Abstracts for the MAA Undergraduate Poster Session

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Organized by

Joyati Debnath
Winona State University

Organized by the MAA

Committee on Undergraduate Student Activities and Chapters

and

CUPM Subcommittee on Research by Undergraduates

Dear Students, Advisors, Judges and Colleagues,

If you look around today you will see about 341 posters and 551 presenters, record numbers, once again. It is so rewarding to see this session, which offers such a great opportunity for interaction between students and professional mathematicians, continue to grow.

The judges you see here today are professional mathematicians from institutions around the world. They are advisors, colleagues, new Ph.D.s, and administrators. We have acknowledged many of them in this booklet; however, many judges here volunteered on site. Their support is vital to the success of the session and we thank them.

We are supported financially by the National Science Foundation, Tudor Investments, and Two Sigma. We are also helped by the members of the Committee on Undergraduate Student Activities and Chapters (CUSAC) in some way or other. They are: Jiehua Zhu; Pamela A. Richardson; Jennifer Schaefer; Lisa Marano; Dora C. Ahmadi; Andy Niedermaier; Benjamin Galluzzo; Eve Torrence; Gerard A. Venema; Jennifer Bergner; Jim Walsh; Kristina Cole Garrett; May Mei; Dr. Richard Neal; TJ Hitchman; William J. Higgins; and Zsuzsanna Szaniszlo. There are many details of the poster session that begin with putting out the advertisement in FOCUS, ensuring students have travel money, online submission work properly, and organizing poster boards and tables in the room we are in today that are attributed to Gerard Venema (MAA Associate Secretary), Linda Braddy (MAA), and Penny Pina (AMS).

Our online submission system and technical support is key to managing the ever-growing number of poster entries we receive. Thanks to MAA staff, especially Margaret Maurer and Maia Henley, for their work setting up and managing the system this year. Preparation of the abstract book is a time-consuming task. Thanks to Beverly Ruedi for doing the final production work on the abstract book.

Thank yous go to Angel R. Pineda (California State University, Fullerton), James P. Solazzo (Coastal Carolina University), Rebecca Garcia (Sam Houston State University), and Dora Ahmadi (Moorehead State University) for organizing an orientation for the judges and authoring the judging form. James also helped reviewing several abstracts.

Thanks to all the students, judges, volunteers, and sponsors. I hope you have a wonderful experience at this years poster session!

Joyati Debnath
Winona State University

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Tudor Investment Corporation and Two Sigma.

Their generosity on behalf of the 2016 Undergraduate Student Poster Session enables students to interact with peers and role models in the mathematical sciences during the largest mathematics meeting in the world.

Support for the student travel grant is provided by the National Science Foundation (grant DMS-1341843).

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Titles, Authors, Advisors and Abstracts

1. Gridline Graphs in Higher Dimensions

Jacob Adams Grand Valley State University
Susanna Lange Grand Valley State University
Advisor(s): Dr. Feryal Alayont, Grand Valley State University

A gridline graph is a graph whose vertices can be realized in $\mathbb{R}^2$ in such a way that the vertices are adjacent if and only if they share a coordinate. This definition can be generalized to n-dimensions by realizing the graph in $\mathbb{R}^n$ and letting two vertices be adjacent if and only if they share at least one coordinate. While it is known that in two dimensions the minimal forbidden induced subgraphs are the claw, diamond, and odd holes with 5 or more vertices, there is no complete forbidden subgraph characterization of gridline graphs in higher dimensions. In this presentation, we investigate local minimal forbidden subgraphs, i.e., those minimal forbidden subgraphs induced by a vertex and its neighbors, of gridline graphs in three dimensions and families of forbidden induced subgraph families that exist in a range of dimensions.

2. Life Annuity and Mortality Fracture

Megan Alaimo Western New England University
Advisor(s): Mr. John Willemain, Western New England

If you are lucky enough to win the lottery, have you considered how you might receive your winnings? An annuity, a series of steady equal payments, is frequently used to pay large sums of money, such as lottery winnings and retirement payments. In this talk, we will discuss the major differences between an annuity certain and a life annuity, including what happens if the annuitant passes away and whether or not life expectancy is taken into account in order to calculate the number of payments. We will also consider how the age and health of the annuitant is taken into account and can affect the present value of the annuity. Finally, we will discuss how the mortality fracture changes the present value of a life annuity.

3. Soft-Contact Lens Hydration Modeling

Austin Alderete George Mason University
Advisor(s): Dr. Daniel Anderson, George Mason University

During typical wear, a soft-contact lens undergoes continuous fluctuation in its water concentration, largely in part due to the evaporation of water at the anterior lens surface during the blink cycle. As time progresses, a hydration gradient is formed across the contact lens which leads to dehydration at the cornea. This is a source of the general discomfort wearers experience, and may also lead to dry eye syndrome and lens adhesion. Previous work has shown that the hydration level throughout a one-dimensional cross section of the lens converges to an oscillatory steady state. We present a simpler model, based on the heat equation, which reproduces this phenomenon as well as a means of predicting the steady state.

4. Impact of Calcium Store Overload on Electrical Dynamics of Cardiac Myocytes

Amanda Alexander Western Washington University
Advisor(s): Dr. Bradford E. Peercy, University of Maryland, Baltimore County

Heart disease is the leading cause of mortality in the United States. One cause of heart arrhythmia is calcium mishandling in cardiac muscle cells. We present a mathematical model of the mechanism by which calcium waves propagate through these cells that accounts for changes in the calcium concentration of the SR, the effects of buffers in the SR, and the effects of voltage differences across the cell membrane. The mathematical model is coded in C and run using parallel computing to efficiently generate simulations of the model; Matlab is utilized to create images indicating the calcium concentration throughout a cardiac cell with respect to time. We found that incorporating a dynamic SR calcium concentration causes the flux of calcium through open CRUs to taper off over the duration of the CRU firing, lowering the likelihood of waves to propagate. Likewise, including the effects of calcium buffers in the SR decreases the free calcium concentration, again decreasing the likelihood of waves to propagate. Additionally, voltage-gated channels are utilized to examine the impact of voltage on calcium dynamics, with results indicating that an increased voltage difference across the cell membrane causes more calcium to be released into the cell.

5. Noise-Induced Stabilization of Stochastic Differential Equations

Tony Allen West Virginia University
Emily Gebhardt Mercyhurst University
Adam Kluball Bethany Lutheran College
Advisor(s): Dr. Tiffany Kolba, Valparaiso University

The phenomenon of noise-induced stabilization occurs when an unstable deterministic system of ordinary differential equations is stabilized by the addition of randomness into the system. Noise-induced stabilization is quite an intriguing and surprising phenomenon as one's first intuition is often that noise will only serve to further destabilize the system. In this paper, we investigate under what conditions one-dimensional, autonomous stochastic differential equations are stable, where we take the notion of stability to be that of global stochastic boundedness. Specifically, we find the minimum amount of noise necessary for noise-induced stabilization to occur when the drift and noise coefficients are power, exponential, or logarithmic functions.

6. Randić Connectivity Indices of Graphs Made of a Cycle and Additional Chords

Jhonny Almeida Montclair State University
Advisor(s): Dr. Aihua Li, Montclair State University

In 1975, Milan Randić introduced the product connectivity index, named as Randić Connectivity Index (RCI), on molecular graphs, to study the chemical properties of branching alkanes. Since then RCI became one of the most successful molecular descriptors for structural-property and structural-activity relationship studies. In this research, I investigate RCI values of certain graphs constructed from a cycle and additional chords. We define the set of all graphs made of the cycle C_n with r additional edges as $D(n, r)$. The focus is on the graphs in $D(n, 1)$ or $D(n, 2)$. For these selected graphs, explicit formulas, patterns and properties of the RCI values have been developed. It is shown that all the graphs in $D(n, 1)$ have the same RCI value. Among all the graphs in $D(n, 2)$, the one admitting the maximal (or minimal) RCI is identified. We further investigate graphs in $D(n, r)$ where r is greater than 2. We identify the method of adding an additional edge such that RCI value does not change.

7. Modeling Wave Phenomena using non-autonomous nonlinear Schrodinger Equations

Gabriel Amador University of Puerto Rico Mayaguez campus
Gerardo Mercado University of Puerto Rico Mayaguez campus
Advisor(s): Dr. Erwin Suazo, University of Puerto Rico Mayaguez campus

In this poster we describe the evolution of solutions of non-autonomous nonlinear Schrödinger equation (NLS) with variable coefficients. To construct these equations we use a Riccati system of equations with selected variable coefficients and explicit solutions as well as generalized lens transformations to transform them into explicit solutions for the standard NLS. We provide an application to fiber optics. Finally, we include a list of equations and their solutions.

8. Better Bounds on the Rate of Non-Witnesses of Lucas Pseudoprimes

David Amirault Lexington High School
Advisor(s): Dr. David Corwin, Massachusetts Institute of Technology

Efficient primality testing is fundamental to modern cryptography for the purpose of key generation. Different primality tests may be compared using their runtimes and rates of non-witnesses. With the Lucas primality test, we analyze the frequency of Lucas pseudoprimes using MATLAB. We prove that a composite integer n can be a strong Lucas pseudoprime to at most $\frac{1}{6}$ of parameters P, Q unless n belongs to a short list of exception cases, thus improving the bound from the previous result of $\frac{4}{15}$. We also explore the properties obeyed by such exceptions and how these cases may be handled by an extended version of the Lucas primality test.

9. Existence of k-Normal Elements in Finite Field Extensions

Loren Anderson North Dakota State University
Advisor(s): Dr. Gary Mullen, Pennsylvania State University

The Primitive Normal Basis Theorem is a powerful result in finite field theory, as it guarantees the existence of a primitive normal element in every field extension. These elements have been well studied due to their applications in areas such as cryptography. Huczynska, Mullen, Panario, and Thomson recently extended the definition of normal elements to k-normal elements, and they posed an open problem of determining the existence of elements "nearest" to primitive normal elements in the sense of having high order and low k-normalcy. We begin by proving that multiplying a k-normal element with an $(n-1)$-normal element in $\mathbb{F}_{p^n}$ over $\mathbb{F}_p$ yields another k-normal element. This allows us to answer the open problem for 0-normal and 1-normal elements of second highest order for a certain class of extensions. Our results yield insights into determining explicit formulas for the number of elements of arbitrary order and k-normalcy, most notably in the case of primitive normal elements.

10. Boltzmann-Type Modeling of Two-Dimensional Grain Growth in Polycrystals

Robert Argus George Mason University
Advisor(s): Dr. Emelianenko, George Mason University

We study the mesoscopic behavior of a grain boundary network and propose a novel two-dimensional model describing the evolution of misorientations. The model obtained is able to capture both small grain disappearances and neighbor switching events. The collision rate parameters involved can be estimated numerically from a large-scale simulation. From this we are able to predict steady-state statistics as well as coarsening rates for normal isotropic grain growth.

11. Dynamics of an N-body Crowd Model

Jeremy Ariche Morehouse College
Juan Claramunt University of Cantabria
Advisor(s): Dr. Vadim Zharnitsky, University of Illinois at Urbana-Champaign

Lately, there has been growing concern regarding the proliferation of people across the world. Specifically, under stressful conditions, dangers of stampeding effects in densely populated areas are imminent. However, little is known about the dynamics of crowd behavior. Thus, we seek an appropriate model utilizing computational methods that accurately simulate the dynamics of a panicking crowd and normal crowd activity. Specifically, we will employ a discrete physics-based model to observe crowd density build-ups as well as energy growth. We find that an explicit relation exists between the energy of the system and the configuration of the system. We also find a relationship between density build-ups and the system configuration.

12. Relative Equilibria of Four Vortices with Three Equal Circulations

Timothy Arnold College of the Holy Cross
Brian Menezes College of the Holy Cross
Advisor(s): Dr. Gareth Roberts, College of the Holy Cross

Using equations derived from fluid mechanics, the vortex problem tracks the motion of a point vortex in space. Since these equations account for the circulations of other vortices in the system, the problem's complexity depends on the number of vortices. We consider the relative equilibria (rigid rotations) of the four-vortex problem with three equal circulations and one arbitrary circulation, m. Using a set-up similar to that of the sister four-body case, we investigate both collinear and symmetric kite solutions. Applying methods from computational algebraic geometry enables us to reduce and factor the defining equations in order to simplify the problem. For example, in the collinear case, a 12th degree polynomial with complicated coefficients in terms of m is factored into the product of four cubics; for the kite configurations, level curves are obtained that classify the different types of possible shapes (convex or concave). Letting the mutual distances vary numerically gives rise to different sets of m-values as well as important bifurcations. Our numerical work leads to the discovery of theorems describing the number and types of possible solutions in terms of the parameter m.

13. Clustering Text Data with a Tandem of BIRCH and k-means

Alvaro Arrospide Fletcher University of Maryland, Baltimore County
Advisor(s): Dr. Jacob, Kogan

The process of data clustering consists of grouping data such that elements in each group are more similar to each other than to those in other groups. Data clustering is used in a broad range of disciplines such as information retrieval, bioinformatics, very-large-scale integration, data mining, and image analysis to name just a few. Text data is usually transformed in high dimensional and sparse vectors. High dimensional data clustering is a useful and practical approach for exploratory data mining. A possible approach to clustering high dimensional data is the k-means algorithm. However, k-means requires that an initial partition is supplied. The clustering scheme Balanced Iterative Reducing and Clustering using Hierarchies (BIRCH) requires a single scan of the data set in order to provide such a partition. We sought to determine the results of applying a tandem of BIRCH followed by k-means to the well-known and publicly available Classic3 data set. We implemented BIRCH and k-means in Java and analyzed the quality of the partitioned dataset at each step. Different orderings of the data set were partitioned so as to determine the stability of this clustering scheme. We also constructed confusion matrices to visualize the partitions generated by BIRCH and k-means.

14. Unilateral and Equitransitive Tilings by Equilateral Triangles of n Different Sizes

Morgan Ascanio University of Washington Bothell
Rebekah Aduddell Texas Lutheran University
Adam Deaton University of Texas at Austin
Advisor(s): Dr. Casey Mann, University of Washington Bothell

A tiling of the plane by polygons is said to be *unilateral* if no two equal sides of polygons meet corner to corner and *equitransitive* if any tile can be mapped via a symmetry of the tiling to any other congruent tile. It has been shown that a unilateral and equitransitive (UE) tiling can be made with any number of sizes of squares. We show that there are exactly two UE tilings by equilateral triangles: one with two sizes of triangles and one with three sizes of triangles.

15. Simulation, visualization and control of quadcopters

Alexander Baez Inter American University of Puerto Rico
Advisor(s): Dr. Carmen Caiseda, Padmanabhan Seshaiyer, Inter American University, George Mason University

At any given moment our world encounters challenging events that affect our natural resources, or life. Many companies, in their search for mitigating the impact of these events have invested in the advancement of technology that will allow them to manage these assignments, including more accurate UAVs (unmanned aerial vehicles). With the use of these vehicles wider perspectives and regions of our planet can be efficiently monitored at a safe distance. The moisture level of crops can be measured, water rafts can be delivered to people in distress at sea or an area can be surveyed for suspects at a fraction of the cost of current methods. These devices, being radio controlled or autonomous, have the great benefit of being maneuvered into difficult regions, but also demand faster response time in the job appointed. A collaborative effort that included probabilistic search and engineering lead us to work on the improvement of UAV's performance while airborne. We have undergone the task of investigating the flight dynamics of this machine. We have implemented and solved the Newton-Euler equations of quadcopter flight using various numerical methods by MATLAB code. A simulation with PD control and visualization was used to model and conduct case studies on the stability of the UAV.

16. Rook Placements and Generalized Eulerian Numbers

Esther Banaian College of Saint Benedict
Jeffrey Davis University of South Carolina
Advisor(s): Dr. Steve Butler, Iowa State University

We consider a generalization of Eulerian numbers which count the number of placements of cn "rooks" on an $n \times n$ board where there are exactly c rooks in each row and each column, and exactly k rooks below the main diagonal. The standard Eulerian numbers correspond to the case $c = 1$. We show that for any c the resulting numbers are symmetric and give generating functions of these numbers for small values of k.

17. Vanishing Viscosity Limits for the Lagrangian Averaged Navier-Stokes Equation

Bethany Barber Creighton University
Advisor(s): Dr. Nathan Pennington, Creighton University

The Lagrangian Averaged Euler and Lagrangian Averaged Navier-Stokes equations are recently derived approximations to the Euler and Navier-Stokes equations, respectively. As the name suggests, the Lagrangian Averaged Navier-Stokes are derived by averaging at the Lagrangian level, and the resulting partial differential equations have more easily controlled long time behavior at the cost of a more complication nonlinear term. In this project we consider the vanishing viscosity problem for circularly symmetric flows.

18. Configuration Spaces of Double Coverings

Daniel Barg Columbia University
Advisor(s): Dr. Yuliy Baryshnikov, University of Illinois at Urbana-Champaign

The configuration space of a covering of a domain $Y \subset \mathbb{R}^d$ with n closed balls of radius r is the space of centers of the covering balls:

$$\mathrm{Cov}_n(r, Y) = \{\vec{x} = (x_1, \ldots, x_n) \in Y^n \mid \forall y \in Y \; \exists \, 1 \leq i \leq n \text{ s.t. } d(y, x_i) \leq r\}$$

We focus on the case when when $Y = I$ is a unit interval, $I = [0, 1]$. Homotopy types of single coverings of I were characterized by Yuliy Baryshnikov (2014). We address this problem for the space of double coverings, defined as

$$\mathrm{DCov}_n(r, I) = \{\vec{x} \in I^n \mid \forall \; y \in I, \; \exists \, 1 \leq j \neq k \leq n \text{ s.t. } \max(|y - x_j|, |y - x_k|) \leq r\}$$

As our main result we prove that for $n = 2k + 1$, $r = \frac{1}{2k}$, $k \in \mathbb{N}$, the space of double coverings of I is homotopic to a graph with a fixed euler characteristic. This euler characteristic completely determines the homotopy type of the configuration space. We also prove some results about single and double coverings of the unit circle.

19. Every n-ary semigroup is binary-subderived

Thomas Barron University of Kentucky
Advisor(s): Dr. Christopher O'Neill, Texas A&M

In universal algebra, an n-ary semigroup (for $n \geq 2$) is an algebra with a single operation of arity n which is associative. Some n-ary semigroups can be constructed from binary semigroups by a process known as derivation; n-ary semigroups constructed in this way are known as "binary-derived". It is known that not every n-ary semigroup is binary-derived. In contrast, our result is that every n-ary semigroup is isomorphic to a subalgebra of a binary-derived n-ary semigroup. (We call this property "binary-subderived".) This guaranteed existence of an ambient binary structure allows the application of many concepts to the n-ary case which previously were restricted to binary structures.

20. Effects of Supplemental Instruction on Student Achievement in an Introductory Statistics Course

Emily Baum Georgia College & State University
Advisor(s): Dr. Brandon Samples, Georgia College & State University

At most universities, an introductory statistics course is required for the majority of the students before they begin their specific major classes. Roughly 25% of undergraduate students at a given university will take a statistics class during a single academic year. Of these students, several will fail to retain the information, making future classes more difficult, or fail to successfully pass the course, increasing the likelihood a student will not graduate on time. Providing academic support through the implementation of a Supplemental Instruction (SI) Program gives students the opportunity to receive free, out-of-class help focused on student achievement in this course. Lead by a SI Leader, students are able to attend sessions to receive conceptual help while reviewing class material, developing study strategies, and collaborating with classmates. We will be focusing on the effects SI can have on student achievement in a statistics classroom. Since statistics is a necessary and important course in several disciplines, proper academic help is crucial for the success of the students. We will share our data analysis for using SI in a statistics course over a 4-year period, providing participants the opportunity to identify the positive effects SI has on student success.

21. Convergence of a Job Market Signaling Game With Agent Estimation of Parameters to the Bayesian Nash Equilibrium

Nathaniel Bechhofer George Mason University
Advisor(s): Dr. Tim Sauer, George Mason University

Using an agent-based simulation, we find that, in many cases, repeated rounds of a job market signaling game yield convergence of agent beliefs and actions to fit a derived Perfect Bayesian Nash equilibrium for the case of abilities drawn from a continuous uniform distribution and corresponding continuous wages (when agents are updating in a Bayesian fashion). Moreover, we find that this convergence is robust to using agent expectations not centered around equilibrium outcomes.

22. Computational Dynamics of a Map with Multiple Stable States

William Bench The College of William & Mary
Advisor(s): Sarah Day, The College of William & Mary

We use numerical, graph, and topological techniques to analyze the dynamics of a map with multiple stable states. Using these methods we analyze the periodic points in the map's period-having and period-doubling cascades. We discretize the phase space and use interval arithmetic to create a graph which reflects the dynamics of the system. We then use information of the graph and computational homology to prove the existence and location of dynamics of the map.

23. Probability of Integer Area Lattice Figures

Carissa Berge-Sisneros Nevada State College
Advisor(s): Dr. Serge Ballif, Nevada State College

We prove that lattice triangles have integer area with probability $\frac{5}{8}$. We use this result to show that any lattice polygon has a greater than 50% probability of having integer area. We then show that if a lattice polygon has integer area, the probability that the area is an even number is greater than 50%. After establishing these two-dimensional results, we prove similar results for lattice pyramids in higher dimensions.

24. First-Order Methods for Linear Optimization

Peter Bernstein Tufts University
Amanda Mayhall Louisiana State University
Anurupa Bhonsale University of Maryland - College Park
Advisor(s): Dr. Dávid Papp, North Carolina State University

We have investigated the use of first-order methods in linear optimization. We developed an algorithm based on gradient descent with directional minimization, and showed that its rate of convergence is linear. Using synthetic data, we demonstrated that when combined with appropriate scaling, our method outperforms Matlab's built-in linear optimizer, and is competitive with state-of-the-art second-order implementations such as SeDuMi and the commercial solver CPLEX.

25. Patterns in the Weak Visibility of Lattice Points

Jack Billings North Central College
Advisor(s): Dr. Neil Nicholson, North Central College

We say that a lattice point Q in a chosen rectangular array of lattice points is weakly visible from a lattice point P, external to the array, if no point other than Q in the array lies on the line segment connecting P and Q. We examine the percentage of points in the array which are not weakly visible by P. As P varies, patterns and symmetries become apparent in these percentages. This is the focus of our research. In conclusion, we investigate the notion that the closer P is to the array, the fewer lattice points in the array, in general, are weakly visible by P.

26. Mathematical Modeling in Ecology: Simulating the Reintroduction of the Extinct Passenger Pigeon (Ectopistes migratorius)

Erin Boggess Simpson College
Alanna Riederer University of Central Oklahoma
Jordan Collignon California State University, Monterey Bay
Advisor(s): Dr. Alex Capaldi, Valparaiso University

The Passenger Pigeon was an iconic species of bird in eastern North America that comprised 25-40% of North American avifauna. Passenger Pigeons went extinct in 1914 due to excessive hunting over the previous 50 years. Current research aims to de-extinct the Passenger Pigeon and someday release the species into its historic range. To determine under which conditions a Passenger Pigeon could survive a reintroduction into a natural habitat, we used two types of models. First, we used a system of delay differential equations to explore the relationship between the young pigeon, adult pigeon, nest predator, and raptor populations. The model incorporates logistic population growth, an Allee effect, and a Holling Type III functional response. Next, we developed a spatially explicit, agent-based model to simulate the population dynamics of the Passenger Pigeon in a number of present-day forest environments. The model incorporates the following stochastic processes: varying availability of food sources, reproduction, and natural death of the Passenger Pigeon. Bio-energetics, tree distributions, and other ecological values were obtained from literature. Results from our simulations suggest that the Passenger Pigeon could survive a reintroduction into a natural environment.

27. Modeling the Spread of Ebola with SEIR and Optimal Control

Harout Boujakjian George Mason University
Advisor(s): Dr. Tim Sauer, George Mason University

Ebola is a virulent disease that has plagued Western Africa, impacting Liberia, Sierra Leone, and Guinea heavily in 2014. Understanding the spread and containment of this disease has become a priority for several countries. We use an SEIR model to simulate the transmission of the disease. The model is validated by data from the World Health Organization. Optimal control theory is used to explore vaccination and quarantine rates to effectively contain the Ebola virus.

28. A New Family of Partial q-Analog Steiner Systems

Grant Bowling University of Michigan
Kavi Duvvoori Brown University
Advisor(s): Dr. Sung-Yell Song, Iowa State University

An important problem in coding theory, projective geometry, and design theory is the construction of q-analog t-designs, t-$(n,k,\lambda;q)$ designs, over the finite field of order q. A t-$(n,k,\lambda;q)$ design is a collection of k-dimensional subspaces, called blocks, of the vector space of dimension n over GF(q), such that each t-dimensional subspace is contained in exactly λ blocks. We give a construction of a family of 2-class Steiner-systems using the technique of field reduction. These designs are new, non-trivial, and have parameters 2-$(rd, 2d, \{1, \begin{bmatrix} r-1 \\ 1 \end{bmatrix}_{q^d}\}; q)$, for positive integers r and d. In addition, the construction generalizes to construct t-$(rd, td, \{1, \lambda_1, \ldots, \lambda_{t-1}; q)$ t-class partial t-designs for any positive integer t<r. Our collection of $2d$-dimensional blocks in GF$(q)^{rd}$ is generated from the set of 2-dimensional subspaces of GF$(q^d)^r$ via the field reduction map. In the one-dimensional case, this construction provides a t-dimensional spread in the projective geometry PG$(rd-1, q)$. While it is not a proper Steiner-system, since no other algebraic techniques are currently known for constructing q-analog Steiner-systems this is an advancement in q-analog design theory.

