a heartbeat. The activity of these types of electrically active cells can be measured through their membrane potential, which is the difference between the intracellular and extracellular voltages. There are two primary types of membrane potential: resting and active membrane potential. The resting membrane potential is the voltage value of a cell when it is at rest and can be measured with the Nernst equation or the Goldman-Hodgkin-Katz equation, depending on the number of ions present. While the active potential is the voltage value for a cell when it is in action and can be measured with the Hodgkin-Huxley equation. This presentation will explore cell membrane potential through the analysis of several key equations.

2:20–2:35

**Dynamics of Escherichia Coli O157:H7 Influenced by Variations in Pathogen Shedding**

Felix Yu

*Texas A&M University*

*E. coli* O157:H7 is a strain of the *E. coli* bacteria that causes severe food-borne illness in livestock and in humans. While this strain can infect humans in a variety of ways, most people become infected after consuming improperly-prepared food. It is assumed that the bacterium has three shedding stages: continuous, latent and intermittent shedding. Whether these stages are mechanisms employed by the bacterium to maximize transmissibility fitness is unknown. We constructed two models to explore the inclusion and exclusion of additional shedding stages. We used ordinary differential equations to represent these two systems. Using the next generation matrix method, we obtained a threshold parameter known as  $R_0$  that signifies whether additional shedding habits are advantageous to the bacterium. We found that the inclusion of latent and intermittent shedding to the continuous shedding yields a disadvantage for *E. coli* O157:H7. Based on the currently estimated parameter values, this result was confirmed by various numerical simulations.

2:40–2:55

**The Relative Impact of Natural Killer and CD8+ T cell**

Yao Messan

*North Carolina A&T State University*

Possible therapeutic interventions in CML include stimulation of natural killer (NK) production and activation of CD8+ T cells as ways to slow down tumor growth. A basic model has been developed, which describes tumor-immune interactions, and focuses on the role of NK and CD8+ T cells in CML tumor surveillance, with the goal of understanding the dynamics of immune-mediated tumor rejection. The three-population model describes tumor-immune cell interactions with Michaelis-Menten dynamics, and incorporates functions describing tumor-immune growth, response, and various interaction rates. Students will discuss various numerical integration techniques for its integration, comparing at least one Euler and one nonstandard finite difference scheme, and design an experiment to use the model to generate numerical data to quantify and analyze the relative impacts of NK cells and CD8+ T cells on tumor growth.

3:00–3:15

**Applications of Image Decomposition in the Description of New Species**

Annie Murphy

*Clarkson University*

When new species are described, much attention is given to the analysis and description of their morphological, or physical, characteristics. These differences may be too subtle to easily be distinguished visually, as in the case of *Calotes versicolor*, a lizard found throughout Southeast Asia. DNA sequencing evidence suggests that the versicolor species may be comprised of multiple distinct species, however this evidence is not conclusive. We used image decomposition and the analysis of the separated texture to extract and analyze data about the lizards' scale structure. This information, coupled with a statistical analysis of our own morphological data was used to determine whether the findings substantiated the claim of the existence of multiple versicolor species presented by the DNA evidence.

3:20–3:35

**Investigation of Second Generation Wavelets**

Karleigh Cameron, Michael James Gustin, John Holden, and Stacy Siereveld

*Central Michigan University*

It is well known that any element in  $L_2$  has a basis expansion. Because of its localization property and fast transform algorithms, a wavelet basis expansion has many applications including speech, images, video, graphics, and engineering. For an infinite or periodic function a traditional wavelet basis works well. However, in many applications the domain of a function is not infinite and functions are not periodic. The need for improvements of wavelet bases introduces the second generation wavelets. We study properties of a lifting operator that serves as a tool to construct these second generation wavelets.

## MAA Session #21

Room: Thoroughbred 5

2:00P.M. – 3:55P.M.