29. A Statistical Inquiry into the Baseball Minor League System

Matthew Breen University of Arkansas
Advisor(s): Dr. Edmund Harriss, University of Arkansas

Throughout nineteen MLB-affiliated leagues, 246 clubs play games and develop players' skills. Taking players drafted out of high school, college and injured players from the professional league, the minor league system exists to transform players with raw talent into pro level players. By tracking the progress of players from similar initial conditions (ex. Similar stats recruited at same point in career) using historically available statistics, we develop a model of normal player development throughout the minor leagues and into the major leagues. By measuring the fit of our model

to untested players, we can identify anomalous developments in players' development which we can categorize by occurrence at various points in careers. Comparing the variance of our untested data from our model will identify player development that isn't measured by our testing data. This anomalous player development, categorized by when it occurs in a player's career can be attributed to specific teams. By quantifying the conditions of the identified teams, we use Monte Carlo simulations in order to determine the efficacy of the specific teams, leading to a conclusion of what makes a good farm team and which teams are the most efficient at turning players from recruits into professional ballplayers.

30. Simulation of Thin Film Equations on an Eye-Shaped Domain with Moving Boundary

Joseph Brosch University of Delaware
Advisor(s): Dr. Tobin Driscoll, University of Delaware

During a normal eye blink, the upper lid moves, and during the upstroke the lid paints a thin tear film over the exposed corneal and conjunctival surfaces. This thin tear film may be modeled by a nonlinear fourth-order PDE derived from lubrication theory. A major stumbling block in the numerical simulation of this model is to include both the geometry of the eye and the movement of the eyelid. Using a pair of orthogonal and conformal maps, we transform a computational box into a rough representation of a human eye where we proceed to simulate the thin tear film equations.Although we give up some realism, we gain spectrally accurate numerical methods on the computational box. It has already been found that this method produces very accurate results for Dirichlet condition for the heat equation. We are able to perform these simulations very quickly (generally in under a minute) using a desktop version of MATLAB.

31. Investigating the Interplay of Argumentation and Mathematics in Classroom Tasks

Megan Brown University of Dayton
Grace Wright Bates College
Steven LeMay University of Connecticut
Advisor(s): Dr. Fabiana Cardetti, University of Connecticut

Our poster showcases the results of a Mathematics Education study carried out this past summer as part of the Math REU program at the University of Connecticut (NSF grant DMS 1262929). Motivated by the current emphasis on argumentation in the new Common Core State Standards for Mathematics, our study focused on analyses of tasks related to the third Standard for Mathematical Practice that highlights argumentation as an expertise all students should develop. We analyzed how argumentation affects the cognitive demand levels of the tasks, as well as how argumentation detracts or contributes to the mathematical content in the tasks. Our findings show a change in cognitive demand levels when tasks were analyzed with and without the argumentation component. In addition, five different themes emerged from our analysis with regards to the interplay of argumentation and mathematical concepts, some of which were common across elementary and high school tasks.

32. A Guaranteed Win: Optimal Strategies for the Region Unknotting Game

Sarah Brown Seattle University
Francisco Cabrera Kalamazoo College
Advisor(s): Dr. Allison Henrich, Seattle University

A region crossing change is an unknotting operation on a knot diagram. We used this operation to invent a game that can be played on knot diagrams: the Region Unknotting Game. Knot games such as ours can be used to demonstrate and discover properties of families of knots. Our team identified winning strategies for the Region Unknotting Game to explore the effects of region crossing changes on twist and torus knot diagrams.

33. The Existence of Solutions to an Even Order Right Focal Boundary Value Problem

Daniel Brumley University of Central Oklahoma
Advisor(s): Dr. Britney Hopkins, Kristin Karber, Thomas Milligan, University of Central Oklahoma

We outline a method for proving the existence of positive solutions to the even order differential equation, $u^{(2n)} = \lambda h\left(t, u, u', u'', \ldots, u^{(2n-1)}\right)$ for $t \in (0,1)$ and $n \geq 2$, satisfying the right focal boundary conditions $u^{(2k)}(0) = 0$ and $u^{(2k+1)}(1) = (-1)^k a_k$ for $k = 0, 1, 2, \ldots, n-1$, where $h : [0,1] \times \prod_{j=0}^{n-1}(-1)^j[0,\infty)^2 \to (-1)^n[0,\infty)$ is continuous, $\lambda, a_0, \ldots, a_{n-1} \geq 0$, and $\sum_{k=0}^{n-1} a_k > 0$. Beginning with a transformation of the even order boundary

value problem into a system of second order differential equations satisfying homogeneous boundary conditions, our method culminates in successive applications of the Guo-Krasnosel'skii Fixed Point Theorem to produce the desired result.

34. Mathematics and Intercultural Competence in Middle School

Megan Brunner State University of New York at Geneseo
Christopher Bennett Sacred Heart University
Kyle Evans University of Connecticut (Mansfield)
Advisor(s): Dr. Fabiana Cardetti, University of Connecticut

Our poster showcases the results of a Mathematics Education study carried out this past summer as part of the Math REU program at the University of Connecticut (NSF grant 1262929). Our study was motivated by the increasingly globalized world that pushes for a greater need to develop intercultural competence (ICC) in children through education. With a focus on Michael Byram's model of ICC, we created two assessment tools — a survey to assess attitudes towards and knowledge of cultures and learning across content areas, and a rubric to assess interactions and reflections — to measure the development of intercultural competence in school students. We also developed lesson plans for 3rd, 6th, and 9th grades that incorporate ICC and the new Common Core State Standards for Mathematics. The results of our study, including all the products, will be displayed in the poster and further explained by the authors.

35. Female Centered Mate Selections As an Explanatory Mechanism for Dimorphic Solutions in a Rock-Paper-Scissors Game

Kelly Buch Southern Illinois University Edwardsville
Abena Annor Florida State University
Advisor(s): Dr. Benjamin Morin, Arizona State University

Side-blotched lizards, *Uta stansburiana*, exhibit trimorphic male throat-colors (orange, blue, or yellow). In terms of mating, the males participate in an apparent game of rock-paper-scissors determined by throat color (i.e., a cyclic dominance chain). Mathematical models of this behavior predict stable monomorphic and trimorphic populations. However, researchers have observed stable dimorphic populations of orange and blue males. Furthermore, it is postulated that the only large-scale, long-term, stable solutions exclude the yellow throat type. We propose a new mathematical model accounting for the female population available for mating that may exhibits such behavior. We discuss the conditions under which particular population configurations are stable and flow attractive. We use these results to motivate conservative methods that may mitigate biodiversity loss by preventing the decline of a particular monomorphic or dimorphic population.

36. Committee Selection with Approval Voting and Hypercubes

Caleb Bugg Morehouse College
Advisor(s): Dr. Francis Edward Su, Harvey Mudd College

This research examines elections of the following form: a committee of size k is to be elected, with two candidates running for each position. Each voter submits a ballot with his or her ideal committee, which generates their approval set. The approval sets of voters consist of committees that are "close" to their ideal preference. We define this notion of closeness with Hamming distance in a hypercube: the number of candidates by which a particular committee differs from a voter's ideal preference. Our approach considers both the combinatorial and geometric aspects of these elections. The goal of the project is to mathematically guarantee higher agreement proportions between voters, which will hold in generality. We establish a tight lower bound for the popularity of the most approved committee, and consider restrictions on voter preferences that may increase that popularity. Then, we conclude by showing that a certain voter characteristic, pairwise intersectionality, can guarantee a committee selection satisfies a majority of the voters.

37. Flowing Through Planar Singularities with Ramps

Colleen Burns University of Arkansas
Advisor(s): Mr. Patrick Wilsiom, University of California, Berkeley

We examine planar flows of curves in $\mathbb{R}^2$ under the curve shortening flow equation $\partial_t X = \partial_s^2 X$, and their associated ramps in $\mathbb{R}^3$. Grayson proved that simple embedded curves evolving by curve shortening flow become increasingly

circular and shrink to a point in finite time. We study non-simple curves for which curvature becomes unbounded during the flow, and discuss the use of space curves (ramps) which project to the given curve in the plane to flow through singularities. We explicitly compute examples of evolving curves with an associated family of ramps, and generate images of both the curve and ramp evolving under curve shortening flow. We explore more general numerical methods for computing the evolution of ramps as curves in space, and take the projection of a ramp into $\mathbb{R}^2$ to determine the evolution of the non-singular plane curve.

38. Minimal Forbidden Subgraphs in Coloring Graphs

Ashley Butts University of the Pacific
Advisor(s): Dr. Heather M. Russell, University of Richmond

Coloring graphs encode the structure of the collection of colorings of a graph. The k-coloring graph of a simple graph G has the proper k colorings of G as its vertex set with edges between colorings if and only if they differ on exactly one vertex of G. We focus on identifying graphs that are not coloring graphs by locating forbidden induced subgraphs — graphs that cannot be an induced subgraph of any coloring graph. Minimal forbidden induced subgraphs are forbidden subgraphs that do not properly contain induced forbidden subgraphs. In our research we found new families — one infinite and one finite — of minimal forbidden subgraphs. A key technique used in our proofs is called edge labeling where we track the vertices of the base graph G that are recolored as one transitions between vertices in the coloring graph via labeling the edges.

39. Graphs of Minimum Order With Dicyclic Automorphism Group

Lindsay Cadwallader University of Connecticut
Peter Huston Otterbein University
Cesar Meza Pomona
Advisor(s): Stephen Graves, The University of Texas at Tyler

A graph with minimum order whose automorphism group is isomorphic to the dicyclic group Dic_n, where $n \geq 2$ is constructed. Write $n = 2^b p_1 p_2 \dots p_r q_1^{m_1} q_2^{m_2} \dots q_s^{m_s}$, where $b \geq 0$, $\{p_1, p_2 \dots p_r\}$ are distinct prime factors of n of multiplicity 1 with $5 < p_1 < p_2 < \cdots < p_r$, and $\{q_1, q_2 \dots q_s\}$ are distinct prime factors of n where the multiplicity of q_i is $m_i \geq 2$ unless $q_i \in \{3, 5\}$, in which case $m_i \geq 1$, with $2 < q_1 < q_2 < \cdots < q_s$. Such a graph has $w = 2^{b+3} + 2\sum_{i=1}^{r} p_i + \sum_{i=1}^{s}(2q_i + q_i^{m_i})$ vertices unless 15 is a factor of n and neither 9 or 25 is a factor, in which case the graph has $w - 3$ vertices.

40. First digit distribution of some families of arithmetic sequences

Zhaodong Cai University of Illinois at Urbana-Champaign
Yuda Wang University of Illinois at Urbana-Champaign
Advisor(s): Dr. A. J. Hildebrand, University of Illinois

It has long been known that sequences such as $\{2^n\}$, $\{n!\}$, $\{F_n\}$ satisfy Benford's law. That is, the frequency of leading digits is not uniform but given by the Benford distribution, $P(d) = \log_{10}(1 + 1/d)$, for $d = 1, 2, \dots, 9$. In our research, we consider the sequences of leading digits arising here more closely and investigate the degree of randomness in their behavior. This is joint work with Yewen Fan and Kevin Kwan, carried out at the Illinois Geometry Lab.

41. Bipyramids and Hyperbolic Volume of Tiling Links

Aaron Calderon University of Nebraska-Lincoln
Nathaniel Mayer Harvard University
Advisor(s): Dr. Colin Adams, Williams College

We generalize a result of Champanerkar, Kofman and Purcell to give a geometric decomposition of the complements of all alternating links whose projection graphs are uniform tilings. This allows us to compute exact volume densities of these links. In the process we generalize the angle structures program of Casson and Rivin to triangulations with a mixture of finite, ideal and ultra-ideal vertices. Finally, we prove the set of possible densities for links in thickened surfaces is a dense subset of $[0, 2v_{oct}]$ where v_{oct} is the volume of the regular ideal hyperbolic octahedron. This is joint work with C. Adams, X. Jiang, A. Kastner, G. Kehne and M. Smith.

42. Lattice Sphere Number

William Campbell University of Texas at Tyler
Zonia Menendez The college of new jersey
Mike Emerick-Cayton UW Bothell
Izaak Berg UW Bothell
Advisor(s): Dr. Jennifer Mcloud-Mann, UW Bothell

Instead of making a knot with a piece of string, we consider knots made from sticks with restricted edge lengths and bonding angles. In particular, we consider lattice knots from natural occurring lattices including the simple cubic lattice, simple hexagonal lattice, face-centered cubic lattice, and two variations of the body-centered cubic lattice. Mathematicians have answered questions like 'how many edges are needed to make a non-trivial knot in a lattice' known as the minimal knotting number of a lattice. We use these established facts along with counting arguments, graph theory, and an exhaustive computerized approach to establish lattice sphere numbers for the lattices mentioned above. We define the lattice sphere number of a lattice L to be the minimum radius of a lattice sphere containing a non-trivial knot.

43. An Exploration of Gravity Dependent Sand Pile Theory

Kevin Campbell Gettysburg College
Advisor(s): Dr. Darren Glass, Gettysburg College

In the common sand pile model, there is an established point on the graph with a set number of grains of sand which will fall onto adjacent piles under certain conditions. The model's rules state that a pile of four or more grains of sand will give one piece of sand to each tile that shares an edge, decreasing the central tile's value by four, and increasing each of the adjacent tiles' values by one. My model will have a few major changes to the standard rules. If a pile is of at least height nine, then one piece goes to each tile touching it; if a tile is of at least height five, then one piece goes to each tile that shares an edge. The other major constraint on my system is that sand will never go from a pile onto a tile with a greater value; that is to say, we are applying gravity to my system so that sand does not flow upwards. Now, this constraint demands that there is an order given to which tiles fall earlier than others, and in what order sand is handed out from the tile. We examine if this new model maintains the fractal patterns made in the common case, and if a pile on the graph can reach a given height greater than nine. Also compared are the radii given by the furthest point from the origin and by the greatest Manhattan distance present.

44. Analyzing the Effects of Topography on High-Altitude Balloon Descent

Elizabeth Carlson Carroll College
Advisor(s): Dr. Eric Sullivan, Carroll College

High-altitude balloons are often used to collect data from the atmosphere, and it is imperative to know where the payload will land in order to recover scientific equipment intact. Building on an experimentally-based prediction model for high-altitude ballooning developed by the author, this project works to incorporate the effects of Montana's mountainous topography on a wind profile calculated via a numerical solution to the Reynolds Averaged Navier-Stokes equations. The trajectory of the balloon and its payload are tracked through the wind field and over the mountainous terrain using Newtonian physics.

45. The Effects of Immunotherapy on Melanoma Cancer

Adam Carty Lee University
Advisor(s): Dr. Debra Mimbs, Lee University

Melanoma is the most serious type of skin cancer, and the risk for developing this type of cancer seems to be increasing. With new advancements in cancer treatment, such as immunotherapy, patients can receive more effective treatment. Immunotherapy boosts a patient's immune system by increasing the amount of immunocytes in the body, and this, in return, should decrease the growth of the tumor. This mathematical model demonstrates how immunotherapy effects the growth of a melanoma tumor based on the size of the tumor and the effectiveness of immunocytes. A dynamical systems modeling software, Stella, has provided a system of difference equations which can be examined to understand the workings of immunotherapy treatment. The importance of this model is that it can predict whether or not this type of treatment is necessary given initial conditions of the melanoma tumor. Limitations to the model and assumptions that have been made are also provided to explain the complexities of this problem and to show that it is difficult to

perfectly represent real-life circumstances. Overall, this model can be used as a tool to help people understand how melanoma tumors grow and how they can be effectively killed.

46. A distance-based method for phylogenetic tree reconstruction using algebraic geometry

Emily Castner Mount Holyoke College
Advisor(s): Dr. Joseph Rusinko, Hobart and William Smith Colleges

Using algebraic geometry and optimization software, we present a new method for phylogenetic quartet reconstruction. Representing tree topologies as varieties and genetic data as points, we determine how well the data fits a Markov model on the associated tree topology by minimizing the distance from the point to the variety. We implement this for the heterogeneous Jukes-Cantor, Kimura 2- and 3-parameter, and general Markov models of evolution. The Kimura 3-parameter model is most accurate on data simulated under the same model. We see that the Jukes-Cantor model is almost as accurate, even with model misspecification on all data, and is by far the fastest.

47. Equal distribution of polynomial discriminants over a finite field

Jonathan Chan Princeton University
Soonho Kwon Princeton University
Advisor(s): Dr. Keith Contrad, University of Connecticut

For a prime power q, we show that the discriminants of monic polynomials in $\mathbb{F}_q[x]$ of a fixed degree m are equally distributed if $\gcd(q-1, m(m-1)) = 2$ when q is odd and $\gcd(q-1, m(m-1)) = 1$ if q is even. A theorem in the converse direction is proved when $q-1$ is squarefree.

48. Lattice/Nonlattice Dichotomy for δ-Disjoint Self-Similar Systems

Phong Chau California State Polytechnic University, Pomona
Advisor(s): Dr. John Rock, California State Polytechnic University, Pomona

In fractal geometry, self-similar sets are constructed as the attractor (or fixed point) a self-similar system, a finite family of contracting similitudes on a complete metric space. In particular, a self-similar system is classified as δ-disjoint if the images of its attractor under the similitudes are separated, pairwise, by at least δ. The research presented here focuses on an aspect of the lattice/nonlattice dichotomy for δ-disjoint self-similar systems. Specifically, simultaneous Diophantine approximation allows for the construction a sequence of lattice self-similar sets to converge to a given nonlattice self-similar set. This result and its consequences are investigated here in the context of δ-disjoint self-similar systems in the hopes of providing framework for studying the box-counting complex dimensions of the corresponding self-similar sets. This research is supported by an NSF grant through the Preparing Undergraduates through Mentoring towards PhDs (PUMP) program, awarded as an Undergraduate Research Group (URG) grant.

49. Curvature approximation for modeling crime fluctuation in Baltimore

Robert Chen Stevenson University
Advisor(s): Dr. Mark A. Branson, Stevenson University

Public transportation is shunned in contemporary society as a means for spreading crime to otherwise "peaceful" neighborhoods. The lack of regulation and safety leads the general populace to seek alternatives to public transit, or even abandon traveling for the safety of their own homes. Discrete curvature algorithms, such as Gauss-Bonnet are employed to analyze and assess the correlation between crime and transportation to better inform public safety officials regarding the influence of public transit on the dynamics of urban lifestyle and crime.

50. NP-Hard Triangle Packing Problems

Amy Chou Massachusetts Institute of Technology
Advisor(s): Mr. James Hirst, Massachusetts Institute of Technology

Packing problems ask whether a set of pieces can be placed inside a target region such that no two pieces overlap. We prove that general triangle packing is NP-hard by proving the existence of three NP-hard cases of triangle packing: (i) packing right triangles into a rectangle, (ii) packing right triangles into a right triangle, and (iii) packing equilateral triangles into an equilateral triangle. Previous works in the design of packing algorithms have assumed that triangle

packing is NP-hard due to the NP-hardness of polygon packing, but this does not justify the NP-hardness of triangle packing. Our proofs mathematically justify that triangle packing is NP-hard using reductions from the NP-hard problems 3-partition and 4-partition. Our hardness results inform programmers the need to design approximation or heuristic algorithms rather than exact algorithms for triangle packing.

51. Random Visibility in Unit Bars

Jiarui Chu Davidson College
Advisor(s): Dr. Laurie Heyer, Davidson College

Two unit bars are visible to each other if an unobstructed vertical sightline can be drawn between them. The study of visibility in bars is motivated by Very-Large-Scale-Integration (VLSI) layout problems, and has applications in robot navigation, hidden-surface removal, and computer-aided software-engineering (CASE) tools. Although there is a rich body of research on visibility in bars, the existing research is mainly done in fields of computational geometry and graph theory. No result has been published on the probability aspects of visibility problems. Our research focuses on three major problems. Assuming n unit bars have uniformly distributed locations within width w, we first conjecture and prove the probability of having zero visibility among the n bars. Then we conjecture the probability density function and expected value for the number of bars required to cover the top bar. At last, we derive the expression for the maximum number of visible pairs among the n bars.

52. Momentum Term for the Modified Spectral Projected Subgradient Method (MSPS)

Samantha Clapp Georgia College and State University
Advisor(s): Dr. Milagros Loreto, University of Washington Bothell, School of STEM

The phenomenon of Zigzagging of Kind I is present in pure subgradient optimization algorithms when, at an iterate p_k, the subgradient direction s_k forms an obtuse angle with the previous movement m_k. Our goal is to identify and correct this phenomenon for the Modified Spectral Projected Subgradient method. We do this by adding a proportion of m_k to s_k; this proportion is called the momentum term and is denoted by τ. Also, we conduct numerical experimentation showing improved numerical results when compared to those of the original MSPS algorithm.

53. Modeling Wave Phenomena using Riccati-System to Construct Nonlinear Schrodinger Equations

Kiara Colon University of Puerto Rico - Mayaguez
Nathalie Luna University of Puerto Rico - Mayaguez
Advisor(s): Dr. Erwin Suazo, University of Puerto Rico - Mayaguez

In this poster, transformations are used in order to construct Nonlinear Schrödinger equations admitting bright, dark and Peregrine soliton solutions. We find explicit solutions for a nonlinear Riccati system with selected variable coefficients to construct the transformations. NLS is the standard model of how light propagates inside of a fiber optic with rich mathematical properties such as being integrable. Some of the most important applications are rogue waves, pulse dynamics in the dispersion-managed fibers and Bose-Einstein condensates.

54. Dynamics and Control of CubeSat Orbits for Distributed Space Missions

Rebecca Conrad University of Denver
Jonathan Burkow Arizona State University
Advisor(s): Ted Yu, California State University Long Beach

CubeSats, standardized nano-satellites, are emerging as practical alternatives to larger satellites. Bringing with them the advantages of cost-efficiency and rapid design cycles, CubeSats present an ideal method for short-term missions of data collection and testing. When placed in orbit in formation, CubeSat clusters have the potential to perform with the capabilities of a much larger satellite. One of the current problems with CubeSat clusters is the challenge of formation flight, which limits mission lifetime due to propulsion constraints. We contribute to the developmental process of maximizing lifetime for a given CubeSat cluster design by minimizing propellant expenditures. Using the impulsive feedback control method developed by Hanspeter Schaub and Kyle Alfriend, we introduce the Relative Thrusting Method as a realistic strategy for formation keeping based on current propulsion capabilities. Our project also encompasses optimizing cluster parameters for geolocation applications.

55. Maximum Positive Semidefinite Nullity and the Tree Cover Number

Brendan Cook Carleton College
Carolyn Reinhart University of Minnesota
Oscar Gonzalez University of Puerto Rico at Rio Piedras
Advisor(s): Dr. Leslie Hogben, Iowa State University

A simple graph is an undirected graph, containing no multiple edges, such that no vertex is adjacent to itself (i.e., no loops). The maximum positive semidefinite nullity of a graph is defined to be the maximum nullity over the set of real positive semidefinite matrices whose ijth entry is zero if and only if vertices i and j are not adjacent in the graph, for $i \neq j$. The tree cover number of a graph is the minimum number of vertex disjoint trees occurring as induced subgraphs that cover all the vertices of the graph. This graph parameter was introduced in 2011 as a tool for the study of maximum positive semidefinite nullity, and little is known about it. It is conjectured that the tree cover number of a graph is at most the maximum positive semidefinite nullity of the graph. While investigating this conjecture, we arrived at several results on tree cover number. Here we present results on the tree cover number of subdivided graphs, graphs resulting from edge removal, and tree cover number for families of graphs. Specifically, we prove the tree cover number for the family of hypercubes is 2. We also characterize when an edge is required in every minimum tree cover.

56. Effects of model error on Sensitivity Analysis Sobol' Indices (SI)

Jeremy Cooley East Tennessee State University
Ken Jutz North Carolina State University
Thomas Bassine University of Connecticut
Advisor(s): Dr. Pierre Gremaud, North Carolina State University

When analyzing high-dimensional input/output systems, sensitivity analysis is commonly performed to identify important variables and reduce the complexity of the problem. To perform sensitivity analysis on fixed data sets, without the possibility of further sampling, we fit a surrogate model to the data. This project explores the effects of model error on sensitivity analysis, using Sobol' indices (SI), a measure of variance contributed by particular variables (first order) and by interactions between multiple variables (total), as the primary measure of variable importance. All analysis is based on data generated by test functions for which the true SI are known. We fit two non-parametric models, Random Forest and Multivariate Adaptive Regression Splines (MARS), to test data, and SI are approximated using R routines. An analytic solution for SI based on MARS basis functions is derived and compared to the actual and approximated SI. Further, we apply MARS and Random Forest to data sets of increasing size to explore error convergence as available data increases. Due to efficiency constraints in the surrogate models, constant relative error is quickly reached despite increasing data size. We find that variable importance and SI are well approximated despite significant model error.

57. Modeling the Interaction Dynamics between Honeybees and Food Availability

Carlos Cruz Loyola Marymount University
Matthew Baca New Mexico Institute of Mining and Technology
Armando Salinas Arizona State University
Carlos Agrinsoni University of Puerto
Advisor(s): Dr. Baojun Song, Montclair State University

The success of honeybee colonies is critical to the United States agriculture with 35% of American diets dependent on honeybee pollination. There are various complex factors that contribute to a colony's failure. Nutritional stressors primarily pertain to food scarcity, lack in diversity of food, and the availability of food with low nutritional value. Previous mathematical models have examined the impact of nutrition and the early recruitment on honeybee population dynamics. In this work, we use a mathematical model to investigate the impact of food scarcity and limited storage space on colony viability, early recruitment rates of workers into foragers, and the influence of these rates on the growth of a colony. We found conditions for the stable coexistence of a honeybee population and food supply as well as conditions for periodic behavior. Through sensitivity analysis we find that a honeybee colony is most sensitive to changes in the rate at which a worker bee encounters food and the rate food is entering the food supply. There are no qualitative differences between using a Holling Type I or Type II functional response in honeybee population persistence when modeling the interaction between a honeybee colony and the availability of food.

58. Minimizing the Cost of Guessing Games

Lindsay Czap Grand Valley State University
Advisor(s): Dr. David Clark, Grand Valley State University

A two-player "guessing game" is a game in which the first participant, the "Responder," picks a number from a certain range. Then, the second participant, the "Questioner," asks only yes-or-no questions in order to guess the number. While the Responder answers these questions, they may lie to the Questioner. Guessing games that can correct lies are error correcting codes, mathematical objects that allow us to detect an error in the information that we receive and correct these errors. We will give basic definitions in coding theory and show how error correcting codes allow us to still guess the correct number even if one lie is involved. We will introduce cost functions to guessing games. Each of the possible answers are assigned a "price." The Questioner will be charged the cost of all of the possible answers that they are asking about. We will discuss optimization methods for minimizing the total cost of a guessing game when a cost function is applied. We prove that, given any cost function, the most cost efficient guessing game takes the form of one of two game constructions.

59. The Topology of Coupled Oscillators

Esha Datta Macalester College
Alec Dunton Harvey Mudd College
Advisor(s): Dr. Lori Ziegelmeier, Macalester College

Coupled oscillators, such as fireflies that flash in unison or metronomes that pulse at the same frequency, have a tendency to synchronize despite differences in initial conditions. In this poster, we outline a study of systems of coupled oscillators governed by the classic Kuramoto model using topological data analysis (TDA). TDA provides a novel approach to study dynamical systems. Our data consists of angular position-velocity information of each oscillator, which can be viewed as a point cloud. We explore key variations in the Kuramoto model, including oscillators' natural frequencies and the network characteristics of the coupling. TDA is then applied to study the emergence and death of topological features such as the number of connected components in the point cloud. We visualize these features over both simulation time and topological scale. We draw connections between the network characteristics assigned to a system of coupled oscillators and the emergent topological structures.

60. Discontinuous standard map dynamics

Tom Dauer Indiana University Bloomington
Meg Doucette University of Chicago
Advisor(s): Dr. Maxim Arnold, University of Texas at Dallas

We study the map $x \to x' = x + \alpha y \pmod 1$, $y \to y' = y + \mathrm{sgn}(x' - 1/2)$, which is related to the extensively-studied Chirikov standard map, sharing several of its properties. On the other hand, since the sign function is discontinuous, KAM theory offers no insight into boundedness of the trajectories. We present both numerical and analytical results on this discontinuous standard map in the case of rational rotation number ($\alpha = p/q$) and integer y_0. We find that for even q all trajectories are bounded, while for odd q there is a unique unbounded trajectory. We also study the asymptotics of the unbounded trajectory, show the existence of certain families of periodic trajectories, and provide evidence for several conjectures.

61. Real-Time Bidding Optimization for Online Advertising

Megan Davis DePaul University
Isaac Bjorke University of California, Santa Barbara
Advisor(s): Dr. Giang Tran, University of Texas, Austin

In the online advertisement industry, real-time bidding (RTB) has emerged as a popular method of website advertisement allocation. Real-time bidding is a mechanism for filling advertisement space on websites. A representative of the website contacts a sell-side platform who holds a second-price sealed-bid auction real-time to sell the ad space to a select number of advertising agents. Sell-side platforms are interested in choosing the revenue-maximizing floor price for each auction. In our research carried out at RIPS IPAM during the summer of 2015, our team established two different learning models for choosing these revenue-maximizing optimal floor prices. These algorithms observe the highest bids received from previous auctions and constantly update an optimal floor price that is submitted to new auctions. Both algorithms are performed on different groups of impressions to ensure proper product-differentiation.

To assist in the efficiency of product-differentiation, we propose potential clustering methods of impression types. After we developed our optimization methods, we ran each algorithm on simulated RTB environments to demonstrate their effectiveness.

62. Ordered factorizations on the integers

Francisco De Jesus University of Puerto Rico - Mayaguez

Advisor(s): Dr. Reyes M Ortiz-Albino, University of Puerto Rico-Mayaguez

The notion of a τ_C-factorization or τ_C-products is an example of what Anderson and Frazier defined as generalized ordered factorizations, but never been studied. We study such type of factorizations and other number theoretical properties as the greatest common τ_C-factor of any two nonzero nonunit element. We will focus on the integers, but this can be easily generalized to a PID. We give examples and characterized some behaviour.

63. Using The Flipped Classroom To Teach Calculus

Torey DeAngelis Westminster College

Advisor(s): Dr. Sararose Lynch, Westminster College

As technology has become more and more prevalent in our world, more and more teachers are integrating it into their instruction. One popular way to use technology in the classroom is to implement the "flipped classroom" model. The purpose of this research is to investigate the effectiveness of this model in a high school mathematics classroom. Specifically, this project details an experience using a flipped classroom model to improve student learning in two high school calculus classrooms.

64. Characterizing the Dynamics of Certain Sequential Dynamical Systems

Colin Defant University of Florida

Advisor(s): Dr. Padraic Bartlett, UC Santa Barbara

A sequential dynamical system (SDS) is a dynamical system defined over a graph in which vertices update sequentially. We present enumeration results concerning periodic points of certain SDS. First, we consider an SDS defined over C_n, the cycle graph on n vertices, using an identity update order and using either the update rule parity_3 or the update rule $(1 + \text{parity})_3$. Let $\alpha_n(r)$ denote the number of periodic points of period r of the SDS defined using parity_3. Similarly, let $\delta_n(r)$ denote the number of periodic points of period r of the SDS defined using $(1 + \text{parity})_3$. We give explicit formulas, derived using Möbius inversion, for $\alpha_n(r)$ and $\delta_n(r)$. As a surprising consequence of these formulas, we find that if we fix r and vary n, then there are only two possible nonzero values of $\alpha_n(r)$ and only one possible nonzero value of $\delta_n(r)$.

For our second enumeration problem, we let η_n denote the maximum number of period-2 orbits that can exist for an SDS defined over a complete graph in which all vertices update via the same rule. We show that η_n is equal to the maximum number of codewords in a binary code of length $n - 1$ with minimum distance at least 3.

65. Domain Representability and Topological Completeness

Matthew DeVilbiss University of Dayton

Advisor(s): Dr. Lynne Yengulalp, University of Dayton

Intuitively speaking, a topological completeness property is a property which generalizes or captures the notion of convergence. A topological space X is domain representable if there exists a continuous directed complete partially ordered set D such that X is homeomorphic to $\max D$. Domain representability is a topological completeness property with applications to computer science that is weaker than subcompactness and stronger than Baire. We explore the relationship between domain representability and some other topological completeness properties. Specifically, we show that the box product of a collection of domain representable spaces is domain representable and if Y is compact and $X \times Y$ is domain representable, then X is domain representable.

66. Marching Locusts: Aggregation and Group Patterns in 1D and 2D.

Sarah DeVore Macalester College
Stephen Schein Harvey Mudd College
Advisor(s): Chad Higdon-Topaz, Lori Ziegelmeier, Macalester College

Large populations of locusts can self-organize into highly coordinated groups called hopper bands that march for many kilometers, consuming all of the vegetation in their path. Eventually, when a hopper band reaches a sufficiently high population density, the group will take flight, at which point it is difficult to apply pest control measures. Thus, it is of interest to understand the conditions under which hopper bands form. We create and use an agent based dynamical system, the Alignment and Intermittent Motion (AIM) model to explore these conditions. Intermittent motion refers to a stochastic stop-go movement pattern observed in collision avoidance studies of locusts. Alignment is described as the weighted average of the headings of a locust's close neighbors. These two modeling inputs produce group-level behaviors characteristic of hopper bands in one and two dimensional numerical simulations, including group morphology and spontaneous direction switching of the group.

67. Weighing Coins and Keeping Secrets: A New Coin Weighing Problem

Nicholas Diaco MIT PRIMES-USA
Advisor(s): Dr. Tanya Khovanova, Massachusetts Institute of Technology

In 2007, a new variety of the well-known problem of identifying a counterfeit coin using a balance scale was introduced in the sixth International Kolmogorov Math Tournament. This paper offers a comprehensive overview of this new problem by presenting it in the context of the traditional coin weighing puzzle and then explaining what makes the new problem mathematically unique. Two weighing algorithms given in a previous work are used to derive lower bounds for the optimal number of admissible situations for given parameters. Additionally, a new weighing procedure is described that is shown to work as well as or better than a few other procedures and can be adapted to provide a solution for nearly any scenario. We also offer a new form of the traditional counterfeit coin problem and provide a lower bound for the number of weighings necessary to solve it.

68. Measurability of Smith-Volterra-Cantor sets

Adrian Diaz-Cabrera California State Polytechnic University
Erin Bullard California State Polytechnic University
Advisor(s): Dr. John Rock, California State Polytechnic University

Smith-Volterra-Cantor sets (SVC sets, also known as fat Cantor sets) are uncountable compact sets on the real line with no interior, many of which have positive length. The research presented here includes a characterization of self-similar SVC sets, a range of parameters that yield generalized SVC sets, and the geometric zeta functions associated with the complements of these sets. Additionally, the box-counting functions and zeta functions of certain SVC sets are analyzed. With this information, notions of of Minkowksi measurability in the cases of both the usual Minkowski content and the inner Minkowski content are determined and discussed in terms of the complex dimensions of these sets.

69. A Scalable Chatroom Providing Full Anonymity Without Loss of Practicality

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Cristian Gutu MIT PRIMES, Joel Barlow High School
Advisor(s): Mr. Albert Kwon, MIT/CSAIL

Existing anonymous communication systems sacrifice anonymity for performance or vice-versa. For instance, the most popular such system, Tor, relies on relays, which are fast but reveal critical information upon in-depth traffic, thus not guaranteeing true anonymity from strong adversaries such as ISPs and government agencies. In contrast, Dining Cryptographer Networks, or DC-Nets, offer users true anonymity at the price of a bandwidth-intensive protocol. For the first part of our research, we designed a protocol improving on DC-Net performance without sacrificing anonymity. Second, we created a chatroom prototype using our protocol. While naive DC-Nets require direct communication between all clients, our protocol has a secure server both performing the most computationally intensive calculations for the clients and serving as a liaison between clients. That is, each client communicated only with the server rather than with all other clients. The communication accordingly improved from $O(n^2)$, as in naive DC-Nets, to $O(n)$. Our protocol is thus able to scale to hundreds of clients with low latency of 248 milliseconds — even when tested across continents — compared to just tens of clients of naive DC-Nets.

70. Optimal Placement of Family Planning Centers

Kiera Dobbs The College of Wooster
Advisor(s): Dr. Matthew Moynihan, The College of Wooster

We address the problem of access to family planning services in the United States. We investigate and implement methods in Operations Research to optimize the location of publicly funded family planning centers in the United States by minimizing travel distance. The solution begins with a designated number of family planning centers for a region of the country. An apportionment algorithm using integer programming is then exercised to allocate centers to all the states in that region based on population and average income. At the state level, a set-covering integer programming model decides the placement for each center that minimizes travel distance, to guarantee that all cities are assigned to a center. Here the travel distance data is weighted based on the average income of a city, to ensure that the minimized travel distance is still reasonable for lower income individuals.

71. Relationship of the Hennings and Chern-Simons Invariant For Higher Rank Quantum Groups

Alexander Doser Iowa State University
McKinley Gray SUNY Geneseo
Winston Cheong Rowan University
Advisor(s): Stephen Sawin, Fairfield University

The Hennings Invariant for the small quantum group, associated to an arbitrary simple Lie algebra at a root of unity, is shown to agree with the Chern-Simons (aka Jones-Witten or Reshetikhin-Turaev) invariant for the same Lie algebra and the same root of unity on all integral homology three-spheres at roots of unity where both are defined. This partially generalizes work of Chen, Yhang and al. which relates the Hennings and Chern Simons invariants for SU(2) and SO(3) for arbitrary rational homomogy three-spheres. In addition it also complements recent work of Habiro and Le equating the Chern-Simons invariant at roots of unity on integral homology three-spheres to a Hennings like invariant defined as a power series.

72. Mathematical Modeling of the James Spinymussel

Marisa Draper James Madison University
Maranda Pepe James Madison University
Dylana Wilhelm James Madison University
Advisor(s): Dr. Anthony Tongen, James Madison University

The James Spinymussel (*Pleurobema collina*) is endangered and at the top of Virginia's conservation list. This species plays a critical role in the environment by filtering and cleaning water while providing shelter and food for macroinvertebrates. However, conservation efforts are complicated by the mussels' burrowing behavior, camouflage, and complex life cycle. The goal of this research was to estimate detection probabilities that could be used to determine whether the species is present in an area and to track individually marked mussels to test for source-sink dynamics. Using existing literature and mark-recapture field data, these goals were accomplished by evaluating dispersion type, clustering trends, odds of detection based on environmental factors, substrate preferences, and matrix population models. These analyses serve as the foundation of mathematical models used to aid in the recovery of the James Spinymussel.

73. Investigating Convergence by Using Cesaro Sequences

Evelyn Easdale California State University, Fullerton
Advisor(s): Dr. Bogdan Suceava, California State University, Fullerton

In the study of sequences of real numbers, the Cesaro-Stolz Lemma is a powerful, classical tool. By using Cesaro sequences, we investigate the polynomial rate with which sequences defined by a recurrence formula of order one approach zero. We illustrate our study through several examples, both classical and new.

74. Modeling Monopoly with Monte Carlo Simulations

Mitchell Eithun Ripon College
Advisor(s): Dr. McKenzie Lamb, Ripon College

Using Python we model the interactions during a two-player game of the *Monopoly*. Optimization techniques such as hill-climbing and Google PageRank allow us to explore the effectiveness of different strategies. More specifically

we compare the performance of players who base their decisions on short-term expected gains with players who have long term strategy agendas. Other topics include the length of the game, the effects of "house rules" and which board spaces are visited most often.

75. Regular Embeddings of Complete Graphs on Powers of Two Vertices

Sophia Elia University of California, Berkeley

Advisor(s): Dr. Bob Jacobsen, Kathryn Mann, University of California, Berkeley

A dessin d'enfant is a bipartite connected graph embedded in an orientable, closed topological surface which divides the surface into a union of open cells. A dessin is fully regular if its automorphism group acts transitively on the directed half edges of the embedded graph. The absolute Galois group of the rational numbers acts faithfully on regular dessins. One category of regular dessins is the regular embeddings of complete graphs. A Biggs map uses a Cayley graph of a finite field to produce the cyclic orderings of edges around the vertices of a complete graph which correspond to a regular map. However, there isn't a constructive way to go from this information to a visual representation of the regular embedding. This project illustrates a fully regular embedding of the complete graph on 8 vertices and locally regular embeddings of the complete graphs on 8, 16, and 32 vertices. We outline a proof of full regularity for the fully regular embedding through computation of the cartographic and automorphism groups.

76. Social Insects and Cheating in Reproduction — An Agent Based Model

Lauren Engel Arizona State University

Advisor(s): Dr. Dieter Armbruster, Arizona State University

Harvester ant colonies typically have anywhere from one to five queens. A queen can control the ratio of female to male offspring she produces, field research indicating that this ratio is genetically hardwired and does not change over time. Further, a queen has an individual reproductive advantage if she has a small reproductive ratio. A colony, on the other hand, has a reproductive advantage if it has queens with large ratios, as these queens produce many female workers to further colony success. We have developed an agent-based model to analyze the "cheating" phenotype observed in field research, in which queens extend their lifespans by producing disproportionately many male offspring. The model generates phenotypes and simulates years of reproductive cycles. The results allow us to examine the surviving phenotypes and determine conditions under which a cheating phenotype has an evolutionary advantage. Conditions generating a bimodal steady state solution would indicate a cheating phenotype's ability to invade a cooperative population.

77. Maximizing the Number of Lattice Points on a Strictly Convex Curve

Brandon Epstein MIT (Research Science Institute)

Advisor(s): Ms. Hong Wong, MIT

We obtain an upper bound for the maximum number of integral lattice points on the graph of a twice differentiable convex function $f : [0, N] \to [0, N^\gamma]$, where $\gamma > 0$ by generalizing an argument for $\gamma = 1$ to all γ between $\frac{1}{2}$ and 2. This method was based on the asymptotes of sums involving the Euler totient function, which we extended to prove the general case. Moreover, we also strengthen upper bounds for smooth convex functions $f : [0, N] \to [0, N]$ with restrictions on higher derivatives. Specifically, we tighten an upper bound on the number of lattice points on a curve with positive first, second, and third derivatives by modifying a method of Bombieri and Pila. We also examine the problem of finding the maximal number of lattice points on a smooth convex curve $y = f(x)$, subject to the condition $1 < f''(x) < 2$. We conjecture that this maximum is attained for the curve $y = \frac{3}{4}x^2$, which has $2\sqrt{\frac{N}{3}} - O(1)$ lattice points. Finding the number of lattice points on curves has a large number of applications, including prime factorization, which is used in modern cryptography to create secure one-way systems for which it is difficult to decode messages without the proper key.

78. Constructing solutions to truncated moment problems and applications to PDE

Paul Estrada CSU Channel Islands
Ty Danet
Advisor(s): Dr. Cynthia Flores, CSU Channel Islands

The classical moment problem was first studied by Tchebyshev, Markov and Stieltjes in the 1870's. They formulated and solved many variations on what Stieltjes called the "problem of moments." The problem is to determine a measure $d\mu(x)$ with a prescribed set of moments:

$$\int_{\mathbb{R}} x^n d\mu(x) = s_n, \tag{1}$$

for all $n = 0, 1, 2 \ldots$ where the values $s_n \in \mathbb{R}$ are called the moments. In 1921, Hamburger answered the question of finding a measure $d\mu(x)$ satisfying a given moment sequence $\{s_n\}_{n\in\mathbb{Z}_+}$. This is a fascinating result because meeting an algebraic condition shows the existence of an analytic object. In physical applications, only the *truncated moment problem* can be formulated. In this poster, we present preliminary results that give insight to the following questions: What is an exact expression for μ solving a given truncated moment problem? For a given truncated moment problem with a solution μ, does there exist a sequence of functions that converges to μ in an appropriate functions space (e.g., $C_0^\infty(\mathbb{R})$ or L^2-based)? This is an effort to explore the synergy between certain PDEs and truncated moment problems.

79. A Synchronized Rectilinear Motion Activity for the College Calculus Sequence

Evan Ezell Maryville College
Advisor(s): Dr. Maria Siopsis, Maryville College

We present a novel approach to introducing the concept of calculating the area between two curves by analyzing student data generated during the "Mirror Game" as described in a paper by Lior Noy, Erez Dekel, and Uri Alon. The Mirror Game consists of two people moving an object along a single axis while attempting to synchronize their movements as closely as possible. We replicate this game with standard physics equipment, creating real-time data for use in class. Students use technology to fit polynomials to their data and then analyze their accuracy by looking at the area between pairs of position curves and velocity curves. In addition to calculating area between curves, we reinforce concepts of position, velocity, and acceleration rectilinear motion. We present the preliminary results of applying this method in a Calculus II course at Maryville College. This data opens the door to use in the entire calculus sequence.

80. Zero Inclusion for the Numerical Range of a Composition Operator

Christopher Felder Butler University
Advisor(s): Dr. Eva Gallardo-Gutierrez, Universidad Complutense de Madrid

This work answers a question posed by P. S. Bourdon and J. H. Shapiro. Namely, that zero is always in the numerical range of a composition operator C_ϕ acting on $H^2(\mathbb{D})$ when ϕ is a parabolic-type self-map of the unit disk $\mathbb{D}$.

81. Integrating Fish Movement in Multispecies Population Modeling

Cody FitzGerald University of New Hampshire
China Mauck Grinnell College
James Keane University of Central Oklahoma
Alexis Gambino Southern Methodist University
Advisor(s): Dr. Minghao Rostami, Burt Tilley, Worcester Polytechnic Institute

A major issue with modeling fish population dynamics is tracking movement of species between regions. Two different veins of models have been explored: data-driven and partial differential equations. A discrete model is formulated that incorporates population growth, predation, and movement based on the relative proportions of a species population. For any population growth/predation model posed, the choice of a transition matrix is non-unique within a family. Additional constraints are needed to close this model. Further, a variety of different PDE formulations is considered,which take into account fish movement, seasonal and long-term temperature fluctuations, and spatial dependence on nutrients. A comparison of these models is discussed.

82. Extending 5-spirals to Fullerenes

Kyle Flanagan St. Mary's College of Maryland
Bethany Molokach **Rachel Olugbemi**
Advisor(s): **Dr. Christina Graves, The University of Texas at Tyler**

Fullerenes are trivalent planar graphs with only hexagonal and pentagonal faces. We are interested in whether certain 2-connected graphs, called fullerene patches, consisting of hexagons and pentagons can be extended to fullerenes. One such collection of fullerene patches is a 5-spiral. A 5-spiral consists of five rows of hexagons, connected at 120° angles when projected onto the hexagonal tessellation. We show that certain families of 5-spirals can be extended to fullerenes by investigating the boundary and dividing the patch into multiple regions. This generates a finite set of inequalities that characterizes when a 5-spiral can be extended with the given process. Moreover, we establish that sufficiently small 5-spirals do not extend to fullerenes, while sufficiently large 5-spirals are conjectured to also not extend. This conjecture, if proven, will complete the classification of 5-spirals.

83. Solving the Unknotting Problem

Brian Foley The College of Wooster
Advisor(s): Dr. John Ramsay, The College of Wooster

The three basic Reidemeister moves have been proven to be the only moves necessary to demonstrate knot equivalence. We propose a set of four generalized moves which demonstrate the equivalence of one knot projection to another projection with an equal or lesser number of crossings. We prove this using a similar strategy to the original Reidemeister proof. Applied inductively, we assert that these four moves are able to monotonically reduce a knot to its minimal crossing projection. The algorithm which applies these moves should thus solve the Unknotting Problem. We also discuss the feasibility and current progress of our computer program written to apply these moves in polynomial time.

84. Stabilization by Noise in $\mathbb{C}^2$-Valued Nonlinear Systems

Lance Ford University of Central Oklahoma
Heather McCain Schreiner University
Advisor(s): Fan Ny Shum, Joe P. Chen, University of Connecticut

The following complex multivariable system has solutions which blow up in finite time:

$$\left\{ \frac{dz}{dt} = -\nu z + \alpha z w \; \frac{dw}{dt} = -\nu w + \beta z w \; z(0) = z_0 \in \mathbb{C} \; w(0) = w_0 \in \mathbb{C}, \qquad \text{where } \nu \in \mathbb{R}^+, \alpha, \beta \in \mathbb{R}. \right.$$

By introducing additive Brownian noise to these equations, we can stabilize the entire system. The noise stabilizes the system by pushing unstable solution curves onto stable flow lines. Recent results have shown how additive noise can stabilize complex single–variable systems. In particular, the system is shown to be ergodic and its solutions converge exponentially fast toward a unique invariant measure. We have numerical results that suggest the same results can be produced for our multivariable system. We numerically verified that the addition of Brownian noise stabilized these formerly unstable solutions. Furthermore, we simulated the invariant measure, which has a heavy-tailed distribution.

85. p-adic Properties of Coefficients of Certain Half-Integral Weight Modular Forms

Claire Frechette Brown University
Advisor(s): Dr. Ken Ono, Emory University

Harmonic Maass forms are composed of two parts: a holomorphic one, called a mock modular form, and a non-holomorphic one, which is actually the period integral of a modular form, which is called the shadow. Given a mock modular form, there is a unique shadow which completes it to a harmonic Maass form, but no obvious relation between the two. Seeking a method to find the shadow, given the mock modular form, Guerzhoy, Kent, and Ono develop a p-adic relation between the two in the case where the shadow is an integral weight newform. However, the same method used in their paper does not directly apply in the half-integral weight case, which we begin to explore here. Studying the parallel cases of Zagier's and Folsom-Ono's grids of weakly holomorphic (respectively weakly holomorphic and mock modular) forms of weights $3/2$ and $1/2$, we investigate their p-adic properties under the action of Hecke operators.

86. Bases for McKay Centralizer Algebras

Lucas Gagnon Macalester College
Advisor(s): Tom Halverson, Macalester College

The finite subgroups of the special unitary group SU_2 have been classified to be isomorphic to one of the following groups: cyclic, binary dihedral, binary tetrahedral, binary octahedral, and binary icosahedral, of order n, $4n$, 24, 48, and 120, respectively. Associated to each group is a representation graph, which by the McKay correspondence turns out to be a Dynkin diagram of type A_{n-1}, D_{n+2}, E_6, E_7, and E_8. The centralizer algebra is the algebra of transformations that commute with tensor powers of these groups. The dimension of the centralizer algebra equals the number of walks on the corresponding Dynkin diagram, beginning and ending at the root node. These dimensions are generalizations of Catalan numbers. In a uniform way, we find bases for these algebras that are in bijection with the walks on the Dynkin diagram. This answers an open question in combinatorial representation theory.

87. Global well-posedness and stability of Beam equation with degenerate damping.

Zeyang Gao Earlham College
Shibei Shen **Jingwei Dai**
Advisor(s): **Dr. Pei Pei, Earlham College**

This is a study of global well-posedness and energy decay of beam equation with degenerate damping. Our primary feature is to investigate numerically the stability (strong and, if possible, uniform) of one-dimensional beam equations and derive theoretical results that support numerical approximations. The main results verify the global existence and uniqueness of weak and strong solutions as well as their continuous dependence on the initial data in appropriate functional spaces. Moreover, based on our numerical approximation, we make a conjecture about the uniform energy decay rate.