2:00–2:15

**Counting the Number of Ways to Draw  $K_{2,n}$** 

Yonghyun Song

*Hamilton College*

Given a complete bipartite graph  $K_{2,n}$ , its geometric realizations are drawings of this graph in the plane using only straight lines for edges. Two geometric realizations are same if one can be obtained from the other by shifting the vertices around without gaining or losing edge crossings. In recent research, a correspondence between permutations of  $S_n$  and geometric realizations of  $K_{2,n}$  allowed a combinatorial perspective on this subject, leading to an algorithm for computing the number of different realizations of  $K_{2,n}$ . This talk proposes a recursive approach by dividing the realizations of  $K_{2,n}$  into those that are decomposable into realizations of complete bipartite graphs with fewer vertices and those that are indecomposable. We present techniques for identifying and counting the indecomposable realizations.

2:20–2:35

**On Induced Planar Graphs**

Timothy Hayes

*Drexel University*

The following will determine whether a graph is an induced planar graph. Consider an arrangement  $A$  of  $n$  lines. All of which may be parallel or concurrent, but not coincident with another. The lines divide the plane into regions. Let a line be broken up into segments dependent on where it intersects other lines. Two regions are said to be adjacent if there is a segment between them. Let  $G_p(A)$  be an induced planar graph to be a graph induced by a line arrangement  $A$  in which every planar region is represented by a distinct vertex. Two distinct vertices will be joined by an edge if and only if the corresponding regions are adjacent. Consider  $G$ , a graph drawing of a random graph. This paper will establish requirements on  $G$ , such that if  $G$  fulfills these requirements, there is an  $A$ , which is also determined, such that  $G_p(A)$  is a stretched out drawing of  $G$ . If there is enough time, an effort to discuss what a graph would have to be like, not a picture of a graph, to be an induced planar graph.

2:40–2:55

**Finding Ramsey Numbers**

Mike Henry, Heather Malbon, and Max Nguyen

*Simpson College*

The Ramsey number,  $R(m, n)$ , is the minimum number of party guests required to have  $m$  guests who all know each other or  $n$  guests who have never met. Surprisingly, very few Ramsey numbers have been found. We will explain how graph theory can be used to study Ramsey numbers and give an overview of which Ramsey numbers are known. We will also share our attempts to find new results through computer programs that use genetic algorithms.

3:00–3:15

**A Graph Theoretical Analysis of the Path-Goal Theory of Leadership**

Matthew Allinder

*DIMACS - Rutgers University*

Using graph theoretical techniques we analyze the effectiveness of the path-goal theory of leadership when applied to the offense of a theoretical football team. A signed digraph is set up based on the relationships between the head coach, offensive coordinator, quarterback, offensive linemen, offensive receivers, and offensive running backs of the team. The signs in the digraph are determined by whether or not the behaviors prescribed by the path-goal theory are acted upon by the leaders of the team. The pulse process, originally developed by Dr. Fred Roberts, is then used to determine how effective the team could be under the various behaviors exhibited by the leaders. Additionally, the connectivity of the team is analyzed to determine a minimum vertex-cut and which arc is the most critical to communication on the team.

3:20–3:35

**Hyperbolic Manifolds as Discretized Configuration Spaces of Graphs**

Michelle Chu

*Emory University*

A discretized configuration space of a graph is a topological space of possible configurations of particles in which multiple particles are not allowed in neighboring edges. We study the discretized configuration spaces of simple graphs which are manifolds of dimensions equal to the number of particles in the configuration. We classify the simple graphs whose discretized configuration spaces are manifolds of dimensions 2 and 3 with this property. We then describe their discretized configuration spaces through their construction by cube-complexes and triangulations by regular simplices. Finally, we discuss their commensurability.