88. One Rook, Two Rook, Red Rook, Blue Rook

Emily Gaub Pacific University
Victoria Prawitz Pacific University
Advisor(s): Dr. Nancy Neudauer, Pacific University

A rook polynomial is a generating function giving the number of ways to place r non-attacking rooks on a given board. Non-attacking rooks are when no two rooks share the same row or column, so these rooks could not attack each other in a real game of chess. Traditionally, the rooks are not distinguishable. We consider the case where the rooks were distinguishable. In particular, sets of rooks of two colors, where rooks of the same color can be in the same row or column and not be considered attacking. We explore the maximum number of distinguishable rooks can be placed on a given board, the number of ways to place non-attacking distinguishable rooks on a board, and expressing those values as a polynomial. Further investigation involves the uses and expansions on a rook polynomial for distinguishable rooks.

89. Lattice Point Visibility on Generalized Lines of Sight

Michelle Gebert University of Wisconsin- Eau Claire
Sara DeBrabander University of Wisconsin- Eau Claire
Advisor(s): Dr. Aba Mbirika, University of Wisconsin- Eau Claire

Lattice point visibility has been a well-studied field since 1971. The classic setting focuses on integer lattice points which lie on straight lines through the origin with rational slopes. Harris, Kubik, and Mbirika generalize this notion of lines of sights to include all curves through the origin given by power functions of the form $f(x) = \frac{a}{b}x^n$ where $\frac{a}{b} \in \mathbb{Q}$ and $n \in \mathbb{N}$. Many questions remain open in this new setting of generalized lines of sights. In the classic setting where $n = 1$, a lattice point is visible if there exists no other lattice point between the origin and the point. It is well-known that the proportion of invisible lattice points is $1 - \frac{6}{pi^2}$, or approximately 40% when $n = 1$. We explore how this proportion changes in a generalized setting as we vary the parameter n. Also in the classic setting, arbitrarily large square patches of invisible lattice points can be found. In the general setting, this is also true. We explore this and other invisible patterns in our generalized setting.

90. Secure Multiparty Computation on a Group

Megan Golbek California State University, Monterey Bay
Advisor(s): Dr. Mike Rosulek, Oregon State University

A function is private if there exists a protocol that is secure, meaning no party can learn any additional information about the other parties' inputs other than what follows from their own input and the function's output. Previous studies have investigated secure multiparty computation with broadcast communication, but this communication does not capture all secure protocols. We investigate secure multiparty computation with point to point communication in order to capture these protocols not captured with broadcasting. We present a secure three party protocol that computes an ordered product on a non-abelian group and begin a characterization of functions that can be securely computed for four or more parties. We present a proof that shows if four or more parties want to compute an ordered group product, then that group must be abelian. Secure multiparty computation has various applications, for example in medical research. Consider the scenario where multiple hospitals wish to share data obtained from their patients for research. Privacy policies can prevent revealing confidential patient information. Secure multiparty computation can create methods for sharing data without revealing any confidential information.

91. Typical Meteorological Year versus Actual Meteorological Year Weather Data: How modern data collection improves previous methods

Jacob Goldberg State University of New York College at Geneseo
Michael Ramsey State University of New York College at Geneseo
Advisor(s): Dr. Amanda Tucker, University of Rochester

Typical Meteorological Year (TMY) data files are intended to be representative of actual weather by carefully piecing together historical weather data to create a "typical" year. These TMY files are employed by those working in the field of building technology and clean energy to gauge how much energy a building will use. This process is currently the standard for industry; however the use of actual weather data might produce more accurate predictions. Actual weather data is increasingly easy to come by, with many stations offering an abundance of data for free or for little cost. We utilize TMY and Actual Meteorological Year (AMY) data along with the Department of Energy's EnergyPlus Model to estimate heating and cooling loads of a residential home in Rochester, NY. We perform statistical analysis to demonstrate that there are striking differences in the predicted energy consumption. In response to a request from our industry partner, Resource Refocus, LLC, we also give ideas for how a more detailed portfolio of energy use might look now that computational power is much larger than it was at the inception of energy modeling.

92. Modeling Cardiovascular Dynamics during Blood Withdrawal

Jacob Goldberg State University of New York College at Geneseo
Mitchel Colebank Clemson University
Madison Kellar Colorado School of Mines
Advisor(s): Dr. Andrea Arnold, Mette Olufsen, North Carolina State University

The body continuously regulates system properties to maintain blood pressure at homeostasis, which prevents fainting or light-headedness during everyday activities. This project aims at understanding how system properties are controlled during blood withdrawal by utilizing a mathematical model to predict cardiovascular dynamics. More specifically, a five-compartment model of systemic circulation is employed and validated against experimental pressure and volume data measured in the hearts of rats. Data was made available from collaborators at the University of Michigan and the Medical College of Wisconsin.

The body's natural responses during blood withdrawal, such as increased heart rate and vasodilation, are represented by model parameters, and the objective of this project is to understand how these parameters change in time to predict experimental observations. Techniques used to estimate time-varying dynamics include piecewise linear splines and ensemble Kalman filtering, with the latter providing a measure of uncertainty in the parameter estimates. Future work aims at applying these techniques to patient-specific modeling and medicine to improve the quality and efficiency of treatment for underlying cardiovascular conditions.

93. Associating Blood Pressure with SNPs through Statistical Mapping

Pedro Henrique Gomes Machado University of Wisconsin - Eau Claire

Advisor(s): Dr. Abra Brisbin, University of Wisconsin - Eau Claire

Genes carry important information about diseases and other traits. In this poster, we apply statistical tests including linear regression to identify SNPs, a type of genetic variant, which are associated with blood pressure in a data set of individuals from Brazil. A baseline analysis of 97223 SNPs on all 22 autosomes using age and sex as covariates finds 209 associations that are nominally significant at the .001 significance level. To refine this signal, we compared these results with the results of other linear regression analyses, including genome-wide or local ancestry as a covariate. Including local ancestry as a covariate results in 61 associations, providing a smaller set of high-priority SNPs to investigate for a potential causal relationship with high blood pressure. We have additionally investigated short intervals containing each associated SNP, and found 28 intervals containing SNPs with significant associations via all three linear regression analyses. These results indicate that using ancestry as a covariate is a powerful technique for prioritizing associations when performing large-scale multiple testing of statistical associations between genetic variants and complex traits such as blood pressure.

94. Finite Frames and the p-Frame Potential

Victor Gonzalez University of Maryland, College Park

Advisor(s): Dr. Kasso Okoudjou, University of Maryland, College Park

Whenever there are more than N vectors in a finite frame — that is, a spanning set — for $\mathbb{R}^N$, the vectors will not all be perpendicular, but they will overlap with one another. One way to measure this overlap is with the p-frame potential. Minima of the p-frame potential are guaranteed to exist, and we attempt to find the minimizers of this potential. We present conjectures about minimizers of the p-frame potential with frames of $M \geq 2$ vectors in $\mathbb{R}^2$ as well as frames of $N + 1$ vectors in $\mathbb{R}^N$. We interpret the former as a geometric configuration and the latter both through an iterative, geometric construction and in the context of two-distance sets.

95. Estimates of Life Table Functions Based on Total Post-Secondary Enrollment

Diana Gonzalez California State Polytechnic University, Pomona

Advisor(s): Dr. Ariel Cintron-Arias, East Tennessee State University

Matrix models are well-known in mathematical demography to describe the time evolution of age groups in populations. Here we propose several matrix models in the context of student enrollment. They are validated against longitudinal observations obtained from the Integrated Postsecondary Education Data System. Least squares methods are implemented (global stochastic optimization schemes) to obtain parameter estimates. Furthermore, we employ Akaike Information Criterion as a statistic for model selection. Bootstrap sampling is carried out to obtain model parameter uncertainty bounds. We discuss the point estimates of student life expectancy, probability of school departure, average number of years in each school group (e.g. sophomore), for fifty-one longitudinal datasets.

96. The Dimensions of the nil-Temperley-Lieb Algebras of Type A and a Variation on the Algebras

Niket Gowravaram MIT PRIMES Program

Advisor(s): Dr. Tanya Khovanova, Massachusetts Institute of Technology

We investigate nil-Temperley-Lieb algebras of type A. We give a general description of the structure of monomials formed by the generators. We also show that the dimensions of these algebras are the famous Catalan numbers by providing a bijection between the monomials and Dyck paths. We show that the distribution of these monomials by degree is the same as the distribution of Dyck paths by the sum of the heights of the peaks minus the number of peaks.

We also investigate a variation on the algebras created by removing one of the relations. In some sense, this variation can be considered as a type B of the other algebras. We give a description of all possible monomials in this second class of algebras. We also show that the dimension of these algebras is the sequence $\binom{2n}{n}$, by showing that the dimension is the Catalan transform of the sequence 2^n.

97. Automated Diagnosis of Melanoma

Anna Grim University of St. Thomas
Advisor(s): Dr. Chehrzad Shakiban, University of St. Thomas

Each year there are more cases of skin cancer than the combined incidence of breast, prostate, lung, and colon cancer. With the growing number of cases each year, there is a rising demand for automated diagnosis of skin lesions rather than invasive biopsy. In this paper, we present an automated diagnosis algorithm that analyzes the color distribution of a skin lesion to determine its diagnosis. We present two methods called global fractal color dimension and the global plane method to measure a skin lesion's degree of global color uniformity. After testing our methodology on approximately 130 tumors, we have been able to successfully diagnose melanoma with an accuracy of 95.71% and 96.15%, respectively.

98. Cyclic Evasion in the Four Bug Problem

Anna Grim University of St Thomas
Advisor(s): Dr. Vadim Zharnitsky, UIUC

We consider the four bug problem where bug j evades bug $j+1$. Based on our experimental and theoretical findings, we have determined that all four bug configurations converge to a stable configuration of either a square or a self-intersecting line. However, a slight perturbation of the square can cause the system to enter a limiting cycle where the configuration does not converge to a stable configuration in finite time.

99. An Infection Spreading Model on Binary Trees

Daniel Guo Homestead High School
Advisor(s): Dr. Partha Dey, University of Illinois at Urbana-Champaign

An important and ongoing topic of research is the study of infectious diseases and the rate at which these diseases spread. Modeling the spread and growth of such diseases leads to a more precise understanding of the phenomenon and accurate predictions of spread in real life. We consider a long-range infection model on a binary tree. Given a spreading coefficient $\alpha > 1$, the time it takes for the infection to travel from one node to another node below it is exponentially distributed with rate $2^{-k}k^{-\alpha}$, where k is the difference in layer number between the two nodes. We simulate and analyze the time needed for the infection to reach layer m or below starting from the root node. The resulting time is recorded and graphed for different values of α and m. Finally, we prove rigorous lower and upper bounds for the infection time, both of which are approximately logarithmic with respect to m. The same techniques and results are valid for other regular n-ary trees, in which each node has exactly n children where $n > 2$.

100. Magnetic Spectral Decimation on the Diamond Fractal

Madeline Hansalik Texas A&M University
Stephen Loew Coe College
Advisor(s): Dr. Luke Rogers, University of Connecticut

The Laplacians for a large class of self-similar fractals and fractal graphs exhibit a property called spectral decimation, in which the spectra of different levels of approximation are related by a dynamical system involving a rational function. Expanding upon the work of Malozemov and Teplyaev [1], we extend some aspects of the spectral decimation method from the Laplacian operator to a magnetic Laplacian operator, and use this to numerically investigate properties of the magnetic spectrum of this operator on the diamond fractal. In particular, we identify the correct unitary transformations and projections to obtain the aforementioned rational functions.

[1] Leonid Malozemov, Leonid; Teplyaev, Alexander "Self-Similarity Operators, Operators, and Dynamics." *Mathematical Physics, Analysis and Geometry,* Vol. 6, #3 (2003).

101. Modeling and Optimizing Newton's Method Through Fractals

Melanie Harrison Lewis University
Elizabeth Langland Lewis University
Advisor(s): Mr. Ray Klump, Lewis University

Computer models allow us to take a closer look at a mathematical idea such as fractals. Fractals are a visual form of a mathematical pattern. Through iterations, a function is made visible with the fractal representation. An algorithm

used to find the value of a root is Newton's Method. Through iteration of Newton's Method, we slowly approximate the value of a root of a polynomial over an initial value or interval. Eventually, this algorithm will converge to a root. Finding this root can be seen graphically and visually through fractals. We have engineered a program to show Newton's Method finding the root by plotting the corresponding fractal. Modeling this method mathematically allows for a visualization of an algorithm and we are able to see how a root is selected. Furthermore, modeling this process through allows us to take a closer look at Newton's Method and how to optimize this process. The purpose of this project is to create a model to look at how we find roots and how they are estimated by an algorithm. We also look at how we can optimize the root-finding process by using quadratic convergence rather than linear and how that differs in fractal analysis.

102. Analyzing Error Associated with Modeling a Swimming Worm

Cassie Hartley James Madison University

Advisor(s): Dr. Eva Strawbridge, James Madison University

This research examines a method of modeling the fluid movement associated with a swimming 1 mm long nematode (C. elegans). By modeling the worm as a sine curve, $\sin(2\pi x - \omega t)$, the induced fluid flow around the worm can be studied using the method of regularized stokeslets. As with any numerical method, there is an error associated with this modeling process. This error is the focus of my research. I compute the arc length of the worm using multiple methods to examine whether this has any impact on error in this modeling process. I compare the output of each in order to study sources of error in our calculations of the movement of the artificial worm. I found that the two methods yielded different results.

103. Mixing Time in Robotic Explorations

Chang He Centre College

Shun Yang Carleton College

Advisor(s): Dr. Yuliy Baryshnikov, University of Illinois at Urbana-Champaign

We consider the mixing time of a point robot that moves randomly in parallel to the coordinate axes in an open domain $B \subset R^2$, which is a collection of rectangles having internal walls restricting the motion of the robot. We call such domain a *gallery*. For a given gallery with s sides and a bottleneck ratio of ϵ, we find that the mixing time for the robot to explore the entire gallery has an upper bound of $O(s^2/\epsilon)$. We also give exact bounds of the mixing time in some special gallery configurations.

104. Implementation of blob basis functions in proton CT reconstruction

Howard Heaton Walla Walla University

Advisor(s): Dr. Reinhard Schulte, Yair Censor, Keith Schubert, LLU, University of Haifa, Baylor University

Proton CT uses physical measurements of protons to reconstruct images of patient parameters that are important for proton therapy. The goal of this work is to demonstrate the efficacy of blob basis functions in comparison to voxel basis functions for proton CT reconstruction and to quantitatively determine the accuracy of resulting images. Mathematical methods were introduced that utilize estimations of the most likely paths (MLPs) traveled by protons through a scanned object to identify the intersection lengths of the MLPs through each blob. Two implementations with different levels of accuracy and computational cost were developed. It was found that, unlike for voxel-based reconstructions, it is not necessary to take all intersected blobs into account. In ongoing work, we are using an algorithm that computes standard iterative projections onto convex hyperplanes to reconstruct an image of the scanned object. Both experimental proton CT data and Monte-Carlo-simulated data will be used to quantitatively evaluate image accuracy and computational efficiency of blob and voxel-based reconstructions. Comparisons will be made between an effective mean chord length approach in voxel-based reconstructions and exact and effective mean chord length approaches in blob-based reconstructions.

105. Knot Tricolorability

McKenzie Hennen University of Wisconsin- Eau Claire
Danielle Brushaber University of Wisconsin- Eau Claire
Advisor(s): Dr. Carolyn Otto, University of Wisconsin- Eau Claire

Our team has been researching the colorability of knots, an invariant used in classification. We focused on the n-Whitehead double and Pure double of knots that have been observed in circular DNA. Representing the knot as a matrix allows us to determine its colorability, and we are interested in knots that are tricolorable. We have found a pattern in the colorabilities of Whitehead doubles of certain knots when twists are added within the link. We have developed conjectures about these, and have proven tricolorability for the Whitehead double of knot 5_1. We are investigating the relationships of different doubling operators and tricolorability.

106. Left, Right, Center Dice Game

Richelle Hites Nevada State College
Advisor(s): Dr. Serge Ballif, Nevada State College

The game of LCR is played with 2 or more players. Each player starts the game with a pile of chips. Players take turns rolling a die with L, C, R, and dot on it. Players will pass chips to the left, right, center or keep the chips. The last player with chips is the winner. In this project we calculate the probability that each player will win games by modeling the game with a probability transition matrix. We show that the last player will have the greatest chance of winning. we also fine the expected length of the games for each number of players and chips. We also investigate which final outcome is most likely and which starting position is most balanced?

107. Image Processing: an Application in Placental Imaging

Madison Hoffman Kenyon College
Robin Belton Kenyon College
Advisor(s): Dr. Elin Farnell, Shawn Farnell, Kenyon College

In recent years, research in the field of placental analysis has demonstrated connections between placental features, such as shape and vascular structure, to the health and development of newborn babies. In an attempt to improve automation of vascular-structure extraction, researchers and medical practitioners have injected placental veins and arteries with colored dye. During the process of dye-injection, placental characteristics may be altered. Using a data set consisting of 19 placentas, we aim to determine the level of similarity between hand-drawn tracings of the vascular structure of placental images before and after dye-injection. In this presentation, we present explorations of various measures of similarity. Initial findings suggest that some important features of placental structure are preserved throughout the process of dye-injection.

108. Shifted Numerical Monoids and the Catenary Degree

Jesse Horton University of Arkansas
Mesa Williams Lee University
Rebecca Conaway Monmouth University
Advisor(s): Dr. Roberto Pelayo, University of Hawaii-Hilo

Numerical monoids (co-finite additive submonoids of $\mathbb{N}$) are known to have non-unique factorizations into irreducible elements. To quantify this interesting factorization theory, the catenary degree measures how many irreducible elements different factorizations for the same element are not shared in common. For a numerical monoid S, the catenary degree of S, denoted $\mathbf{c}(S)$, is the largest catenary degree of any element. In this poster, we explore numerical monoids of the form $M_n = \langle n, n + r_1, n + r_2, \ldots, n + r_k \rangle$ with shift parameter n. We show that the sequence of catenary degrees $\{\mathbf{c}(M_n)\}_{n \in N}$ for M_n has an eventual quasi-linear behavior. In the process of studying this catenary degree sequence, we prove a broader result of interest in commutative algebra: the Betti elements and minimal presentations of these shifted numerical monoids have eventually periodic structure with period r_k.

109. Unraveling Celtic Knots

Shawna Howard Westminster College
Advisor(s): Dr. David Offner, Westminster College

Celtic knots have fascinated both artists and mathematicians alike for centuries. A Celtic knot may be thought of as a laser which bounces off the edges of a border until it returns to its point of origin. In this presentation, we will examine what is known about Celtic knots in a rectangular $p \times q$ panel in regards to the number of components within a panel and mirrors. We will use these facts to develop a method for predicting when a mirror will split a component which crosses itself in a relatively prime panel. We will also use what is known for rectangular panels to explore the mathematical properties of other Celtic knot boundaries.

110. A Stock Market Predictor based on a modified Neural Network Algorithm

Jason Howell Western Connecticut State University
Advisor(s): Dr. Xiaodi Wang, Western Connecticut State University

Artificial neural networks are well known tools for data mining in different fields. In this research paper we will develop a modified neural network algorithm to be used to predict stock market values and compare the results to other algorithms. In general neural networks are systems that receive data, interpret it, then make predictions. Over time the network can be trained so that it can make better predictions. Here we show how this modified neural network can predict stock market values better using various market indicators as input.

111. Reliability Polynomials of Simple Graphs having Arbitrarily many Inflection Points

Christopher Hunt Vassar College
Danielle Blackwell
Advisor(s): Dr. David Milan, The University of Texas at Tyler

The reliability polynomial of a graph is the probability that the graph remains connected if each edge is included with independent probability p, as a function of p. Such polynomials are always increasing and thus cannot have any critical points (except at 0 and 1), and so a natural question to consider the inflection points of such polynomials. It was initially conjectured by Colbourn that there could be at most one, and Brown, Koc, and Kooij found examples of graphs whose reliability polynomials have two inflections points. Christina Graves and David Milan were able to show that there exist non-simple graphs with reliability polynomials that have at least n inflection points for each positive integer n, but the graphs resulting from this construction had many multiple edges. By generalizing the construction and making use of some properties of the reliability polynomial of the complete graph, we show that this result can be achieved using only simple graphs, and provide concrete examples with up to five inflection points.

112. Locking and Unocking Sparse Configurations of Hard Disks in Convex Shapes

Esther Hunt George Fox Unversity
Emily Black Wesleyan University
Advisor(s): Dr. Yuliy Baryshnikov, University of Illinois Urbana-Champagne

In our research, we investigated sparse configurations of hard disks in convex shapes. It is commonly believed that hard disks lining smooth convex shapes are locally jammed but unlocked, as with bearing balls in a circle. However by modeling various constructed boundaries, we discovered possible locking and unlocking regimes in the configuration space.

113. Mathematical Modeling of the NF-κB signaling pathway using PySB

Geena Ildefonso University of Central Florida
Advisor(s): Dr. Carlos Lopez, Vanderbilt University

Nuclear Factor-κB (NF-κB) is a transduction pathway centering around transcription factors that regulate gene expression in response to environmental stimuli. Aberrant activation of NF-κB has been linked to inflammation, and improper immune development. In addition, NF-κB has a pivotal role in the initiation and progression of several cancers. Importance of NF-κB to these pathologies has led to the development of many mathematical models over the past decade, motivated by the need to gain a detailed quantitative and predictive understanding of biological systems. In particular, Tay et al.'s model successfully recapitulates key aspects of NF-κB signaling, and in turn has yielded

insights into the pathway's structure, dynamics, and function. Recently, modeling frameworks such as PySB have emerged, which construct mathematical rule-based models of biochemical systems as computer programs. PySB facilitates reusable, shareable, and transparently developed biological models. Here, we implement Tay et al.'s model of NF-κB signaling in PySB, and examine the pathway's ability to control programmed cell death through regulation of anti-apoptotic signals. We also intend to link the model with other models of apoptosis and necrosis to better understand cell fate outcomes in cancers.

114. Fantasy Football Prediction Using Bayesian Hierarchical Modeling

Adam Jauregui California State University, Fresno
Advisor(s): Dr. Steve Chung, California State University, Fresno

Fantasy football modeling has seen a sudden spike in interest in the past few years from academic researchers. Their objective has been to model an NFL quarterback's fantasy points for each NFL game in hopes of making accurate fantasy point predictions. In line with this growing body of research, we propose a Bayesian hierarchical model to estimate an NFL quarterback's fantasy points and compare that to a Frequentist approach, testing the predictions of both models on the 2014 NFL season. Overall, the Bayesian hierarchical model outperformed the Frequentist model for the 20 NFL quarterbacks tested. In addition, we tested every quarterback's "inflation-adjusted" fantasy points to compensate for the fact that quarterbacks typically score higher fantasy points in today's game than in the past.

115. Combinatorial Games of No Strategy

Caleb Ji The Bear Creek School
Advisor(s): Dr. Tanya Khovanova, MIT

We study a particular class of combinatorial games called Games of No Strategy, which are games whose winners are predetermined. These games were first named and studied by Dr. James Propp, and we continue his work here. Finding the number of ways to play such games often leads to new combinatorial sequences and involves methods from analysis, number theory, and other fields. In the game Chocolate Break, we use a recursion to calculate the number of ways to play, which leads to four new sequences in the OEIS. We also use tools such as the Law of Quadratic Reciprocity and hypergeometric functions to study the divisibility and periodicity properties of these sequences. For the game Planted Brussel Sprouts, a variation on the well-known game Sprouts, we find the number of end-states through a bijection a and obtain a new proof that the number of ways to play is equal to the number of spanning trees on n vertices, Finally, in the game Binary Fusion, we show through both algebraic and combinatorial proofs that the number of ways to play generates Catalan's triangle.

116. Modelling the Evolutionary Dynamics of the Mutating Human Immunodeficiency Virus within and between T-cells

Talon Johnson Morehouse College
Advisor(s): Dr. Shelby Wilson, Morehouse College

Human immunodeficiency disorder, or HIV, is a sexually transmitted disease that weakens one's immune system by compromising helper T cells, ultimately resulting in the development of AIDS. Like many viruses, HIV has a high degree of error in viral DNA replication and synthesis, thus resulting in mutations. These mutations lead to a large number of quasi-species, in which in the human immune system can't mount an effective response of these mutations. A mathematical model of nonlinear differential equations with piecewise constants highlights the pessimistic effects that HIV population growth has on the T-cell population. We will give results of model that follows four populations: the alternate version T- cell model, latent (infected) T-cell population model, active T-cell population model, and dynamics of the HIV virus population model. Furthermore, we will modify the model to reflect of evolving mutations of an HIV infection and immune response.

117. Hopper Bands: Locust Aggregation

Ryan Jones Harvey Mudd College
Advisor(s): Andrew Bernoff, Harvey Mudd College

Locusts cause massive amounts of plague and famine wherever they travel. Locust swarms eat through vegetation and croplands as they travel, causing devastation wherever they go. Once a critical density is reached, aggregating locusts

form hopper bands that extend over many kilometers. These bands have an average density of 50 locusts/m^2 and can range from 20-120 locusts/m^2. Merging hopper bands lead to higher densities, which causes the swarm to become airborne. This is a problem, since flying swarms are much more costly to control than hopper bands. Presented here is a new model directed towards locust behavior. With minimal assumptions, this model elicits biologically accurate locust behavior. The goal of this model is to give insight into how these hopper bands are created, so that efforts can be made to prevent their formation.