3:40–3:55

**Quantum Walk on Graphs and Perfect State Transfer**

Rachel Elizabeth Bachman

*Clarkson University*

We study the time evolution of a particle performing a quantum walk on a finite, simple, undirected and connected graph  $G$  with adjacency matrix  $A$  representing the respective edges. A unitary matrix,  $U$ , is defined as  $U = e^{-itA}$  using the Taylor series expansion of the exponential function being applied to the matrix  $-itA$ . More specifically, given two vertices  $x$  and  $y$  of  $G$ , we ask if there is a positive time  $t$  where the  $(x, y)$ -entry of  $U$  has unit magnitude. In this case, we say that  $G$  has the property of “perfect state transfer” (PST) at time  $t$  from vertex  $x$  to vertex  $y$ . The main question of interest is to characterize graphs which have this PST property and to provide constructions of new families of such graphs. A specific focus of our work is on PST graphs with large diameter. This question has attracted considerable interest in the context of quantum information and computation.

## PME Session #7

Room: Thoroughbred 7

2:00P.M. – 3:55P.M.

2:00–2:15

**Using Pictures to Study Mathematical Beliefs and Attitudes in Middle Tennessee**

Stephanie Jessie

*Austin Peay State University*

We study beliefs and attitudes of some middle Tennessee students towards mathematics learning using mathematical pictures drawn by the students themselves. An analysis of these pictures for encoded mathematical beliefs showed in general, that we could divide the belief structure and attitudes of students into categories that were consistent with established categories. However, there emerged a new category, that showed a belief system inconsistent with known categories.

2:20–2:35

**An Extension of the Hiring Problem**

Nicole Harp

*St. Norbert College*

The Hiring Problem, also known as the Secretary Problem, is a classic probability problem that looks at the optimal strategy for hiring the best-qualified candidate from a pool of applicants. It is dependent on the fact that an applicant cannot be revisited once he or she has been interviewed. This talk will take a look at the mathematics associated with a variation of the original problem.

2:40–2:55

**Assessing the Risk of an Oil Spill in the Arctic**

Abigail Wendricks

*St. Norbert College*

Drilling in the arctic region is a topic which has been discussed at length in the American legislature. Current legislation proposes the placement of 20 oil rigs in the Arctic Ocean. We are interested in estimating the probability of a major oil spill if this plan is enacted. We use data on drilling productivity, the frequency and severity of past spills from oil rigs and oil transport to estimate the probability and impact of an oil spill in the Arctic ecosystem.

3:00–3:15

**Learning About Collective Memory from Query Bursts**

Anthony DeCino

*University of California, Los Angeles*

Query logs provide detailed and useful insight about individual users, but they also provide an unprecedented level of detail about the collective behavior of user communities. Quantitative analysis of such collective behavior can inform search engine design, web content provision, and social scientific theory development. This project is specifically interested in collective memory, namely memory that is common to the members of a community and often encoded in social artifacts and public discourse. Event-driven query bursts can be identified even in the highly aggregated, and approximate, Google Trends data. This project compares several possible models for the decay of these bursts through Bayesian model selection and found overwhelming support for asymptotic decay that follows the power law model rather than the exponential model. As some memory of an event is necessary to frame a relevant query, this research interprets the results as the “forgetting functions” of collective memories.

3:20–3:35

**The Euler-Lagrange Equation and Application of Variational Calculus in Physics**

Justin Simpson

*University of North Florida*

In 1900, David Hilbert compiled a list of 23 unsolved problems, two of which focus on the Calculus of Variations. In addition, this topic is applicable to a variety of physical situations, such as the Brachistochrone problem, Lagrangian Mechanics, and geodesics. The intent of this presentation is to introduce a general audience to variational calculus by deriving a fundamental necessary condition known as the Euler-Lagrange Equation and applying to a problem.