118. A Discrete Age-Structured Model of Newt Population Dynamics During a Period of Drought

Marjorie Jones Pepperdine University
Advisor(s): Dr. Courtney Davis, Timothy Lucas, Pepperdine University

We investigate the impact of California's historic drought on the persistence of a native amphibian, the California newt (*Taricha torosa*), in the Santa Monica Mountains. We construct mathematical models that capture the observed decline of California newt populations in drought conditions. We develop a set of nonlinear difference equations that incorporate probabilities for newt maturation and survivorship to model each life stage of the newt. In the model, newt fecundity is dependent upon variable rainfall and stream characteristics. We ground our model biologically with local newt population data collected since 1992. We use the model to predict how future drought patterns could impact these newt populations. The model's prediction of newt decline or persistence given potential future drought patterns can inform the need for restorative measures to prevent local extinction of newt populations. Based on model simulations we predict how the number of available newt egg-laying sites varies with rainfall. Also, we see that even with severe drought, newt populations can rebound if the drought is sufficiently short. We plan to extend our model to explore the impact of drought or flood conditions on invasive predators such as crayfish.

119. Removing ocular artifact from electroencephalogram data utilizing eye-tracking technology

Rachel Kaale Simpson College
Christopher Hanson Simpson College
Matthew Hayden Simpson College
Advisor(s): Dr. Donald Evans, Simpson College

Electroencephalogram (EEG) data is often riddled with unwanted artifact including, but not limited to eye blinks and eye movements, interference from AC electrical devices, changes in skin potentials, and muscle activity. These artifacts can make it very difficult to accurately interpret EEG data. Therefore, this research has been dedicated to creating a novel approach for ocular artifact removal in EEG data that utilizes eye-tracking technology, is simpler than other widely-used methods, and requires only a small number of EEG channels. These goals were accomplished with the Simpson College Eye-Track Algorithm (SCETA) which was developed in MATLAB. This algorithm utilized matrix mathematics and statistical analysis to accurately remove artifacts created by eye blinks. During the fall of 2015, an IRB approved study was conducted to test the technique and it was found that while SCETA was not as effective as some other artifact correction techniques such as Independent Component Analysis, its speed, versatility, and applicability to data sets with small numbers of channels make it a strong option as an ocular artifact correction technique.

120. Multi-Peg Towers of Hanoi

Kruz Kalke University of Arkansas
Advisor(s): Dr. Ernst Leiss, University of Houston - Computer Science

The Towers of Hanoi puzzle continues to present interesting, unsolved problems in discrete mathematics and computer science. While there is no proof that the Frame-Stewart algorithm is optimal for any number of pegs, here we give computational tools to generate the sequence of numbers needed to optimize this algorithm. Another goal of this project is to prove worst-case claims for certain restricted movement graphs when using parallel moves. We compare our results to sequential movements and show how parallelism gives different bounds for least-optimal movement graphs.

121. Improved Lower Bounds on Kissing Numbers in 25 through 31-Dimensional Euclidean Space.

Kenz Kallal Boston University - PROMYS
Advisor(s): Dr. Henry Cohn, Microsoft Research Institute

This project is concerned with constructing large kissing configurations in 25 through 31-dimensional Euclidean space, which are sets of non-overlapping unit spheres in $\mathbb{R}^n$ for $25 \leq n \leq 31$ all externally tangent to the unit sphere S^{n-1}.

Before our work, the current greatest lower bounds for the kissing numbers in these dimensions were obtaining by finding a set S with $|S| = 480$ of minimal vectors of the Leech Lattice Λ_{24} such that for any distinct $x, y \in S$, we have $\langle x, y \rangle \leq 1$. Then, a probabilistic argument based on the automorphism group of the Leech lattice gives a collection of disjoint subsets of size decreasing from $|S|$ with the same property. Cohn, Jiao, Kumar, and Torquato proved that these subsets can be used to explicitly construct kissing configurations in dimensions 25-31, which leads to the current bounds. We introduce a new algorithm structured around simulated annealing, achieving $|S| = 488$. Moreover, we use another greedy construction to increase the lower bounds on each one of the disjoint subsets of Λ_{24}. The result of both of these constructions is an increase in the lower bounds for the kissing numbers in dimensions 25 through 31.

122. Probabilistic Analysis of Maximum Soliton Length in the Box-Ball System

Karthik Karnik Boston University - PROMYS
Mikaeel Yunus Boston University - PROMYS
Advisor(s): Dr. Lionel Levine, Cornell University

Cellular automata are dynamical systems that, unlike ordinary physical systems, derive complexity from a simple set of rules. The box-ball system is a solitonic cellular automaton in which the movement of balls governs a one-dimensional particle physical universe. We determine that in the steady state of the generalized infinite box-ball system, the positions of consecutive filled boxes will form solitons of increasing length. Using the remarkable analog of these solitons to physical wave-particles, we present a simple algorithm to fully characterize solitons in the steady state. We implement this algorithm using a program in Python 3.0 to analyze the maximal soliton length for a given probability p that a box is filled in a BBS of initial length n. By applying existing notions in probability theory and using the concept of the random walk, we prove empirical bounds on the order of the expected maximum soliton length of a BBS for various values of p. Our proofs extend to determining the order of the highest point of the reflecting random walk for varying p.

123. The Characterization of Vectors with the Haar Property

Terrence Kelleher Bridgewater State University
Advisor(s): Dr. Shannon Lockard, Bridgewater State University

With every passing day, the volume of digital communication within our society grows larger and larger. Therefore the need for finding effective ways to send a message and ensure its reception is growing just as rapidly. One way to do this is by using error correcting codes; the mathematics behind one such code is the focus of this project. Let $\mathbf{v}$ be some vector in $\mathbb{R}^n$ and let D_{2n} be a matrix representation of the dihedral group of order $2n$. We define $D_{2n} * \mathbf{v}$ to be the set of vectors resulting from every matrix in D_{2n} being multiplied by $\mathbf{v}$ on the right. We say that $\mathbf{v}$ has the Haar property if and only if all subsets of order n of $D_{2n} * \mathbf{v}$ are linearly independent. It has been shown that vectors with the Haar property exist in all $\mathbb{R}^n$ where n is odd. Here we give the characteristics of vectors in $\mathbb{R}^3$ that have the Haar property. We also provide some conjectures about vectors in $\mathbb{R}^5$ that have the Haar property. It has also been shown that when n is even, vectors in $\mathbb{R}^n$ do not have the Haar property. Thus for even n, we will define a modified Haar property and explore characteristics of vectors in $\mathbb{R}^4$ satisfying the modified property.

124. A Mathematical Model to Study the Joint Effects of Genetics and Diet on Obesity

Victoria Kelley James Madison University
Demetrius Rhodes University of South Carolina Beaufort
Advisor(s): Dr. Carlos Castillo-Chavez, Arizona State University

Obesity has become one of the most pervasive epidemics facing North America today. According to the Centers for Disease Control and Prevention, more than one-third of adults in the United States are obese, and approximately 17% of children and adolescents are obese. Previous studies show that a particular SNP in the fat mass and obesity associated *FTO* gene is associated with the expression of obesity. We build a system of non-linear ordinary differential equations that considers both genetic and environmental effects on populations with three distinct genotypes. The autosomal dominant allele is A, therefore individuals who have the genotypes AA and Aa express the *FTO* gene. Equilibria analysis and simulation results show that over a long period of time, when the birth frequency of each genotype is dependent on current allele frequencies, the dominant allele is outbred by the recessive gene allele. Simulation results further show that having the allele A has a stronger impact on obesity than the environment. This study provides a

significantly new insight into the synergistic impact that genetics and diet play on obesity. Note that with genetic inheritance, environment makes no significant impact on the prevalence of obesity in the long term.

125. An explicit solution formula for the discrete Schrodinger equation on a half line lattice

Susan Kemboi University of Texas at Arlington
Advisor(s): Dr. Tuncay Aktosun, University of Texas at Arlington

The discrete Schrödinger equation

$$-\psi_{n+1} + 2\,\psi_n - \psi_{n-1} + V_n\,\psi_n = \lambda\,\psi_n, \qquad n \geq 1,$$

is a second-order difference equation used to describe the quantum mechanical behavior of a particle of energy λ in the force field of a real-valued potential V_n. The goal is to obtain a large class of explicit solutions to this difference equation in a systematic way in terms of elementary functions of the discrete variable n. This is done in two stages. In the first stage the direct scattering problem is analyzed, and in the second stage the inverse scattering problem is analyzed. In the direct problem, given potential values V_n the corresponding scattering matrix is determined. In the inverse problem, given the scattering matrix the corresponding potential values are determined. An explicit solution formula is developed by choosing the scattering matrix as a rational function of the parameter z, where λ and z are related as $\lambda = 2 - z - z^{-1}$.

126. Counting MV Assignments of the Origami Snake Tessellation

Kristen Kenney Western New England University
Advisor(s): Dr. Thomas Hull, Western New England University

The origami snake tessellation is a corrugation-patterned fold that, unlike the Miura-ori or the square twist tessellation, contains vertices of degree four and six. This makes it challenging to determine the number of valid mountain-valley (MV) assignments of the snake tessellation, or the number of ways which the tessellation can be folded flat. We present upper and lower bounds on $S_{m,n}$, which denotes the number of valid MV assignments for an $m \times n$ snake tessellation. We also describe our efforts for finding a closed formula for $S(m, n)$. This work was supported by the NSF ODISSEI grant EFRI-1240441.

127. Consensus vs. fragmentation in a model of opinion dynamics

Ratna Khatri George Mason University
Advisor(s): Dr. Matt Holzer, George Mason University

We consider a continuous version of the Krause model of opinion dynamics. Interaction between agents either leads to a state of consensus, where agents starting out with random initial opinions converge to a single opinion as time evolves, or to a fragmented state with multiple opinions. We linearize the system about a uniform density solution and predict consensus or fragmentation based upon the most unstable mode of the dispersion relation. Analytical predictions are then compared to numerical simulations.

128. Analyses of a Bayesian Language Learning Model for Young Children

Katie Khuu University of California, Irvine
Advisor(s): Dr. Lisa Pearl, University of California, Irvine

Little is known about what children are learning in the early stages of linguistic development. As a case study, I investigate how children first learn grammatical categories, like noun and verb. This process begins around twelve months, when they are capable of using: (i) distributional cues such as which words appear together, and (ii) communicative cues like what utterance types (e.g., questions, statements, or commands) words appear in. I apply a promising learning strategy using both of these cues to an age-appropriate dataset in a Bayesian Hidden Markov Model. To evaluate whether the strategy was successful, I compare the inferred categories to adult categories and evaluate how useful the inferred categories are. Using qualitative analyses, I find that the inferred categories do not match up to the adult categories. As a result, I apply the perplexity measure to examine the predictability of the corpus given the inferred categories. My main findings suggest that what children would learn with this strategy is still very useful, even if it is not the adult-level knowledge yet. These results can inform expectations about what typically developing children should be able to learn about language early on.

129. Task Performance Dynamics in Social Insects

Katherine Kincade Arizona State University
Advisor(s): Dr. Yun Kang, Arizona State University

Division of Labor amongst social insects is frequently discussed in regards to the colony's worker population. However, before a colony achieves a worker population, a queen is required to perform all of the tasks necessary for her survival: foraging, building the colony, and brood care. A simple ODE model was developed through the use of a framework of replicator equations in dynamical environments to investigate how queen ants perform and distribute all of the tasks necessary for her and her colony's survival by incorporating individual internal thresholds and environmental stimulus. Modifications to the internal threshold, risk of performing the task, and the rate of increase of the environmental stimulus were also explored. Because of the simplicity of the model, it could also be used to measure the task performance of larger populations of social insects. However, the model has only been applied to the data collected from Pogonomyrmex barbatus single queen ants.

130. Investigating Microbial Communities in DCM-Contaminated Groundwater using 16S rRNA Gene Analysis and Functional Metagenomics

Veronica Kirchner Juniata College
Advisor(s): Dr. Kimberly Roth, Regina Lamendella, Juniata Collegge

Microbial community analysis was performed on groundwater samples from a mature remediation site at which an undetermined quantity of methylene chloride (aka dichloromethane or DCM) was released. Samples were collected during four quarterly events from 10 wells that ranged in DCM contamination level. 16S rRNA gene analysis was completed to identify and quantify microbial populations present in the samples. Statistical analyses were used to reveal trends between OTU abundance and environmental metavariables, including DCM concentration. Spearman's correlations suggested that DCM-degrading OTUs were highly correlated with DCM concentration in the sample. Biomarker analysis identified enriched OTUs based on Kruskal-Wallis statistical values. Adonis and anosim results confirmed the significance of the Principle Coordinate Analysis plots, which showed distinct clustering of the samples based on DCM concentration and location. Data visualization techniques then were used to show the trends found through computational analysis. Results indicated that DCM concentration contributes to the shaping of the microbial community, particularly at high concentrations. Future work will implement functional metagenomics to identify and quantify the relative abundance of functional genes present.

131. Bounded Tiling-Harmonic Functions on the Integer Lattice

Jacob Klegar Choate Rosemary Hall
Advisor(s): Dr. Sergiy Merenkov, CCNY-CUNY

Tiling-harmonic functions are a class of functions on square tilings that minimize a specific energy. These functions may provide a useful tool in studying square Sierpinski carpets. In this paper we show two new Maximum Modulus Principles for these functions, as well as a proof that the set of tiling-harmonic functions is closed. One of these Maximum Modulus Principles is used to show that bounded infinite tiling-harmonic functions must have arbitrarily long constant lines. Additionally, we give three sufficient conditions for tiling-harmonic functions to be constant and numerical bounds on the constant in an analogue of Harnack's Inequality. Finally, we explore comparisons between tiling and graph-harmonic functions, especially in regards to oscillating boundary values.

132. Zero Divisor Graphs of Commutative Rings

Mitchell Klein University of St. Thomas
Michael Driscoll University of St. Thomas
Advisor(s): Dr. Michael Axtell, University of St. Thomas

In our research we linked existing ideas of zero divisor graphs and annihilator class graphs of commutative rings to a new graph construction called the associate class graph. The goal of the research is to determine what ring-theoretic information these 3 graphs can provide. We show how these three graphs are not sufficient to always distinguish non-isomorphic commutative rings, and we classify when the zero divisors of a commutative ring form an ideal based solely on the associate class graph. In addition, we pose several open questions for future research.

133. A Study of Games Over Finite Groups

Daniel Kondratyuk Boise State University
Stephanie Potter Boise State University
Advisor(s): Ms. Liljana Babinkostova, Marion Scheepers, Boise State University

There are many fruitful connections between the research areas of game theory and cryptography. For example, game theory was used to solve cryptographic information exchange problems, such as secret sharing and secure multiparty computation (SMPC). Our research involves a study of two-player games defined over several algebraic structures. These structures are used as mathematical platforms for numerous modern cryptosystems. The aim of this research is to identify algebraic structures providing a defense against more recently developed protocol-based attacks while still supporting other security objectives. Our methods include investigation of games defined over various fundamental finite groups, followed by investigation of the effects of mathematical constructions on the strategic features of the games. Our research gives a complete analysis of certain classes of games defined over the finite abelian groups. We also give partial results for nonabelian groups; we conjecture that the problem for arbitrary groups is NP-complete.

134. Relaxed coloring of sparse graphs

Michael Kopreski College of William and Mary
Advisor(s): Dr. Gexin Yu, College of William and Mary

A graph G is $(d_1, d_2, \ldots, d_t)$-colorable if its vertices may be partitioned into subsets $V_1, V_2, \ldots, V_t$ such that for a given d_i, the maximum degree $\Delta(G[V_i]) \leq d_i$. We study this relaxed coloring of graphs with bounded maximum average degrees. In this paper, we give new upper and lower bounds for the maximum average degree of $(1, \ldots, 1, 0, \ldots, 0)$-colorable graphs when there are $a \geq 2$ 1-improper and $b \geq 1$ proper color classes.

135. Classifying 7-Dimensional Indecomposible Solvable Lie Algebras With Niradical Isomorphic to $A_{5,1} \oplus \mathbb{R}$

Daniel Kosten Grand Valley State University
Advisor(s): Dr. Firas Hindeleh, Grand Valley State University

This poster is the third in a series that examine seven-dimensional solvable Lie Algebras with a six-dimensional niradical. Low-dimensional solvable Lie Algebra classification started back in 1963 by Mubarakzyanov. and were completely classified up to dimension six. A general theorem asserts that if g is a solvable Lie Algebra of dimension n, then the dimension of its maximum nilpotent ideal (called the nilradical) is at least $\frac{n}{2}$. For the seven-dimensional algebras, the nilradical's dimension could be 4, 5, 6 or 7. The four- and seven-dimensional nilradical cases were classified. We examine the six-dimensional niradical case. We first looked for the six-dimensional nilpotent algebras and found 32 algebras. The first case was completed in 2014, and the second case was completed in 2015. In this project we focus on the class where the nilradical is isomorphic to a direct sum of the five-dimensional algebra $A_{5,1}$ and the one dimensional algebra denoted by $A_{5,1} \oplus \mathbb{R}$.

136. Modeling mumps outbreaks: Incorporating delay in stochastic, continuous time models

Brianna Kozemzak Saint Mary's College
Advisor(s): Dr. Elizabeth Wolf, Saint Mary's College

Mumps is a single stranded RNA virus that is transmittable through respiratory secretions. Review of literature reveals a delay of approximately 2-4 weeks between exposure to mumps virus and the onset of parotitis and other associated symptoms. We create and explore various stochastic, continuous time models adapted from the SIR disease model, some of which satisfy the Markov Property and others of which incorporate fixed or normally distributed delay. Such models are developed for simulation of mumps outbreaks among small communities to investigate the significance of incorporating delay in disease models and to address questions concerning mumps outbreaks among highly vaccinated populations. Preliminary results show significant differences in the course of mumps outbreaks in mathematical models that incorporate delay and those that do not.

137. Applications and mathematical challenges of digital image mosaicking

Hanna Kristensen Pepperdine University
Advisor(s): Dr. David Strong, Pepperdine University

We created an automatic algorithm that creates a mosaic of user-supplied digital images to reproduce a user-supplied single larger digital image. In creating the mosaic of smaller images to reproduce the larger image, the simplest approach in selecting which image to place in each larger image pixel location is to select the one image of all smaller images whose overall color or grayscale levels best matches that of the larger image at that pixel location. This result can be improved by implementing different best-fit techniques for each pixel location depending on the characteristics of the surrounding pixels, for instance, placing a high texture image in a location with a high standard deviation in pixel color. Other additional considerations arise in choosing a best fit, including: reshading the smaller images to better match the color or grayscale value at each large image pixel location; reproducing essentially solid areas in the larger image; creating smoothness between smaller images in the mosaic for a more visually pleasing large image; ensuring that all of the user-supplied supplied images are used at least once in the larger image, or possibly using all of the smaller images essentially an equal number of times; and numerical issues such as integer types in Matlab.

138. Rainbow Numbers with Respect to 2-Matchings and 3-Matchings

Juergen Kritschgau Bates College
Advisor(s): Dr. Michael Young, Iowa State University

Our results focus on the rainbow numbers of the various graphs with respect to M_2 and M_3. We find the rainbow numbers for all graphs with respect to M_2. From then on out, the number of troublesome cases increases for rainbow numbers with respect to M_3. We prove that the rainbow numbers of trees with a diameter of 6 or greater have $rb(T, M_3) = \Delta + 2$. We extend this result to all graphs with diameter 6 or greater. Our results suggest that $rb(G, M_3) = \Delta + 2$ for unconnected graphs G; this is an area for further study.

139. A Volume Argument for Tucker's Lemma in 2-dimensions

Beauttie Kuture Pomona College
Oscar Leong Swarthmore College
Christopher Loa University of Tennessee, Knoxville
Advisor(s): Dr. Francis Su, MSRI-UP

Sperner's Lemma is a combinatorial result that can be used to prove Brouwer's fixed point theorem and has many useful applications in economics. Recently, McLennan and Tourky provided a novel proof of Sperner's Lemma using a volume argument and a linear deformation of a triangulation. We adapt a similar argument to prove Tucker's Lemma on a triangulated 2-dimensional cross-polytope with the condition that its extreme points have distinct labels. However, the technique used in McLennan-Tourky's argument does not directly apply because such deformation would distort the volume of the cross-polytope. So, we remedy this by inscribing the cross-polytope in its dual polytope, triangulating it, and considering how the volumes of the deformed simplices behave. We then generalize the argument to apply to triangulated cross-polytopes whose vertices do not have distinct labels.

140. Maximum or Minimum RCI Values of Graphs Consisting of Trees with a Hexagonal Base

Giancarlo Labruna Montclair State University
Advisor(s): Dr. Aihua Li, Montclair State University

In this research, we investigate trees with a hexagonal base and their Randic Connectivity Index (RCI) values. We first focus on graphs with specific trees attached. The set of all such graphs with n vertices is named as $\Gamma(n)$. We give explicit RCI formulas for all of the graphs in $\Gamma(n)$. We determine among all of the graphs in $\Gamma(n)$ which graphs admit maximum or minimum RCI values. We show that the graph with the path P_n as the attached tree has the maximum RCI value. Correspondingly, the graph with the attached star S_n admits the minimum RCI value. We further investigate any tree attached to a hexagon and generalize the result from the previous special case.

141. On the Catenary Degree of Numerical Semigroups Generated by a Generalized Arithmetic Sequence

Dana Lacey North Central College
James Pangelinan University of Guam
Rachel Domagalski Central Michigan University
Marly Cormar
Advisor(s): Dr. Roberto Pelayo, Brian Wissman, University of Hawaii-Hilo

We give a closed form for the catenary degree of any element in a numerical monoid generated by a generalized arithmetic sequence in embedding dimension three. While it is known in general that the largest and smallest nonzero catenary degrees are attained at Betti elements, the current literature contains no information about the other realizable catenary degrees. By classifying each element in terms of its Betti element divisors, we identify all the catenary degrees achieved and where they occur. In addition, our research provides the dissonance number and the period value, even though previous works have shown only that the catenary degree is periodic using a non-existential proof.

142. An Exploration into Deriving and Simulating Stochastic Delay Differential Equations Using a Case Study in Pork Production

John Lagergren East Tennessee State University
Advisor(s): Dr. Michele Joyner, East Tennessee State University

In this poster presentation, we derive and simulate a stochastic differential equation (SDE) model based on a continuous time Markov chain (CTMC) model for pork production in North Carolina. We then confirm the validity of the model by comparing it to the deterministic solution and CTMC simulations. Then, we add fixed and random delays to the CTMC model and derive a stochastic delay differential equation (SDDE), in order to more accurately simulate reality. As above, we then confirm the validity of the SDDE model by comparing it to the delay differential equation (DDE) and the CTMC model with delays. Moreover, we investigate the differences between the models with and without delays. Finally, we explore applications of the SDDE beyond the derivation.

143. Leamer Monoids and the Huneke-Wiegand Conjecture

Miguel Landeros California State Polytechnic University, Pomona
JunHao Ren University of Guam
Karina Pena California State Polytechnic University, Pomona
Advisor(s): Dr. Roberto Pelayo, Brian Wissman, University of Hawaii at Hilo

The Huneke-Wiegand Conjecture is a long standing open problem in Commutative Algebra regarding torsion submodules of tensor products. The recent work of P. Garcìa-Sanchez and M. Leamer provides an approach towards a partial solution of the Huneke-Wiegand Conjecture by constructing a new class of monoids, S_Γ^s, known as Leamer monoids, whose elements correspond to arithmetic sequences contained in a numerical monoid Γ. In their work, P. Garcìa-Sanchez and M. Leamer show that special cases of the Huneke-Wiegand Conjecture are equivalent to the existence of irreducible elements of the form $(x, 2)$ in S_Γ^s. In this poster, we focus on identifying these irreducible elements in Leamer monoids derived from symmetric numerical monoids generated by both an arithmetic and generalized arithmetic sequence. We highlight a visual technique developed to identify and prove the existence of irreducible elements of the form $(x, 2)$ in S_Γ^s. Furthermore, we give an explicit description of these irreducible elements for any Leamer monoid derived form a symmetric numerical monoid whose generators form an arithmetic or generalized arithmetic sequence.

144. Prime Two-Ball Juggling Patterns

Jacob Landgraf Michigan State University
Advisor(s): Dr. Steve Butler, Iowa State University

Juggling patterns can be described by a closed walk in a (directed) state graph, where each vertex (or state) is a landing pattern for the balls and directed edges connect states that can occur consecutively. The number of such patterns of length n is well known, but a long-standing problem is to count the number of prime juggling patterns (those juggling patterns corresponding to cycles in the state graph). For the case of $b = 2$ balls we give an expression for the number of prime juggling patterns of length n by establishing a connection with partitions of n into distinct parts. From this

we show the number of two-ball prime juggling patterns of length n is $\big(\gamma - o(1)\big)2^n$ where $\gamma = 1.32963879259\ldots$. For larger b we show there are at least b^{n-1} prime cycles of length n.

145. Density of sets with distance restrictions

Dana Lapides Lafayette College
Hannah Turner University of Texas at Austin
Advisor(s): Dr. Tamas Keleti, Eotvos Lorand University (ELTE)

A 1-avoiding set is a set of points in $\mathbb{R}^n$ such that no two points in the set are exactly distance 1 from one another. Erdős conjectured that the maximum density of such a set in $\mathbb{R}^2$ is strictly less than 1/4. Some fairly good bounds have been established for these sets, but Erdős's conjecture has yet to be proven for arbitrary 1-avoiding sets. 1-avoiding sets can be generalized to $[d, \delta]$ interval-avoiding sets, with one-avoiding sets being the special case when $d = \delta = 1$. We use an equivalence relation on the points of $\mathbb{R}^n$ to present results that help to describe arbitrary $[d, \delta]$-avoiding sets.

146. Infinite Automata: A new computational model

William Laprade Westfield State University
Advisor(s): Dr. Walter Lawrence Griffith, Westfield State University

Since its invention in 1936 by Alan Turing, the Turing machine has been an important tool in the development of our understanding of algorithms and the power of computation. However the Turing machine is not without its limitations. Tasks such as solving the Halting problem or constructing non-computable numbers cannot be done using Turing machines. Our solution to these limitations is to use, Infinite Automata, a new computational model, which we define as a deterministic machine that consists of an infinite sequence of Turing machines. We will show that infinite automata have the power to do many tasks that Turing machines cannot.

147. New geometries for cellular automata

Phyllis Lau San Jose State University
Samanvitha Basole San Jose State University
Advisor(s): Tim Hsu, San Jose State University

A cellular automaton is a collection of "cells" arranged in a geometric pattern (often the Cayley graph of a group) in which the state of each cell evolves according to some rule based on the current state of its immediate neighborhood. For example, Conway's well-known Game of Life is a cellular automaton on the group $\mathbf{Z} \times \mathbf{Z}$ (i.e., the square grid) that is defined by a simple rule, but can nevertheless simulate a universal Turing machine. We investigate cellular automata on the group $\langle a, b, c \mid 1 = a^2 = b^2 = c^2 \rangle$ (i.e., the infinite free trivalent tree). Specifically, using both experiments done with interactive software of our own design and theoretical methods, we describe ways in which natural generalizations of Conway's Game of Life to the infinite trivalent tree seem to be limited in their computational power by the geometry of the tree. We also describe possible future directions.

148. The Symmetry and Structure of Coloring Graphs

Farquhar Lauren Bethany Lutheran College
Benjamin Buckner Lee University
Advisor(s): Dr. Heather M. Russell, University of Richmond

The k-coloring graph of G is defined as the graph whose vertex set is all the proper k-colorings of G with edges between colorings if and only if they differ at precisely one vertex of G. Our research seeks to determine properties that indicate whether a graph is realizable as a coloring graph. In particular, we prove results about the automorphism group of a coloring graph and divisibility rules for subgraphs of coloring graphs. We also show that connected 3-coloring graphs are biconnected in addition to detailing structural results for k-coloring graphs with cut vertices and coloring graphs isomorphic to I_n.

149. Characterizing Primitive Nondeficient Number with a Given Number of Primes

Huyen Le Cal Poly Pomona
Fany Salazar Cal Poly Pomona
Advisor(s): Dr. Mitsuo Kobayashi, Cal Poly Pomona

In the first century, Philo of Alexandria wrote that the world was created in 6 days and the moon orbits the earth in 28 days because 6 and 28 are perfect numbers. These numbers also belong to the larger set of primitive nondeficient numbers (PNDs). A natural number is primitive non-deficient if the sum of the reciprocal of its factors is greater than or equal to 2 while any proper subsum is less than 2. A formula describes primitive non-deficient numbers with two prime factors. We study the question of characterizing PNDs with three prime factors.

150. Prime Labeling of Trees with the Gaussian Integers

Hunter Lehmann Seattle University
Andrew Park Seattle University
Advisor(s): Dr. Steven Klee, Seattle University

A graph on n vertices is said to admit a prime labeling if we can label its vertices with the first n natural numbers such that any two adjacent vertices have relatively prime labels. Here we extend the idea of prime labeling to the Gaussian integers, the complex numbers whose real and imaginary parts are both integers. We begin by defining an ordering on the Gaussian integers that lie in the first quadrant. Using this ordering, we show that several families of trees as well all trees of order at most 72 admit a prime labeling with the Gaussian integers.

151. Proving Tucker's Lemma With a Volume Argument

Oscar Leong Swarthmore College
Beauttie Kuture Pomona College
Christopher Loa University of Tennessee - Knoxville
Advisor(s): Dr. Francis Su, Harvey Mudd College

Sperner's Lemma is a statement about labeled triangulations of a simplex. McLennan and Tourky (2007) provided a novel proof of Sperner's Lemma by examining volumes of simplices in a triangulation under a piecewise linear deformation. We adapt a similar argument to prove the 2-dimensional Tucker's Lemma on a triangulated cross-polytope P. The McLennan-Tourky technique does not directly apply because the natural deformation may distort the volume of P. We remedy this by inscribing P in its dual polytope, triangulating it, and considering how the volume of deformed simplices behave.

152. Poster withdrawn.

153. Axisymmetric Swarm Equilibria

Zhaoqi Li Macalester College
Weerapat Pittayakanchit Harvey Mudd College
Advisor(s): Dr. Chad Topaz, Andrew Bernoff, Macalester College, Harvey Mudd College

Herbivorous animals tend to form groups as a survival mechanism against carnivores. One of the most popular models in the literature describes this situation as particles interacting kinematically via an attractive-repulsive Morse potential. We study a continuum description of this model, a nonlocal partial differential equation describing the time evolution of a population density field. Using the calculus of variations and stability arguments, we predict the outcome of the swarm. We compare these continuum results to two other models: the original particle model, and a model of interacting circles which exploits radial symmetry of the particle model. These models closely match the continuum results. The solution is circular symmetric and in some parameter regimes has a singularity (where the density becomes infinite) at the edge of the support. This result casts doubt on the validity of the Morse potential as a model for interactions in biological swarms, which have finite density.

154. The Double Gaussian Isoperimetric Problem

Jason Liang University of Chicago
Advisor(s): Frank Morgan, Williams College

In the Euclidean plane, a circle solves the isoperimetric problem: it provides the least-perimeter way to enclose a given area. In the plane with Gaussian weight or density, a straight line solves the isoperimetric problem: it provides the least-weighted-perimeter way to enclose a region with prescribed weighted area. (Because the density dies off exponentially, the line has finite weighted length and the half-plane on either side has finite weighted area.) We consider the isoperimetric problem where the density is the sum of two Gaussian densities and prove that when the densities have equal variances, the isoperimetric regions on the line are rays.

155. Obstructions to Convexity in Neural Codes

Caitlin Lienkaemper Harvey Mudd College
Advisor(s): Dr. Anne Shiu, Texas A&M Univerity

How does the brain encode the spatial structure of the external world? One way is through hippocampal neurons called place cells, which become associated to convex regions of space known as their receptive fields: each place cell fires at a high rate precisely when the animal is in the receptive field. The firing patterns of multiple place cells form what is known as a convex neural code. How can we tell when a neural code is convex? To address this question, Giusti and Itskov identified a local obstruction, defined via the topology of a code's simplicial complex, and proved that convex neural codes have no local obstructions. Curto et al. proved the converse for all neural codes on at most four neurons. Via a counterexample on five neurons, we show that this converse is false in general. Additionally, we describe our work in classifying neural codes on five neurons, supported by our enumeration of connected simplicial complexes on five vertices. Finally, we discuss the relationship between convex sets and good covers.

156. r-Completeness of Sequences of Positive Integers

William Linz Texas A&M University
Edna Jones Rose-Hulman Institute of Technology
Advisor(s): Dr. Peter Johnson, Auburn University

A strictly increasing sequence of positive integers (a_n) is said to be (weakly) *complete* if every sufficiently large positive integer is representable as a sum of distinct terms of (a_n). Complete sequences have been well-studied, and many specific examples, such as the sequence of powers of two ($a_n = 2^{n-1}$), are known. We introduce a new notion of r-completeness by defining a sequence (a_n) to be *r-complete* if every sufficiently large positive integer is representable as the sum of r or more distinct elements from (a_n). For example, the sequence $(2^{n-1})_{n\geq 1}$ is complete, but not 2-complete. We establish a number of results related to r-complete sequences. In particular, for any positive integer r we construct an example of a sequence which is r-complete, but not $(r+1)$-complete.

157. Numerical Approximations for Pollutant Transport in the Senegal River

Austin Little Morehouse College
Advisor(s): Dr. Christina Frederick, Georgia Institute of Technology

Many computational challenges arise when modelling pollutant transport in a river due to irregular geometries of real geographical structures. Jagged boundaries can introduce unwanted artifacts, such as numerical instability and non-physical solutions. In this research, pollutant transport in a river is simulated using a model for shallow water equations coupled with an equation for the concentration of a pollutant. The system of hyperbolic differential equations is implemented in MATLAB using a Lax-Friedrichs method that uses explicit finite differencing. Three approaches are considered to eliminate these unwanted artifacts. A mathematical approach involves applying a mapping from the original river geometry to a cartesian coordinate system and deriving the equations in the new coordinate system. Numerically, the issue could be addressed by modifying the numerical scheme to include ghost points. Physical considerations, such as adapting the physical parameters in the shallow water equations, are also discussed. The solutions are validated by comparison with solutions that are discretized on a fine grid. River geometry information from the Senegal River is extracted from graphical information systems (GIS) datasets.

158. An Equivalence Between the Polytabloid Bases and Specht Polynomials for Irreducible Representations of the Symmetric Group

Hengzhou Liu University of Wisconsin-Eau Claire
Advisor(s): Dr. R. Michael Howe, University of Wisconsin-Eau Claire

Let n be a positive integer and let λ be a semi-standard Young tableau whose shape is a partition of n. In this poster we demonstrate a map ϕ from the vector space of polytabloids obtained from λ to the vector space of polynomials in n variables that intertwines the action of the symmetric group S_n. We also obtain an equivalence between the bases elements for the irreducible representations of S_n in the polytabloid space and the Specht polynomials as described in Specht, W. (1935), "Die irreduziblen Darstellungen der symmetrischen Gruppe", *Mathematische Zeitschrift* 39 (1): 696–711.

159. On some notions of co-maximal number theory

Denise Lopez University of Puerto Rico - Mayaguez
Advisor(s): Dr. 'Reyes M Ortiz-Albino, University of Puerto Rico - Mayaguez

The theory of τ-factorization was defined in 2004 by Anderson and Frazier. It generalized most of the type of factorization studied, including the co-maximal factorizations by McAdams and Swon. We Study the co-maximal factorizations on the integers using Anderson and Frazier notation. Our main interest is to find several notions of number theory with respect to this new type of factorization. In this presentation, we give a characterization of the co-maximal factorizations, the greatest common divisor and least common multiple.

160. On r-hued coloring of planar and sparse graphs.

Kate Lorenzen Juniata College
Joshua Thompson Iowa State University
Advisor(s): Dr. Rong Luo, West Virginia University

For integers $k, r > 0$, a (k, r)-coloring (or *r-hued k-coloring*) of a graph G is a proper vertex k-coloring such that every vertex v of degree $d(v)$ is adjacent to at least $min\{d(v), r\}$ different colors. The *r-hued chromatic number*, denoted by $\chi_r(G)$, is the smallest integer k for which G has a proper (k, r)-coloring. The *maximum average degree* of G, $mad(G)$, is defined as the maximum of the average degrees taken over all subgraphs of G. For planar graphs, it is known that $\chi_2(G) \leq 5$ and we investigated $\chi_3(G) \leq 8$. We prove the following results by the discharging method. For any graph G, if $mad(G) < \frac{12}{5}$, then $\chi_3(G) \leq 6$; if $mad(G) < \frac{7}{3}$, then $\chi_3(G) \leq 5$; if G has no C_5 component and $mad(G) < \frac{8}{3}$, then $\chi_2(G) \leq 4$.

161. Configuration Spaces of Hinge Structures and Projective Geometry

Ben Lowe University of Chicago
Conrad Kosowsky
Advisor(s): Dr. Yuliy Baryshnikov, University of Illinois Urbana-Champaign

Milgram and Trinkle gave a more or less complete description of the configuration spaces of linearly immersed polygonal circles in $\mathbb{R}^3$ with fixed side-lengths, or polygonal linkages. Spaces of such configurations subject to the additional constraint of fixed angles between consecutive edges are not well studied, however, despite relevance in robotics and molecular biology. The first step to studying these configuration spaces is better knowledge of when the hinge structure, viewed as a robotic arm with revolute joints, infinitesimally loses full range of motion. We show that this occurs exactly when there exists a plane distribution given by a 1-form of the form $-ydx + xdy + cdz$, up to a rotation and a translation, that the polygonal hinge structure, considered as a piecewise linear curve, integrates. With this result, we give descriptions of some spaces of 'singular' configurations. We also prove a result in projective geometry about 5-tuples of points in $\mathbb{R}^3$.

162. The Application of Massey Methods in Rank Aggregation

Lingbo Lu Davidson College
Advisor(s): Dr. Timothy Chartier, Davidson College

Rank aggregation is an important tool for combining information from ranked lists with efficiency and accuracy. During the research, we worked to apply Massey Method, a well-known method for ranking sports teams, into an

algorithm for aggregating ranked lists. We also compare the results from the Massey method with that from Borda Count, a traditional method for rank aggregation. We firstly used Web data and aggregated the lists of top 100 results from Google and Bing, when searching the same keyword. However, possibly due to a lack of same entries that appear in both lists, there is a lack of monotonic correlation between the results from the two methods. We then switched to aggregate the ranks for the 351 teams in 2015 NCAA men's basketball. There is a high correlation between the outcomes from the two methods. We then compare the aggregated ranks with that from actual game results, and worked to improve the methods by adding weights to the ranked lists.

163. Accurate Solution and Gradient Computation for Interface Problems

Kevin Luna Northern Arizona University
William Reese North Carolina State University
Nnenna Anoruo University of South Carolina
Justin Rivera Wentworth Institute of Technology
Advisor(s): Dr. Zhilin Li, North Carolina State University

There are numerous applications of ODE/PDE boundary value problems such as the Navier-Stokes equations for bubble simulations, or the Stefan problem modeling the interface between ice and water, that are known as interface problems. For interface problems, the free boundary or moving interface depends on the gradient of the solution. For these problems it is important to obtain both accurate solutions and accurate derivatives of the solution. Standard numerical methods for ODE/PDEs generate accurate solutions, but less accurate derivatives. During our research, we explored ways to compute solutions and derivatives to ODE/PDE BVP interface problems accurately at the same time, particularly using a finite difference discretization of Li's Immersed Interface Method. In our research, a new result was found, specifically, that the computed derivatives at the interface of interface problems using standard one-sided finite differences were only first order accurate in general, while the two sided approach using the Immersed Interface Method was second order accurate in general. This conclusion is new and significant in this area. Multiple examples were examined that allowed us to confirm these findings. A proof for this result has yet to be found, and is a possibility for future work.

164. Radio Number for Eighth Power Paths

Joanna Machuca California State University, San Bernardino
Agustin Mendoza California State University, San Bernardino
Advisor(s): Dr. Min-Lin Lo, California State University, San Bernardino

Let G be a connected graph. For any two vertices u and v, let $d(u, v)$ denote the distance between u and v in G. The maximum distance between any pair of vertices of G is called the diameter of G and denoted by $\text{diam}(G)$. A *radio labeling* of G is a function f that assigns to each vertex a label from the set $\{0, 1, 2, 3 \ldots\}$ such that the following holds for any vertices u and v: $|f(u) - f(v)| \geq \text{diam}(G) - d(u, v) + 1$. The span of f is defined as $\max_{u,v \in V(G)}\{|f(u) - f(v)|\}$. The *radio number* of G is the minimum span over all radio labelings of G. The *eighth power* of G is a graph constructed from G by adding edges between vertices of distance eight or less apart in G. In this presentation we will discuss the process of determining the radio number for the eighth power paths and our findings.

165. Mathematical Modeling of Hydrogen Gas Diffusion into Crack and its Consequential Growth

Andrew Madsen University of North Georgia
Advisor(s): Dr. Alla Balueva, University of North Georgia

In this study, we model a non-ideal sink of hydrogen diffusion into a crack and its consequential growth in metal over time. First, we study how the crack grows in an ideal-sink approximation, which is when we don't take into account that hydrogen inside the crack can diffuse back into the material. In both cases, we start with the equation of state for the ideal gas, $PV = mRT$, and sequentially derive the gas pressure P, the crack volume V, and the gas mass m. For the gas mass m, we calculate the gas flux through the crack, $Q(t)$. To obtain a result when a non-ideal sink is taken into account, we come up with a different method of calculating the gas flux, $Q(t)$, to include the loss of hydrogen inside the crack. After formulating the integral equation for the crack radius, we then differentiate both sides to reduce it to a differential equation. After solving the differential equation, we finally obtain a closed form solution how the radius of the crack depends on time.

166. Arithmetic Mirror Symmetry from Reflexive Polytopes

Chris Magyar University of Wisconsin - Eau Claire
Advisor(s): Dr. Ursula Whitcher, University of Wisconsin - Eau Claire

For decades physicists have sought for a way to unify relativity and quantum mechanics. A top candidate for this unified field theory is string theory. In string theory, we extend our usual four dimensional space time model into higher dimensions by attaching six dimensional Calabi-Yau varieties to every point in our known four dimensions. This construction gives rise to some interesting mathematics that we explore in lower dimensional cases. We construct dual algebraic varieties from pairs of reflexive two and three dimensional lattice polytopes. These varieties exhibit the phenomenon of mirror symmetry predicted by string theorists for dual Calabi-Yau varieties in higher dimensions. We use concepts from algebraic geometry, number theory, and combinatorics to examine one-parameter families of these varieties. Counting points of these elliptic curves over finite fields, we demonstrate a number-theoretic mirror symmetry relationship holds for five pairs of elliptic curve families. We link these results to classic arithmetic results on elliptic curves. We compare our results to the case of K3 surfaces. We use Picard-Fuchs differential equations, which describe the periods of our varieties as we vary their parameters, to relate the K3 surfaces to elliptic curves.

167. Lower volume bounds for a class of hyperbolic 3-orbifolds

Jessica Mallepalle Arcadia University
Joseph Melby University of Minnesota Morris
Jennifer Vaccaro Olin College
Advisor(s): Dr. Shawn Rafalski, Fairfield University

A hyperbolic 3-orbifold is a 3-dimensional space with a geometric structure that obeys the laws of hyperbolic geometry. A key invariant of a hyperbolic 3-orbifold is its hyperbolic volume. For this project, we use estimation techniques due to Agol, Storm, and Thurston to provide lower bounds on the volume of a class of hyperbolic 3-orbifolds, based on whether they contain certain types of 2-dimensional hyperbolic suborbifold.

168. Quantum Computing. Device Independent Quantum Key Distribution

John Mangles Creighton University
Advisor(s): Dr. Randy Crist, Creighton

Quantum key distributions (QKD) have become a topic of significant interest in quantum computing. A QKD is a technique in quantum cryptography for creating a completely random key for transferring information. The potential for truly secure data transfer is an important area of work in quantum cryptography, but it is subject to noise in the system that can disturb the key. Thus there is a need for a completely device independent quantum key distribution protocol. We will present several examples of QKD and some results related to device independent QKD accomplished in a noisy environment.

169. Nonlinear dynamics in biochemical networks

McKenna Manning Arizona State University
Advisor(s): Dr. Eric Kostelich, Arizona University

This project focuses on biochemical networks, specifically on dual feedback motifs and their associated periodic behavior. Our model based on previous work by Kholodenko et al in 2006. We consider two compounds and two enzymes; the enzymes catalyze reactions that convert one compound into the other thus preventing the extinction of either compound. We find that with the addition of a forcing term, the limit cycle behavior exhibits period doubling. Period doubling is a well-known path to chaos and we investigate whether there are conditions in which our model exhibits chaotic behavior. Our future work consists of implementing the Belousov-Zhabotinskii reaction and the Oregonator model to help us find chaos in our model.

170. A fractional diffusion model for electrical signal propagation in cardiac muscle tissue

Mae Markowski George Mason University
Advisor(s): Dr. Harbir Antil, George Mason University

It is well known that cardiac muscle tissue is heterogeneous. In order to better account for the spatial complexity of the region, we propose a novel time-space fractional diffusion equation with a Caputo fractional time derivative of order

$\gamma \in (0, 1)$, and a fractional Laplacian operator of order $s \in (0, 1)$. Ultimately, we will numerically approximate the model with a finite element method approach to for the fractional Laplacian, coupled with a finite difference scheme for the Caputo time derivative.

171. Symbolic Methods for Characterizing Point Periodicity in Dynamical Systems

Samuel Marks Princeton University
Roshan Padaki Boston University: PROMYS
Kara Luo Boston University: PROMYS
Advisor(s): Dr. Victor Rotger, Universitat Politecnica de Catalunya

The main focus of the study of dynamical systems is to understand the behavior of points $x \in A$ under iteration of some function $f : A \longrightarrow A$. One powerful method for studying the behavior of points is to represent their orbits symbolically. In particular, if $\{A_i | i \in \mathfrak{I}\}$ is a partition of A into disjoint subsets, then we can define the sequence of $T(x)$ so that $T(x)_n = i$, where $f^n(x) \in A_i$. If $T(x) \neq T(y)$ whenever $x \neq y$, then we call the partition $\{A_i\}$ *sharp*. Although there exists a sharp partition for any dynamical system, partitions that are coarser in the sense that they have smaller index sets $\mathfrak{I}$ encode more information about point periodicity under iteration of f. We develop methods for finding coarser sharp partitions, and we apply these partitions to characterize point periodicity for a variety of dynamical systems. Finally, we investigate topologically conjugate systems in order to broaden the implications of our results.

172. Substitutions and Similar Rauzy Fractals

Austin Marstaller The University of Texas at Dallas
Meghan Malachi
Advisor(s): Dr. Benjamin Itza Ortiz, Universidad Autonoma del Estado de Hidalgo

Let A be an alphabet and let S and T be substitutions under A. There exist certain classes of substitutions that can be delineated by a fractal, specifically a Rauzy fractal. We attempt to characterize substitutions over a three-letter alphabet that produce Rauzy fractals that are translations of one another. The methods we employ are analyzing the "staircase" constructed from the sequences created by the substitutions as well as thoroughly examining the structure of those sequences. Through these methods, we have found that should a substitution be Right-Conjugate to another, they will generate Rauzy fractals which are translations of one another. We wish to find more interesting characteristics of substitutions which generate similar Rauzy fractals.

173. Sums of Permutations in Cryptographic Hash Functions

Michelle Mastrianni Carleton College
Nicholas Lacasse
Bai Lin
Dmitriy Khripkov
Advisor(s): Ms. Liljana Babinkostova, Boise State University

Grostl, a finalist in the NIST SHA-3 hash project competition, is a cryptographic hash function whose simple and elegant design inspires continued interest in its security features. Within Grostl is a compression function built from two distinct permutations of bytes, both of which are heavily based on the Rijndael-like AES block cipher. In each iteration of the compression function, the one-line notations of the permutations (P and Q) are summed together bitwise. The summation $P \oplus Q$ is a new function which may or may not itself be a permutation. Our research, motivated by the Grostl compression function, focuses on when and how often the sum of two permutations over a cyclic group is a permutation (or close to a permutation). From a security standpoint, this question is important because if the sum of two permutations in a compression function itself approximates a permutation, the overall hash function is more likely to be collision resistant. We give enumeration results over cyclic groups and finite fields of prime power order. We give enumeration results over cyclic groups and examine the effects of iterated functional composition of these permutation sums, thereby laying the foundation for continued research of these questions for Galois fields.

174. Classifying 7-Dimensional Indecomposible Solvable Lie Algebras With Niradical Isomorphic to $H \oplus \mathbb{R}^3$.

Christopher Mattoon Grand Valley State University
Advisor(s): Dr. Firas Hindeleh, Grand Valley State University

This poster is the second in a series that examine seven-dimensional solvable Lie Algebras with a six-dimensional niradical. Low-dimensional solvable Lie Algebra classification started back in 1963 by Mubarakzyanov. and were completely classified up to dimension six. A general theorem asserts that if g is a solvable Lie Algebra of dimension n, then the dimension of its maximum nilpotent ideal (called the nilradical) is at least $\frac{n}{2}$. For the seven-dimensional algebras, the nilradical's dimension could be 4, 5, 6 or 7. The four- and seven-dimensional nilradical cases were classified. We examine the six-dimensional niradical case. We first looked for the six-dimensional nilpotent algebras and found 32 algebras. The first case was completed in 2014. In this project we focus on the class where the nilradical is isomorphic to a direct sum of the three-dimensional Heisenberg algebra and a three-dimensional abelian algebra denoted by $H \oplus \mathbb{R}^3$.

175. Cops and Robbers on Infinite Graphs

Elizabeth Matys Gettysburg College
Jordan DuBeau Middlebury College
Advisor(s): Dr. Robert Bell, Michigan State University

Consider a connected graph G. The game of Cops and Robbers is played on G with two players C and R. C first places her cop(s) on the graph and then R places his robber on the graph. The two players alternate turns (C and then R) by either moving any number of their pieces to an adjacent vertex or staying still. The weak copnumber of a graph refers to the fewest number of cops it takes to guarantee that the robber never visits one vertex infinitely many times without being captured. Using the weak copnumber, we explore extensions of known results about Cops and Robbers on finite graphs to infinite settings. For example, theorems regarding copnumbers of graph products (Tošić, Maamoun, and Meyniel) are extended to infinite graphs. We give examples of results that do not generalize to the infinite setting and present progress towards extending the result that $c(G) \leq 3$ for every planar graph G (Aigner and Fromme) to infinite graphs.