3:40–3:55

**Oregon Blackberry Invasion Analyzed by Spatial Stochastic Modeling**

Heather Johnston

*Western Oregon University*

Nonnative species are disruptive to natural ecosystems. The Himalayan Blackberry in Western Oregon is of particular concern due to the massive barriers the plant creates and its stubbornness of eradication. A spatially stochastic model is used to predict the spread of the blackberry in Oregon, taking into account factors such as elevation, annual rainfall, temperature, urban sprawl and black bear ranges. This presentation covers individual research in mathematical ecology done at the Texas A&M Math REU 2011.

## PME Session #8

Room: Thoroughbred 8

2:00P.M. – 3:55P.M.

2:00–2:15

**Nonabelian Groups With Perfect Order Subsets**

Hongying Zhao

*College of Saint Benedict*

Let  $G$  be a finite group. Define an *order subset* of order  $n$  to be the set of all elements in  $G$  that have order  $n$ . A group  $G$  is said to have *perfect order subsets* if the size of every nonempty order subset of  $G$  is a divisor of  $|G|$ ; in this case we say  $G$  is a *POS-group*. A fair amount of research has been done on finite abelian POS-groups, but relatively little work has been done on nonabelian POS-groups. In this talk we will discuss our progress in investigating properties of nonabelian POS-groups.

2:20–2:35

**Abundancy Index Outlaws**

Katherine Moore

*Kenyon College*

The abundancy index function  $I : \mathbb{N} \rightarrow \mathbb{Q} \cap [1, \infty)$  is given by  $I(n) = \frac{\sigma(n)}{n}$ , where  $\sigma(n)$  is the sum of divisors function. Rational numbers greater than one in the range of  $I$  are called *abundancy indices* and those not in the range of  $I$  are called *abundancy outlaws*. In general, it is very difficult to identify a given rational as an index or an outlaw. In this talk we will present a couple of classes of abundancy outlaws and illustrate a way of visualizing the outlaws among the set of rational numbers greater than one.

2:40–2:55

**Cryptography and Quantum Computing**

Mario Sracic

*Youngstown State University*

We will introduce the Sakalauskas, Tvarijonas and Raulynaitis Key Agreement Protocol (STR-KAP) which involves the formulation of a secret key over an infinite, non-abelian group  $\Gamma$  with a representation  $\Gamma \rightarrow GL_d(\mathbb{F}_{p^k})$ , between two communicators, Bob and Alice. An examination of a classical brute force attack on the STR-KAP is presented as well as an analysis of new quantum circuits designed to crack the Protocol. We will compare the complexity of the quantum circuits to classical computing techniques to find if they result in a more efficient means of breaking the STR-KAP.

3:00–3:15

**Novel Approaches for Solving the Collatz Conjecture Using Fractional Set Positions**

Richard Freedman

*Wake Forest University*

The Collatz Conjecture ( $3x + 1$  Problem) states that for any initial positive integer value, the orbits formed by the following discrete dynamical system contain a 1: if the current number  $x$  is odd, then the next number is  $3x + 1$ ; otherwise, the next number is  $\frac{x}{2}$ . This conjecture remains unproven and Paul Erdős once said that “Mathematics is not yet ready for such problems.” At MathFest 2010 we introduced set positions, a coding process that revealed several interesting properties about these orbits. One observation was that infinitely many orbits could be represented by a single sequence of respective fractional set positions which are a subtuple of set positions. This allows us to define a conjecture involving fractional set positions that is equivalent to the Collatz Conjecture. After proving this equivalence, this presentation will consider new methods for proving (or disproving) our conjecture which would also prove (or disprove) the Collatz Conjecture.

3:20–3:35

**Intersecting Hyperbolas and Ellipses**

Kirill Lazebnik

*State University of New York at Genesco*

Given two conics centered at the origin, we give a formula for the area of intersection of their interiors. This formula is invariant under matrix transformations. This calculation is important in finding the number of lattice points within this region, in the spirit of Gauss’ Circle Problem.