176. Homology and the Coupled Patch Model

Cristin Mayes Hampton Universtiy
Advisor(s): Dr. Sarah Day, College of William & Mary

The Expedition in Training, Research, and Education for Mathematics and Statistics through Quantitative Explorations of Data (EXTREEMS - QED) is a summer program sponsored by the National Science Foundation (NSF) here at the College of William and Mary. The following is a detailed report on my homological and coupled-patch research.

177. Exploring Performance and Convergence Speed of Gradient Descent and Accelerated Gradient Descent Algorithms for Total Variation Image Denoising and Image Inpainting

Michael Mbaba Morehouse College
Advisor(s): Dr. Maryam Yashtini, Georgia Institute of Technology

The Gradient Descent and Accelerated Gradient Descent Algorithms are two of the most prominent algorithms utilized and researched in the field of Convex Optimization. Image Denoising and Image Inpainting are two methods to recover images and minimize noise and alterations in the field of Image Processing. The Total Variation model reconfigures the problem of noise reduction and image recomposition as a convex optimization problem where the convex function to be minimized is composed of the numerical difference between the noisy image and the clean image. Our initial research was to determine the effect of the regularization parameter, lambda, on the performance of the algorithms in accurately inpainting and denoising the selected images. In addition, subsequent research analyzed the effect of the size of the impainting domain and amount of noise on both the performance and convergence speed of the two algorithms. Our ongoing research is concerning the effect of the mathematical type of random noise introduced on the performance and convergence of the two algorithms studied. We are conducting experiments to determine if Poisson or Cauchy noise would generate a statistically significant variation in the performance and convergence speed for an equal amount of noise.

178. A Characterization of Four Interval Wavelet Sets

Christopher McDonald Bridgewater State University

Advisor(s): Dr. Vignon Oussa, Bridgewater State University

Wavelets are mathematical tools used to represent signals such as audio files, pictures, videos, and various other types of data. At this point, Wavelet Theory is a mature subject, and the literature contains a body of techniques used to design wavelets. One of these techniques relies on the construction of wavelet sets (sets tiling the real line by successive integral shifts and dyadic dilations.) In this project, we obtained a complete characterization of wavelet sets of four intervals. This extends results of wavelet experts M. Bownik and K. Hoover who recently characterized wavelet sets of two and three intervals. More precisely, we showed that there are three possible configurations of wavelet sets of four intervals, and for each configuration, we can provide an explicit characterization of the corresponding wavelet sets. Among various mathematical objects used in our characterization, the symmetric group S_4 (the group of 24 permutations on 4 symbols) appears as a central tool. Furthermore, special attention is paid to the case of symmetric wavelet sets. That is, wavelet sets which are up to boundary points, invariant under multiplication by negative one.

179. Probabilistic Search Problem for UAV's in Uncertain Environments

Kathleen McLane George Mason University

Advisor(s): Dr. Padmanabhan Seshaiyer, George Mason University

The use of unmanned aerial vehicles (UAVs) to accomplish tasks is a fast growing field in technology today for search and rescue operations. This multidisciplinary area requires precise mathematical modeling and evolution of probabilistic algorithms. In this project, we consider a model for a UAV that investigates a decision-making framework using probabilistic search algorithms. Bayes filters are implemented to detect the presence of targets in the search area through calculations of belief functions expressed via conditional probabilities. These belief functions are then combined into an iterated refinement algorithm to help determine the location of the target. The proposed mathematical models are validated computationally for benchmark applications including searching a savannah in Africa for poachers to identifying residues of pesticides in crops.

180. Forcing the Fold: Developing a Minimal Forcing Set Algorithm

Samuel McLaren Western New England University

Tianna Procon Western New England University

Advisor(s): Dr. Thomas Hull, Western New England University

This project explores the mathematics behind origami. Specifically, it explores single-vertex folds of varying degrees and their respective minimal forcing sets as follows: each crease is assigned to be either a mountain or a valley fold. A forcing set then is a subset of creases that force all the other creases to fold according to their labels. The minimal forcing set is the smallest of such. During this research our objective is to form and prove an algorithm allowing a user to find the minimal forcing set of creases for any degree of a flat-foldable singe-vertex fold without needing to exhaust all possibilities. This will be a useful tool in making the design and construction of self-folding structures more efficient and economical.

181. Hidden Markov Chain based PageRank Algorithm

Scott McOmber Western Connecticut State University

Advisor(s): Dr. Xiaodi Wang, Western Connecticut State University

Google is the world's number one search engine. Anything you search on the web can be found through Google. The Google PageRank algorithm is one of the main algorithms used in calculating a page's rank. The algorithm was created by multiple scientists to rank scientific journals and later was used with a weighted system and integrated to rank pages on the internet. This algorithm is very complex and difficult to use for people who do not understand certain things in advanced mathematics. In this research we apply the hidden Markov chain and some properties of linear algebra to reduce the complexity of the original PageRank algorithm.

182. Tropical Embeddings of Metric Graphs

Adan Medrano Martin del Campo Massachusetts Institute of Technology
Advisor(s): Mr. Sylvain Carpentier, Massachusetts Institute of Technology

Every graph Γ can be embedded in the plane with a minimal number of edge intersections, called its classical crossing number cross (Γ). In this paper, we prove that if Γ is a metric graph it can be realized as a tropical curve in the plane with exactly cross (Γ) crossings, where the tropical curve is equipped with a metric via the lattice length. Our result has an application in algebraic geometry, as it enables us to construct a rational map of non-Archimedean curves into the projective plane, whose tropicalization is almost faithful when restricted to the curve's skeleton.

183. Quadratic Prime-Generating Polynomials Over $\mathbb{Z}[i]$

Monta Meirose Morningside College
Frank Fuentes Seattle University
Advisor(s): Dr. Erik Tou, University of Washington

The quadratic polynomial $x^2 + x + 41$ is prime for $x = 0, 1, \ldots, 39$. For this reason, it is called a prime-generating polynomial. Many other prime-generating polynomials have been discovered by computer searches, and their efficiency at producing primes can be predicted in some special cases. In this talk, we find and classify prime-generating polynomials $f(z)$, where the variable and coefficients are permitted to be Gaussian integers. Many of the same criteria for efficiency may be generalized from integer polynomials, though without a natural ordering of Gaussian integers there are some surprising differences. Since Gaussian polynomials live in a two-dimensional space, some symmetry can be observed—rotations and reflections, as well as translations, dilations, and combinations of these create more complicated families of polynomials. Our results so far have led us to polynomials that have a high efficiency on a region near 0.

184. Canonical triangulations of rational knot complements and their fundamental groups

Dean Menezes University of Texas at Austin
Advisor(s): Dr. Neal Stoltzfus, Louisiana State University

We present a fast method of computing a "nice" form for a canonical triangulation for rational knots described by Sakuma and Weeks for the case of the twist knots. We apply this method to compute various other knot invariants such as the fundamental group and the invariant trace field based on this canonical triangulation.

185. Hybrid Model of Newt and Crayfish Populations in Santa Monica Mountain Streams with Manual Crayfish Removal

William Milligan Emory University
Advisor(s): Dr. Courtney Davis,Tim Lucas, Pepperdine University

Global amphibian populations are declining, and invasive species contribute to the decline in many species. Predation by the invasive crayfish, *Procambarus clarkii*, on the California Newt, *Taricha torosa*, has led to local extinction in some Santa Monica Mountain streams. Elimination of crayfish is difficult, thus the goal of this project is to determine if and to what level of human intervention could extend the time to extinction of the newts or create coexistence between the species. We mathematically model native newt population dynamics using discrete, time-delayed equations, and we model invasive crayfish populations continuously. The interactions between the species are described using Michaelis-Menten enzyme kinetics and Beddington-DeAngelis predation in a hybrid model. Computer simulations demonstrate that intermittent trapping schedules can extend time to extinction if crayfish are trapped when both the frequency and rate of removal are sufficient. We predict newt persistence levels in the absence of crayfish trapping and for simple trapping schedules.

186. Error indicators in the adaptive finite element method

Talin Mirzakhanian California State Polytechnic University, Pomona
Diana Gonzalez California State Polytechnic University, Pomona
Advisor(s): Dr. Ryan Szypowski, California State Polytechnic University, Pomona

Partial differential equations are used to model numerous physical phenomena, from fluid dynamics to structural mechanics. Producing an analytic solution to these equations is often intractable, and so we must resort to approximation

techniques. In this project, we discuss the adaptive finite element method. This method relies on producing a mesh which is refined in such a way that the error is reduced in each iteration, ultimately producing a solution within any desired tolerance (provided the computational time and power is available). The adaptive algorithm takes the form of a set of substeps solve, estimate, mark, and refine repeated until convergence. We focus on a step in which the error is approximated, and this approximation is what is used to guide the refinement procedure. A number of possible error indicators are discussed and compared on various problems.

187. Rainbow Ramsey Theory on the Integers

Boyd Monson Grinnell College
Henry Ehrhard Grinnell College
Yifei Zhang Grinnell College
David Kraemer Grinnell College
Advisor(s): Dr. Joseph Mileti, Grinnell College

We explore a polychromatic analogue of Rado's Theorem, a classic result in Ramsey Theory. An r-coloring of $\mathbb{N}$ is a function that maps each natural number to one of r different integers, or "colors." By contrast, an r-bounded coloring of $\mathbb{N}$ is a function that maps the natural numbers to infinitely many colors, each of which is used at most r times. Where one may think of an r-coloring as using a handful of infinitely long crayons, one may think of an r-bounded coloring as using an infinite set of tiny crayons. Rado's Theorem characterizes the conditions for when a homogeneous linear system yields a monochromatic solution (a vector whose entries are colored identically) for all r-colorings of $\mathbb{N}$. In the r-bounded case, we instead seek polychromatic solutions; that is, vectors whose entries are each colored distinctly. We prove a polychromatic analogue of Rado's Theorem in the special case of a single equation, and we provide results building towards an analogous statement of the general version of the theorem.

188. Traffic Simulations in Santa Barbara County, California using Modified Numerical Methods and Burgers Equation

Armando Morales California State University Channel Islands
Advisor(s): Dr. Cynthia V. Flores, California State University Channel Islands

The goal of this research project is to develop a mathematical model for traffic simulation. The classical Burgers Equation, $u_t + uu_x = 0$, is used as our model and is derived by assuming the traffic's velocity and density are continuous functions, and furthermore captures the phenomenon of shock and rarefaction wave formation. Numerical methods were investigated and applied to linear and non-linear partial differential equations (PDE's). Presented in this poster are the modified numerical methods that were the key to approximating solutions to Burgers Equation and the numerical outcome in a traffic flow model. The traffic simulation takes place on the northbound US 101 freeway from Camarillo, California to Santa Barbara County on approximately 11 miles of highway. Data from the Caltrans Performance Measurement System coming from different sensors along the highway collect information such as traffic flow, velocity, and occupancy. We were able to compare the numerical results from our simulations to the known data and improve the classical Lax-Wendroff method to approximate solutions to the Burgers Equation.

189. Rank of Recurrence Matrices

Roman Morales St. Edward's University
Advisor(s): Dr. Jason Callahan, St. Edward's University

A recurrence matrix is a matrix whose entries are the terms of a sequence defined by a recurrence relation. A recurrence relation is an equation that recursively defines a sequence of numbers, once one or more initial terms are given. The rank of a matrix is the maximum number of linearly independent columns or rows of the matrix. In 2014, Christopher Lee and Valerie Peterson proved that the maximum rank of a recurrence matrix is the order of the corresponding recurrence relation but that for order-two recurrence relations the rank drops whenever the ratio of the two initial terms of the recurrence relation is an eigenvalue of the relation. Using the method of fundamental solutions, we generalize their result for order-two relations by finding the only other case in which rank drops. We then discuss more recent results by Sebastian Bozlee that determine the rank of recurrence matrices by considering whether the relation can be written to have lower order in hopes of extending our results to recurrence relations of orders higher than two.

190. Mathematical Modeling of Epidemic with Exposed Group.

Diana Morales University of Central Arkansas
Alma Malibekova University of Central Arkansas
Advisor(s): Dr. Long Le, University of Central Arkansas

Our research focuses on mathematical modeling of an epidemic within a population. It consists of a system of four nonlinear ordinary differential equations that represent the groups of potential, exposed, infected and removed populations. The goal of the research is to find and study the endemic stability state. Three steady states for the system are found and their stability is analyzed using Routh-Hurwitz criterion. Using the found criterion, stability of flu, measles, and ebola were analyzed.

191. Limiting Distributions for Open Topological Markov Chains

Peter Morfe Cooper Union
Christopher Ianzano Stony Brook University
Elizabeth Yoo Columbia University
Advisor(s): Dr. Mark Demers, Fairfield University

Open dynamical systems are models of physical systems in which mass or energy is allowed to escape from the system. Central questions involve the existence of conditional equilibria (measures that are invariant under the dynamics conditioned on non-escape) which can be realized as limiting distributions under the dynamics of the open system. We study this problem in the context of topological Markov chains, a class of symbolic dynamical systems with a wide variety of applications. Under a combinatorial condition on the Markov chain, we study transfer operators associated with positive recurrent potentials and prove the existence of a spectral gap on a natural function space. This implies the existence (and uniqueness in a certain class) of limiting distributions which represent conditional equilibria for the open system. We also prove a relation between the escape rate from the system and the entropy on the survivor set (the set of points that never enters the hole).

192. Matrix Completion Problems for Sylvester Equations $AX - XA^T = 0$ and $AX - A^TX = 0$.

Kirsten Morris Georgia College
Advisor(s): Dr. Geoffrey Buhl, California State University Channel Islands

A matrix completion problem examines a partial matrix composed of specified and unspecified entries and determines if this partial matrix can be completed to satisfy some given property. We determine if partial matrices can be completed to satisfy the Sylvester equations $AX - XA^T = 0$ or $AX - A^TX = 0$. If a partial matrix X with a certain pattern of specified and unspecified entries can be completed for almost any matrix A to satisfy $AX - XA^T = 0$ or $AX - A^TX = 0$, we call the pattern *admissible*. Using the Kronecker product, we rewrite these matrix equations as linear equations and examine the structure of the nullspace of these linear equations to characterize patterns as admissible or not. Through this approach we obtain a complete characterization of admissible and inadmissible patterns for $AX - A^TX = 0$ and a partial characterization for $AX - XA^T = 0$.

193. Shallow Waves in Density Stratified Bilinear Shear Currents

Theresa Morrison San Diego State University
Advisor(s): Dr. Christopher Curtis, San Diego State University

In this poster we examine the role of nonlinearity on the evolution of surface and internal layers in density stratified fluids with steady but different shear currents in each stratified layer. Our work addresses two physically motivated parameter sets which display a range of nonlinear phenomena. We also show, when the difference between the vorticities in each layer is sufficiently large and of different signs, large amplitude nonlinear phenomena, particularly along the internal layer, emerges. Dispersive shock wave and solitary wave phenomena appear in the parameter regimes examined in this work. Our results show that jumps in density and vorticity generate strong nonlinear responses, and therefore sea state models should account for these variations in order to improve their predictive capabilities.

194. Region Alternating Knots

Colin Murphy Seattle University
Ra'Jene Martin Denison University
McKenna Renn University of Washington
Advisor(s): Ms. Jennifer Townsend, Bellevue College

Colin Adams introduced the notion of almost alternating knots, which are non-alternating knots that have a projection in which one crossing change is required to create an alternating diagram. We extend this notion in conjunction with Ayaka Shimizu's work on region crossing changes to develop the idea of a region almost alternating knot. This is defined as a knot where there exists a diagram such that a single region crossing change will produce an alternating diagram and no alternating projection exists. We discuss families of knots that are region almost alternating and their characteristics, such as their relation to almost alternating knots, their behavior in a connected sum, bounds on region dealternating numbers, and warping span. Our results resolve an open question about the maximum warping span of knots.

195. Palindromic *n*th Power Sums

Ann Murray Nebraska Wesleyan University
Advisor(s): Austin Mohr, Nebraska Wesleyan University

The number 434 is a palindromic number with the special property that it can be written as the sum of consecutive squares of positive integers: $434 = 11^2 + 12^2 + 13^2$. We present an algorithm implemented in Sage that lists the palindromic numbers that can be written as the sum of consecutive nth powers of positive integers. This problem is a generalized version of the 125th Project Euler problem.

196. Cellular Models of Canine Parvovirus

Brittany Myers University of Central Oklahoma
Advisor(s): Sean Laverty, University of Central OKlahoma

Parvovirus is a virus that infects actively dividing cells in many animals, including dogs. Canine Parvovirus type 2 (CPV2) has two forms, intestinal and cardiac, which often kill young dogs when they become infected. We built differential equations models to better understand how CPV2 infects host cells. The model includes viruses, infected cells, target cells, protected cells, and antibodies. Since there are multiple ways antibodies inhibit infection, we built two different models to study the different types of antibody response. Based on our model results, we propose the most effective method for fighting off CPV2 after infection.

197. Cap-Matching for Cryptographic Protocol Analysis

David Myers Oberlin College
Advisor(s): Dr. Chris Lynch, Clarkson University

The Dolev-Yao intruder is the most powerful intruder possible in a network; it can overhear, intercept, and send any message. Therefore, the intruder's knowledge is limited only by the formal properties of the cryptographic ol in plprotocace. In order to determine the intruder's knowledge in a given protocol, we introduce **Cap**-matching problems and give algorithms which decide them. **Cap**-matching problems are similar to finding the largest affine subspace of a given form of some affine subspace, but with one technical difference.

Given a term s and a **Cap**-term T, the **Cap**-matching problem $s \stackrel{?}{\in} T$ is solved by the largest set of substitutions σ for which $s\sigma$ is of the form given by T. We give an algorithm which solves **Cap**-matching problems for syntactic equality, and algorithms which solve **Cap**-matching problems for associativity, homomorphism, and commutativity when s is linear. We then use these algorithms to deduce intruder knowledge in the Dolev-Yao model. In the syntactic theory we treat encryption as a black box; in the other equational theories we begin to model properties of the cryptographic algorithms themselves.

198. Application of Curvature Analysis as a Methodology of Modeling Crime in Baltimore

Abdallah Naanaa Stevenson University
Advisor(s): Dr. Mark Branson, Stevenson University

The death of Freddie Gray created a stir in Baltimore that led to violent protests. We analyzed the curvature of the city using six algorithms and the metric of public transportation time in order to estimate a generalized location of crime. We further hypothesized that areas of positive curvature would correspond to crime locations because these areas are challenging to diffuse from. The cityscape of Baltimore was modeled three-dimensionally using grid points to emulate a mesh. Each point within the mesh had neighboring points found and we fit curvature values to them to define the closest value to the crime location. We determined that some of these algorithms gave consistent calculations that correlated positively curved areas with the locations of crime.

199. Edge Detection with Noisy and Interrupted Data

Rachel Nahon Arizona State University
Alena Chang Arizona State University
Advisor(s): Dr. Anne Gelb, Arizona State University

Data are acquired as samples of Fourier coefficients in applications such as synthetic aperture radar (SAR) imaging and magnetic resonance imaging (MRI). Scientists are often interested in identification or extracting important features of interest, but may not be interested in full image reconstruction. This is a very difficult problem, because Fourier data provide global information while the features themselves are typically local. The problem can further be complicated when the data are corrupted or unusable in certain frequency ranges. In this investigation we reduce the problem to one dimension and attempt to recover its discontinuity locations from given noisy and corrupted Fourier data.

Many algorithms have been developed to detect edges in piecewise smooth functions from a finite sampling of Fourier data. In this investigation we employ the concentration factor edge detection [Gelb and Tadmor, 1999]. We adapt this algorithm to consider the case where some of the collected data are unreliable and should not be used. The algorithm is flexible so that it can accommodate specific constraints and prioritize different goals, for example, if it is better to recover some false edges or to miss some detections. Numerical results are provided.

200. Some Nonsimple Modules for Centralizer Algebras of the Symmetric Group

Yukihide Nakada Allegheny College
Kelly Pohland Allegheny College
Advisor(s): Dr. Craig Dodge, Allegheny College

James classified the simple modules over the group algebra $k\Sigma_n$ using modules denoted D^λ where λ is a partition of n. In particular, he showed that D^λ is simple or zero for every partition λ and, furthermore, that for every simple $k\Sigma_n$-module S there exists a partition λ such that $D^\lambda \cong S$. This paper is an extension of a paper of Dodge and Ellers in which they studied analogous modules $\mathcal{D}^{(\lambda,\mu)}$ over the centralizer algebra $k\Sigma_n^{\Sigma_l}$, where λ is a partition of n and μ is a partition of l. For every positive prime p, we find counterexamples to their conjecture that the $k\Sigma_n^{\Sigma_l}$-modules $\mathcal{D}^{(\lambda,\mu)}$ are always simple or zero, where k is a field of characteristic p. We also study the relationship between $\mathcal{D}^{(\lambda,\mu)}$ and $\mathrm{Hom}_{k\Sigma_l}\left(D^\mu, \mathrm{res}^{\Sigma_n}_{\Sigma_l} D^\lambda\right)$ in special cases.

201. A fast approximation algorithm for solving the set cover problem for multiple queries

Ashwin Narayan Williams College
Natalia Postawa Adam Mickiewicz University in Pozna
Advisor(s): Dr. Alejandro Morales, UCLA

Efficient retrieval of information is of key importance when using big data systems. When a customer requests data, a query is sent to a router which returns the machines containing the requested data. Ideally the smallest set of machines required is returned, which reduces load and energy consumption. Mathematically, this is the set cover problem, which is NP-hard, making the routing process a balance between optimality and time cost. There exists an efficient greedy approximation algorithm for routing a single query, but there is currently no better method for processing multiple queries than running that algorithm repeatedly. For big data systems, this method is impractical. In fact, the current baseline method used is to rout a query to all machines and choose the ones that respond fastest as a cover. We give a number of methods that improve the runtime of routing multiple queries in both real-time and non-real-time cases.

We first cluster queries, using a variant of known clustering algorithms and then process the queries as clusters using a novel processing algorithm. Our experimental results show that this significantly reduces the set cover time cost, while still preserving optimality. Our methods will contribute to efficient database retrieval methods.

202. Sums of Reciprocals of Irreducible Polynomials over Finite Fields

Spencer Nelson St. Lawrence University
Advisor(s): Dr. Sam Vandervelde, St. Lawrence University

We investigate monic irreducible polynomials over a finite field of specified order and provide some results regarding the sum of the reciprocals of such polynomials. Monic irreducible polynomials are closely related to many combinatorial objects such as necklaces (equivalence classes of strings under cyclic rotation) and Lyndon words (sequences of symbols taken from a non-empty finite set). They are also an important object in the study of the algebraic structure and the security of many symmetric key ciphers, including the Advanced Encryption Standard — cipher used by the U.S. government to protect classified information. Our study is built on the well known result by L. Carlitz from 1935 in which he provided an intriguing formula for sums involving the reciprocals of all monic polynomials of a given degree over a finite field of specified order and which we revisit in this paper.

203. Families of Cyclic Codes over Finite Chain Rings

Andrew Nemec Texas A&M University
Advisor(s): Dr. Andreas Klappenecker, Texas A&M University

A major difficulty in quantum computation and communication is preventing and correcting errors in quantum bits. The most popular solution to this problem is to use quantum stabilizer codes because they can be constructed from self-orthogonal classical cyclic codes, which have traditionally been studied over finite fields, but have recently been generalized to various classes of rings. We investigate the conditions for self-orthogonality for cyclic codes over finite chain rings, in particular the families of quadratic residue and BCH codes, and then derive bounds for the minimum distance on the codes.

204. Information embedding based on Pseudo Quantum Signal in M-band Wavelet Domain

Hieu Nguyen Western Connecticut State University
Advisor(s): Dr. Xiaodi Wang, Western Connecticut State University

Privacy information is being a big concern during our modern world. Many companies, governments are developing different mathematical algorithms to protect their privacy information. Due to the recent development of Wavelet Transform and Pseudo-Quantum Signal, we are proposing the new scheme of Mathematical Cryptography to help protect the privacy information among the internet. In this research, we first digitalize our textual data into three or more matrices to form a "color image". Color image consisted of the combination of three or more colors. We then perform M-Band Wavelet Transformation to each of three matrices of this "color image". Then, in the approximation part of each transformed matrix, we embed another "secret information" using "Pseudo-Quantum Signal". After the embedding, we perform the inverse the M-Band Wavelet Transformation and combine all the color channels together to create an color image consists of three or more different textual data with "secret message" in each color channel.

205. Dual Schubert Polynomials

Eric Nie MIT PRIMES, Westborough High School
Advisor(s): Dr. Pavel Galashin, Massachusetts Institute of Technology

We first introduced some notation and background information relating to Schubert polynomials before we proceed to present our new results relating to dual Schubert polynomials. Postnikov and Stanley found a method for determining the entries in the inverse Schubert-Kotska matrix for 312-avoiding permutations and 213-avoiding permutations. We consider the following question: for which $\sigma \in S_3$ the corresponding entries of the inverse Schubert-Kostka matrix $K^{-1}_{\omega,a}$ belong to the set $\{-1, 0, 1\}$ for all σ-avoiding permutations ω and all sequences a. Postnikov and Stanley showed that this is true for $\sigma = 312$ and $\sigma = 213$. We show that this is false for $\sigma \in \{123, 231, 132\}$ and then make some progress towards proving it for the remaining case $\sigma = 321$.

206. Are you getting your fair share of social security benefits?

Jeremy Robert Nielsen University of Wisconsin-Eau Claire
Advisor(s): Dr. Zhixin Yang, University of Wisconsin-Eau Claire

In this article, we studied the actuarial fairness of social security benefits from different perspectives. We addressed the present bias issue in our work and presented three ways to motivate retirees to have delayed retirement payment. On the one hand, we generalized Sanders and other scholars work by using more reasonable dynamic systems for risk free interest and mortality rates, on the other hand, we demonstrated how to use lump sum and bonus system methods to attract retirees so that they would claim their benefits in a smart way.