3:40–3:55

**Variations on the Collatz Conjecture**

Robert Short

*John Carroll University*

The Collatz Conjecture states that we can obtain 1 by choosing any positive integer  $n$  and iterating the function

$$f(n) = \begin{cases} 3n + 1 & n \text{ odd} \\ \frac{n}{2} & n \text{ even} \end{cases}$$

a sufficient number of times.

About a year ago, I came across the conjecture, and it immediately captured my attention. After many failed attempts at proving the conjecture, I looked at variations of the function in an attempt to find a common property. In this talk, I will describe the Collatz Conjecture and some of my more interesting variations.

## J. Sutherland Frame Lectures

2011	Margaret H. Wright	<i>You Can't Top This: Making Things Better with Mathematics</i>
2010	Nathaniel Dean	<i>Incomprehensibility</i>
2009	Persi Diaconis	<i>The Mathematics of Perfect Shuffles</i>
2008	John H. Conway	<i>The Symmetries of Things</i>
2007	Donald E. Knuth	<i>Negafibonacci Numbers and the Hyperbolic Plane</i>
2006	Donald Saari	<i>Ellipses and Circles? To Understand Voting Problems??!</i>
2005	Arthur T. Benjamin	<i>Proofs that Really Count: The Art of Combinatorial Proof</i>
2004	Joan P. Hutchinson	<i>When Five Colors Suffice</i>
2003	Robert L. Devaney	<i>Chaos Games and Fractal Images</i>
2002	Frank Morgan	<i>Soap Bubbles: Open Problems</i>
2001	Thomas F. Banchoff	<i>Twice as Old, Again, and Other Found Problems</i>
2000	John H. Ewing	<i>The Mathematics of Computers</i>
1999	V. Frederick Rickey	<i>The Creation of the Calculus: Who, What, When, Where, Why</i>
1998	Joseph A. Gallian	<i>Breaking Drivers' License Codes</i>
1997	Philip D. Straffin, Jr.	<i>Excursions in the Geometry of Voting</i>
1996	J. Kevin Colligan	<i>Webs, Sieves and Money</i>
1995	Marjorie Senechal	<i>Tilings as Differential Gratings</i>
1994	Colin Adams	<i>Cheating Your Way to the Knot Merit Badge</i>
1993	George Andrews	<i>Ramanujan for Students</i>
1992	Underwood Dudley	<i>Angle Trisectors</i>
1991	Henry Pollack	<i>Some Mathematics of Baseball</i>
1990	Ronald L. Graham	<i>Combinatorics and Computers</i>
1989	Jean Cronin Scanlon	<i>Entrainment of Frequency</i>
1988	Doris Schattschneider	<i>You Too Can Tile the Conway Way</i>
1987	Clayton W. Dodge	<i>Reflections of a Problems Editor</i>
1986	Paul Halmos	<i>Problems I Cannot Solve</i>
1985	Ernst Snapper	<i>The Philosophy of Mathematics</i>
1984	John L. Kelley	<i>The Concept of Plane Area</i>
1983	Henry Alder	<i>How to Discover and Prove Theorems</i>
1982	Israel Halperin	<i>The Changing Face of Mathematics</i>
1981	E. P. Miles, Jr.	<i>The Beauties of Mathematics</i>
1980	Richard P. Askey	<i>Ramanujan and Some Extensions of the Gamma and Beta Functions</i>
1979	H. Jerome Keisler	<i>Infinitesimals: Where They Come From and What They Can Do</i>
1978	Herbert E. Robbins	<i>The Statistics of Incidents and Accidents</i>
1977	Ivan Niven	<i>Techniques of Solving Extremal Problems</i>
1976	H. S. M. Coxeter	<i>The Pappus Configuration and Its Groups</i>
1975	J. Sutherland Frame	<i>Matrix Functions: A Powerful Tool</i>

Pi Mu Epsilon would like to express its appreciation to the American Mathematical Society, the American Statistical Association, and to the Committee for Undergraduate Research, the Society for Industrial and Applied Mathematics, the SIGMAA-Environmental Mathematics and BioSIGMAA for the sponsorship of the Awards for Outstanding Presentations. It would additionally like to thank the National Security Agency for its continued support of the student program by providing subsistence grants to Pi Mu Epsilon speakers.

## MAA Lectures for Students

2011	Roger Nelson	<i>Math Icons</i>
2010	Sommer Gentry	<i>Faster, Safer, Healthier with Operations Research</i>
2009	Colm Mulcahy	<i>Mathemagic with a Deck of Cards on the Interval Between 5.700439718 and 806581751709438785716606368564037 6697528950544088327782400000000000</i>
2008	Laura Taalman	<i>Sudoku: Questions, Variations and Research</i>
2007	Francis Edward Su	<i>Splitting the Rent: Fairness Problems, Fixed Points, and Fragmented Polytopes</i>
2006	Richard Tapia	<i>Math at Top Speed: Exploring and Breaking Myths in Drag Racing Folklore</i>
2005	Annalisa Crannell & Marc Frantz	<i>Lights, Camera, Freeze!</i>
2004	Mario Martelli	<i>The Secret of Brunelleschi's Cupola</i>
2004	Mark Meerschaert	<i>Fractional Calculus with Applications</i>
2003	Arthur T. Benjamin	<i>The Art of Mental Calculation</i>
2003	Donna L. Beers	<i>What Drives Mathematics and Where is Mathematics Driving Innovation?</i>
2002	Colin Adams	<i>"Blown Away: What Knot to do When Sailing" by Sir Randolph "Skipper" Bacon III</i>
2002	M. Elisabeth Pate-Cornell	<i>Finding and Fixing Systems' Weaknesses: The Art and Science of Engineering Risk Analysis</i>
2001	Rhonda Hatcher	<i>Ranking College Football Teams</i>
2001	Ralph Keeney	<i>Building and Using Mathematical Models to Guide Decision Making</i>
2000	Michael O'Fallon	<i>Attributable Risk Estimation: A Tale of Mathematical/Statistical Modeling</i>
2000	Thomas Banchoff	<i>Interactive Geometry on the Internet</i>
1999	Edward G. Dunne	<i>Pianos and Continued Fractions</i>
1999	Dan Kalman	<i>A Square Pie for the Simpsons and Other Mathematical Diversions</i>
1998	Ross Honsberger	<i>Some Mathematical Morsels</i>
1998	Roger Howe	<i>Some New and Old Results in Euclidean Geometry</i>
1997	Aparna Higgins	<i>Demonic Graphs and Undergraduate Research</i>
1997	Edward Schaefer	<i>When is an Integer the Product of Two and Three Consecutive Integers?</i>
1996	Kenneth Ross	<i>The Mathematics of Card Shuffling</i>
1996	Richard Tapia	<i>Mathematics Education and National Concerns</i>
1995	David Bressoud	<i>Cauchy, Abel, Dirichlet and the Birth of Real Analysis</i>
1995	William Dunham	<i>Newton's (Original) Method - or - Though This Be Method, Yet There is Madness</i>
1994	Gail Nelson	<i>What is Really in the Cantor Set?</i>
1994	Brent Morris	<i>Magic Tricks, Card Shuffling and Dynamic Computer Memories</i>
1993	Richard Guy	<i>The Unity of Combinatorics</i>
1993	Joseph Gallian	<i>Touring a Torus</i>
1992	Peter Hilton	<i>Another Look at Fibonacci and Lucas Numbers</i>
1992	Caroline Mahoney	<i>Contemporary Problems in Graph Theory</i>
1991	Lester Lange	<i>Desirable Scientific Habits of Mind Learned from George Polya</i>

**MAA Committee on Undergraduate Student Activities and Chapters**

Linda Braddy, Chair  
Tarrant County College

Jennifer Bergner  
Salisbury University

Daniel Birmajer  
Nazareth College

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