207. Covering Sets for Rectangles in the Lattice

William Noland North Central College
Advisor(s): Dr. John Goldwasser, West Virginia University

The famous Turan-type problems study the maximum fraction of a structure one may select without selecting certain forbidden configurations. Stated in terms of complements, our problem is: given a set S of rectangles of specified dimensions, we want to determine the minimum density of a set A of points in $\mathbb{Z} \times \mathbb{Z}$ such that every copy in $\mathbb{Z} \times \mathbb{Z}$ of any rectangle in S has at least one of its four vertices in A; in this case we say that A is a covering set for the rectangles in S. It is trivial that covering all axb rectangles requires precisely $1/4$ of the lattice. Our first result is that the covering density for 1×1 and angled $\sqrt{2} \times \sqrt{2}$ squares is also $1/4$. The primary focus of our work was on covering two different sizes of axis-aligned rectangles. Covering both axb and bxa rectangles requires just $1/4$ of the lattice (no more than just $a \times b$), though the patterns which do so vary with the relative parity of the dimensions. We also have results on covering pairs of squares, which lead to a general conjecture in that regard. Finally, we have determined the exact required covering density required for $a \times c$ and $a \times d$ rectangles.

208. An Extension of the Accelerated Random Search Algorithm to Constrained Black-Box Global Optimization

Luigi Nunez Saint Joseph's University
Kayla Varela Saint Joseph's University
Advisor(s): Dr. Rommel Regis, Saint Joseph's University

The Accelerated Random Search (ARS) algorithm for bound constrained black-box global optimization has been shown to converge to the global optimum with probability one faster than Pure Random Search (PRS) when the objective function is continuous. We develop an extension of ARS called Constrained Accelerated Random Search (CARS) that is meant for black-box global optimization problems with inequality constraints. Under certain conditions, CARS can also be shown to converge to the global optimum with probability one. Moreover, numerical experiments show that CARS also outperforms the constrained version of PRS on a wide variety of test problems. To further improve its performance on computationally expensive global optimization problems, we use radial basis function (RBF) surrogate models to approximate the objective and constraint functions in the implementation of CARS. Numerical experiments on test problems and application problems and comparisons with alternative optimization methods also show that CARS assisted by RBF surrogates is a promising method for computationally expensive constrained black-box global optimization.

209. Conic Sections within Math-Art Designs

Stina Nyhus Utah Valley University
Advisor(s): Dr. Violeta Vasilevska, Utah Valley University

This presentation will show how math, origami folds, and GeoGebra can be used together to construct amazing math-art designs. Many authors (T. Hull, D.P. Scher, S. Smith, etc.) have researched origami folding in connection with the conic sections and used dynamic software to demonstrate those folds. Work done by V. Vasilevska has explored and animated segments of tangents to parabolas in creating math-art designs. We expand on those methods by animating segments of tangent lines to ellipses and hyperbolas. It will be shown how geometry was used to develop the construction methods and to create several GeoGebra animation tools. These methods and tools are used to find a best-fitting conic section within a constraining polygon. GeoGebra will be used to demonstrate the construction and animation of the tangent segments that create beautiful works of geometric art.

210. Orthogonal Polynomials for Plasma Physics

Jessica Oehrlein Franklin W. Olin College of Engineering
Bud Denny Arizona State University
Maria Jesus Munoz Lopez Trinity College Dublin
Advisor(s): Dr. Susana Serna, Universitat Autonoma de Barcelona

Control of the thermonuclear fusion process for the production of energy is a major scientific challenge worldwide. Experiments are being carried out on nuclear fusion using hydrogen plasma with the goal of reaching ignition. Ignition is the point at which more energy is produced from the reaction than was required to initiate it. At present, models predict ignition when it is not occurring. The development of more accurate numerical tools for these plasma models could lead to improvements in experiments. Kinetic theory in plasma physics uses a variety of numerical methods that involve expanding functions into orthogonal polynomials. We investigate one useful family of such polynomials, called Maxwell polynomials. We first study the Maxwell polynomials analytically by considering the polynomial coefficients, three-term recurrence relation, and differential equation. We then turn to a numerical approach to study applications to plasma physics. In particular, we expand initial condition distributions using Maxwell polynomials and conclude that fewer Maxwell polynomials than Laguerre polynomials are required for such expansions. In addition, we use Maxwell polynomials to find the imaginary component of the dielectric function of the quantum Lenard-Balescu equation.

211. Mathematical Model for Time to Neuronal Apoptosis Due to Accrual of DNA DSBs

Annabel Offer Texas Tech University
Chindu Mohanakumar University of Florida
Advisor(s): Dr. Carlos Castillo-Garsow, Eastern Washington University

We propose a mechanism to explain neuronal aging by tracking the number of non-transient DNA double-strand breaks (DSBs) and repairs over time that may lead to apoptosis. Neuronal apoptosis depends on the amount of space between DSBs as well as time. We derive three models to track the effect of neurodegeneration: a system of autonomous Ordinary Differential Equations (ODEs), a probability model to track the spatial requirement, and a stochastic model that incorporates both the ODE temporal dynamics and a spatial probability model. Using these models, we estimate a distribution for the lifespan of a neuron and explore the effect of parameters on time to death. We identify three possible causes of premature neuronal apoptosis: problems with coding critical repair proteins, issues with the neuron detecting DSBs, and issues with the neuron responding to DSBs.

212. Minimality of Uniform Splitting Families

Colin Okasaki Harvey Mudd College
Gabriel Currier Pomona College
Advisor(s): Dr. Liljana Babinkostova, Marion Scheepers, Boise State University

Our research is focused on the maximum sizes of minimal uniform splitting families: families in which every element is necessary to maintain its splitting property. We describe exact upper bounds on the sizes of minimal $(m, 2)$ splitting families, and some looser bounds for more general splitting families. The importance of the theory of splitting families is twofold. Their study is closely related to some classical results in set theory applied to topology such as the result that the real line can be split into two disjoint sets in such a way that every Cantor set in the real line intersects both sets. They are also used to build efficient algorithms for attacking the low Hamming weight discrete log problem - a specific case of an otherwise computationally infeasible problem used in many cryptographic schemes.

213. Tangency and Structure in the Schmidt Arrangement of the Eisenstein Integers

Evan Oliver University of Colorado, Boulder
Advisor(s): Katherine Stange, University of Colorado, Boulder

Actions on the extended real line by Mobius transformations of $\mathrm{PSL}_2(\mathcal{O}_K)$ where $\mathcal{O}_K$ is the ring of integers of an imaginary quadratic field K, produce an image in the complex plane called a Schmidt arrangement. This poster specifically observes occurrences of tangency within the Schmidt arrangement generated by the Eisenstein integers. There exists a recursive formula to generate an Apollonian-like circle packing of tangent circles within this Schmidt arrangement. We also study tangencies in the Schmidt arrangement created by a congruence subgroup.

214. A Computational and Explicit Exploration of the St. Petersburg Paradox

Alexander Olivero Butler University

Advisor(s): Dr. William Johnston, Butler University

This poster displays a sample distribution, generated from both a computer simulation (for large n) and explicitly calculated (for smaller n), that is not governed by the Central Limit Theorem and, in fact displays chaotic behavior. To our knowledge, the explicit calculation of the sample distribution function is new. This sample distribution function comes from an exploration of the historic, St. Petersburg Paradox, an idea that has perplexed mathematicians for hundreds of years.

215. Exploration of Tesler Matrices

Jason O'Neill University of California, Los Angeles

Advisor(s): Mr. Alejandro Morales, University of California, Los Angeles

Tesler matrices, introduced by Glenn Tesler to study Macdonald Polynomials, were recently rediscovered by James Haglund to study Diagonal Harmonics. They were shown to be a solution to the Kostant Partition Function. However, there is no known formula for the number of Tesler matrices, which we denote as $T(1^n)$. At the start of this project, the best bounds were $n! \leq T(1^n) \leq 2^{\binom{n}{2}}$. A Tesler matrix, $A = [a_{i,j}]$ is an upper-trianglular matrix with nonnegative integer entries such that $d_k = 1$ for all k where d_k is defined as: $d_k = (a_{k,k} + a_{k,k+1} + \cdots + a_{k,n}) - (a_{1,k} + a_{2,k} + \cdots + a_{k-1,k})$. We introduce an enumerative tool called the *Diagonal Polynomial* to improve the bounds on $T(1^n)$. We explore sets defined in a similar manner as Tesler matrices except with d_k not necessarily 1. We develop methods for generating lower bounds with Tesler matrices and then apply a method to an idea from Representation Theory to improve the lower bound of the dimension of the zero weight space of a certain infinite dimensional Verma module answering a question posted on math overflow. Finally, we examine Prime and Connected Tesler matrices and maps by Paul Levande between Tesler matrices and permutations.

216. Mathematical Modeling and Analysis of Mental Disorder Diagnostics

Andrea Oranday Trinity University

Advisor(s): Dr. Eduardo Cabral Balreira, Trinity University

The Diagnostic and Statistical Manual of Mental Disorders (DSM) serves as the golden rule book for diagnosing mental disorders. Due to similar diagnostic criteria and great varieties in symptom combinations, filing a patient under the ideal disorder can be subject to human error. In this project a computerized system was developed for assigning a probability that a patient has a mental disorder given a set of diagnosed symptoms. In this way, diagnoses can be corrected or confirmed so the best method of treatment can be identified. Additionally, analysis was conducted on how the order of detection of symptoms influences the diagnosis. Methods were developed to find optimal diagnostic interviews for each anxiety disorder and were compared with the standard, Anxiety Disorder Interview Schedule (ADIS). Said methods were validated with case studies based on the DSM-IV-TR criteria. The results established, from a mathematical perspective, that the ADIS does not necessarily provide the optimal path to diagnosis and there exists disorder specific alternatives.

217. High Resolution Multi-Nested Simulations of Twin Tropical Cyclones

Che Ortega Arizona State University

Advisor(s): Dr. Eric Kostelich, Arizona State University

In May 2002 a rare atmospheric event occurred in which counter-rotating twin tropical cyclones straddling the equator evolved in the Indian Ocean. High-resolution simulations of this event were conducted using a nested Numerical Weather Prediction model which is driven by a global model that provides data for the initial and boundary conditions. The simulation results show that evolutions of the cyclones are better resolved as the resolution increases. It was found that each cyclone produced a potential vorticity field and trajectory that affected the behavior of the other. This interaction was confirmed by idealized simulations of cyclone evolutions in single and twin configurations.

218. Modeling the consequences of reduced vaccination coverage on the spread of measles

Guillermo Ortiz Bridgewater State University
Advisor(s): Dr. Irina Seceleanu, Kevin Rion, Bridgewater State University

A recent trend of vaccine skeptics opting out of immunization programs has led to a resurgence of measles cases in the US. In this project we created a stochastic model simulating the spread of measles in a closed population with the goal of studying the long-term consequences of decreased vaccination levels on the number and duration of measles outbreaks. We employed a non-homogeneous Markov model with SEIR states - Susceptible, Exposed, Infected, and Recovered, in which the transition probabilities of the dynamic driving events from one state to the next are determined by a variety of distributional assumptions. We used our model to simulate the dynamics of the spread of measles using different resistivity levels in the population, and thus are able to illustrate the influence of pockets of unvaccinated people on the incidence of the disease. Furthermore, our model incorporates public health intervention factors such as quarantining and immunizing individuals in response to a measles outbreak, and thus can be used to inform health policies for preventing and controlling the spread of measles in the US.

219. Rings, Completions, and Strange Formal Fibers

Nina Pande Williams College
Sarah Fleming Williams College
Lena Ji Columbia University
David Schwein Brown University
Advisor(s): Susan Loepp, Williams College

Let R be a Noetherian ring with exactly one maximal ideal. We can define a metric on R based on its maximal ideal and complete R with respect to that metric. The relationship between a ring R and its completion can be studied through the natural map from the prime ideals of the completion of R to the prime ideals of R given by intersecting ideals of the completion with R. If $\mathfrak{p}$ is a prime ideal of R, the inverse image of $\mathfrak{p}$ under this map is called the formal fiber of R at $\mathfrak{p}$. The dimension of the formal fiber of R at $\mathfrak{p}$ is the length of the longest chain of prime ideals the formal fiber of R at $\mathfrak{p}$ contains. For a typical ring R, the dimension of its formal fiber at a particular prime ideal $\mathfrak{p}$ is equal to $n - 1 - \operatorname{ht}\mathfrak{p}$ where n is the Krull dimension of R. We show that there are excellent unique factorization domains with the unusual property that the dimensions of their formal fibers do not follow this pattern. We show that, in fact, the dimensions of the formal fibers at the zero ideal and height one prime ideals can be exactly controlled over a large range of values.

220. Linnik's Theorem for Sato-Tate Laws on Elliptic Curves with Complex Multiplication

Peter Park Princeton University
Advisor(s): Dr. Ken Ono, Jesse Thorner, Emory University

Let $E/\mathbb{Q}$ be an elliptic curve with complex multiplication (CM), and for each prime p of good reduction, let $a_E(p) = p + 1 - \#E(\mathbb{F}_p)$ denote the trace of Frobenius. By the Hasse bound, $a_E(p) = 2\sqrt{p}\cos\theta_p$ for a unique $\theta_p \in [0, \pi]$. In this paper, we prove that the least prime p such that $\theta_p \in [\alpha, \beta] \subset [0, \pi]$ satisfies

$$p \ll \left(\frac{N_E}{\beta - \alpha}\right)^A,$$

where N_E is the conductor of E and the implied constant and exponent $A > 2$ are absolute and effectively computable. Our result is an analogue for CM elliptic curves of Linnik's Theorem for arithmetic progressions, which states that the least prime $p \equiv a \pmod{q}$ for $(a, q) = 1$ satisfies $p \ll q^L$ for an absolute constant $L > 0$.

221. On Higher Order Difference Equations with both Advance and Delay

Jacob Parsley Tennessee Tech University
Advisor(s): Dr. Lingju Kong, University of Tennessee at Chattanooga

A higher order difference equation defined on the integers with the p-Laplacian operator and containing both advance and delay is presented. Some criteria are given for the existence of infinitely many anti-periodic solutions of the equation. Several consequences of the main theorems are presented, as well as a couple of examples to illustrate the applicability of the results. This research is a result of the presenter's participation in a 2015 NSF-funded REU at the University of Tennessee Chattanooga.

222. A Fibonacci analogue of the Stirling numbers

Roshil Paudyal Howard University
Advisor(s): Dr. Jeffrey Remmel, University of California, San Diego

In this work, we present a Fibonacci analogue of the Stirling numbers of the first and second kind. If we let $(x)_{\downarrow_0} = (x)_{\uparrow_0} = 1$ and for $k \geq 1$, $(x)_{\downarrow_k} = x(x-1)\cdots(x-k+1)$ and $(x)_{\uparrow_k} = x(x+1)\cdots(x+k-1)$, then the Stirling numbers of the first and second kind are the connections coefficients between the usual power basis $\{x^n : n \geq 0\}$ and the falling factorial basis $\{(x)_{\downarrow_n} : n \geq 0\}$ in the polynomial ring $\mathbb{Q}[x]$. We study the combinatorial properties of Fibonacci analogues, $\mathbf{Sf}_{n,k}$ and $\mathbf{cf}_{n,k}$, of the Stirling numbers which are defined by the equations

$$x^n = \sum_{k=1}^{n} \mathbf{Sf}_{n,k}\ (x)_{\downarrow_{F,k}} \text{ and } (x)_{\uparrow_{F,n}} = \sum_{k=1}^{n} \mathbf{cf}_{n,k}\ x^k$$

where $(x)_{\downarrow_{F,n}} = x(x-F_1)\cdots(x-F_{k-1})$ and $(x)_{\uparrow_{F,n}} = x(x+F_1)\cdots(x+F_{k-1})$ are the Fibonacci analogues of the falling factorial and rising factorial bases. In particular, we develop a new rook theory model to give combinatorial interpretations to the numbers $\mathbf{Sf}_{n,k}$ and $\mathbf{cf}_{n,k}$.

223. Spectral Inference of a Directed Acyclic Graph Using Pairwise Similarities

Allison Paul La Salle Academy
Advisor(s): Mr. Soheil Feizi, Massachusetts Institute of Technology

A gene ontology graph is a directed acyclic graph (DAG) which represents relationships among biological processes. Inferring such a graph using a gene similarity matrix is NP-hard in general. Here, we propose an approximate algorithm to solve this problem efficiently by reducing the dimensionality of the problem using spectral clustering methods. We prove that the original problem can be simplified to the inference problem of overlapping clusters in a network. We then solve the simplified problem in two steps: first we infer clusters using a spectral clustering technique. Then, we identify possible overlaps among the inferred clusters by identifying maximal cliques over the cluster similarity graph. We illustrate the effectiveness of our method over various synthetic networks in terms of both the performance and computational complexity.

224. Pattern Avoidance in Task-Precedence Posets

Lucy Pepin University of Wisconsin-Eau Claire
Mitchell Paukner University of Wisconsin-Eau Claire
Jarred Wieser University of Wisconsin-Eau Claire
Advisor(s): Dr. Manda Riehl, University of Wisconsin-Eau Claire

Classical pattern avoidance was first studied on permutations. We have researched an extension to a new structure: multiple task-precedence posets with three levels, which we will call diamonds. The vertices of each diamond are assigned labels which are compatible with the poset. A corresponding permutation is formed by reading these labels by increasing levels, and then from left to right. We used Sage to form enumerative conjectures for the associated permutations avoiding collections of patterns of length three, which we then proved. We have discovered several interesting bijections between: diamonds avoiding 132 and certain generalized Dyck paths, diamonds avoiding 132 and 213, and diamonds avoiding 231 and 312. We have also found the generating function for descents in these permutations for the majority of collections of patterns of length three. In the near future, we aim to find closed formulas for avoiding 231 (we currently have a recursive formula) and avoiding 321 and their descent generating functions. Furthermore, an interesting application of this work can be found when we our task-precedence posets represent warehouse package fulfillment by robots, in which case avoidance of both 231 and 321 ensures we never stack two heavier packages on top of a lighter package.

225. Time-Inhomogeneous Branching Processes

Mark Perlman The University of Chicago
Nick Bhattacharya The University of Texas at Austin
Advisor(s): Dr. Dmitry Dolgopyat, University of Maryland

A branching process Z_n is a probability model for a population defined by $Z_0 = 1$ and $Z_{n+1} = \sum_{j=1}^{Z_n} X_{n,j}$ where $X_{n,j}$ are i.i.d. copies of X_n, the offspring distribution at time n. In the classical time-homogeneous theory, where

$X_n = X$ is independent of n, two results hold regarding the long-term behavior of the process: first, extinction (Z_n is eventually 0) occurs with probability one if and only if $E(X) \leq 1$; second, a normalized process conditioned on survival, $\frac{Z_n|Z_n\neq 0}{E(Z_n|Z_n\neq 0)}$, converges to an exponential distribution if $E(X) = 1$. We extend these results to the time-inhomogeneous case for both discrete- and continuous-time branching processes. The specific case when the generating function is fractional linear lends itself to explicit calculations. Then, general results follow from bounding by fractional linear processes. We find that extinction is sure if and only if $E(Z_n)^{-1}$ is not summable; and the scaled, conditioned process converges to an exponential if both $\sum_{i=1}^{n} E(Z_i)^{-1}$ and $E(Z_n)\sum_{i=1}^{n} E(Z_i)^{-1}$ tend to infinity.

226. Triangulating Almost-Complete Graphs

Kim Pham University of California, Irvine
Advisor(s): Dr. Padraic Bartlett, University of California, Santa Barbara

It is known that a complete graph K_n can be written as an edge-disjoint union of triangles, provided a few trivially necessary conditions (namely, the number of edges is a multiple of 3, and the degree of each vertex is even). Therefore, it is natural to wonder if a similar result would hold for almost-complete graphs; that is, graphs on n vertices such that every vertex has degree of at least $(1 - \epsilon)n$. Nash-Williams (1970) conjectured that there exists some $\epsilon > 0$ such that these graphs admit a triangle decomposition. In this presentation, we will discuss our proof of some cases of this result. Our proof uses techniques from Latin squares, design theory, and "trades" on graphs. No prior experience with these concepts will be necessary to follow this talk.

227. Logistic Patch Models for Transmission Dynamics of Ebola Virus Epidemic

Tin Phan Arizona State University
Advisor(s): Dr. Yang Kuang, Arizona State University

Mathematical models are necessary tools to help forecast the recent Ebola epidemic that occurs in West Africa. Often researchers approach this problem using variation of the compartmental model SIR (Susceptible, Infected, Recovered); however, we use a class of logistic patch models, which are derived from the well-known logistic equation by incorporating various migration rates between patches that individually exhibit logistic growth. Each model is fit to multiple data sets to compare the reliability in forecasting via error estimation and parameter confidence intervals as functions of trained data points. The basic reproduction number is also estimated. We conclude that the patch models improve the short-term forecasting but produce erratic behavior in long term forecasting in comparison with the logistic model.

228. Hall's Juggling Theorem by rook placements

Scarlitte Ponce California State University Monterey Bay
Advisor(s): Dr. Steve Butler, Iowa State University

We give an algorithmic proof of Hall's Juggling Theorem, any collection of throws whose average is a whole integer can be rearranged to form a valid juggling pattern by reinterpreting the problem with rook placements. Juggling patterns can be described by listing the series of throws that the pattern requires, $t_1 t_2 \dots t_n$ where at time $\equiv i \pmod{n}$ we throw the ball so it will land t_i beats in the future. A sequence of throws can be juggled if and only if there are no collisions. For a valid pattern of period n, the average of the throws, $\frac{1}{n}(t_1 + t_2 + \cdots + t_n)$, is a whole number that corresponds the amount of balls needed.

Hall's Juggling Theorem *Given throws $t_1, t_2, \dots, t_n$ satisfying $t_1 + t_2 + \cdots + t_n \equiv 0 \pmod{n}$, there is some permutation π so that $t_{\pi(1)} t_{\pi(2)} \dots t_{\pi(n)}$ is a valid pattern.*

We give a proof of this result based on a general proof of Hall's Theorem due to Joe Buhler and Ron Graham. We proceed by first showing how juggling patterns correspond to rook placements on an $n \times n$ board and then give an algorithm of placing the rooks.

229. Mathematically Modeling Cancer Metastasis through mechanical properties detected by a Microfluidic Microcirculation Mimetic

Sruti Prathivadhi Creighton University
Advisor(s): Dr. Andrew Ekpenyong, Creighton University

Accounting for over 90% of cancer deaths, metastasis is a complex process by which cancer translocates to organs away from the primary tumour site. Unfortunately, existing cancer drugs do not target metastasis as our understanding

of the field is limited. Thus, there is an urgent need for anti-metastasis therapy. In our project, we mathematically model the mechanical properties of metastasizing cancer cells during their circulation in blood vessels. Specifically, we consider the fluid dynamics of the microcirculation using a microfluidic platform which mimics capillary constrictions of the pulmonary and peripheral microcirculation. Using the Navier-Stokes equation, finite element analysis and COMSOL Multiphysics (registered) simulations, we extract the elastic and viscuous properties of the cancer cells, subjected to various chemotherapeutic drugs. The mechanical properties enable us to assess, in a non-invasive manner, the pro- and anti- metastatic effects of these cancer drugs. Our work is a first step towards establishing cell mechanics as a readout to assist in effective anti-metastatic drug development.

230. Statistical analysis of a case-control Statistical Analysis of a Case-Control Alzheimer's Disease: a Retrospective Approach with Sufficient Dimension Reduction

Rebecca Rachan North Central College
Subodh Selukar University of North Carolina, Chapel Hill
Trevor Adriaanse Bucknell University
Meshach Hopkins University of Maryland, Baltimore County
Advisor(s): Dr. Kofi Adragni, University of Maryland, Baltimore County

Alzheimer's Disease is a neurological disorder chiefly present in the elderly that affects functions of the brain such as memory and logic, eventually resulting in death. There is no known cure for Alzheimer's and evidence points to the possibility of a genetic link. This study analyzes microarray data from patients with Alzheimer's disease and disease-free patients in order to evaluate and determine differential gene expression patterns between the two groups. The statistical problem stemming from this data involved many predictor variables with a small sample size, preventing the use of classical approaches from being effective. We turn to a novel three-step approach: first, we screen the genes in order to keep only the genes marginally related to the outcome (presence of Alzheimer's); second, we implemented a sparse sufficient dimension reduction to retain only predictors relevant to the outcome; lastly, we perform a hierarchical clustering method to group genes that exhibit mutual dependence. We adapted this methodology from Adragni et. al and expand on their work by optimizing the existing R code with parallel capabilities in order to enhance performance speed. Thus, our results reflect both an analysis of the microarray data and a performance study of the modified code.

231. New Results on Ramsey Multiplicity and Graph Commonality

Sarvasva Raghuvanshi Research Science Institute
Advisor(s): Mr. James Hirst, MIT

If a graph G has v vertices, a *copy* of G inside a larger graph K is a subgraph $H \subset K$ on v vertices such that $G \subseteq H$. Similarly, we define an *anticopy* of G to be a subgraph $H \subset K$ on v vertices such that $G \subseteq \bar{H}$. A graph F is common if and only if the minimum density of copies and anticopies of F in any graph G is $2^{1-|E(F)|}$, where $|E(F)|$ denotes the number of edges in the graph F. Note that this minimum is attained when G is a random graph with edge density one-half. In this paper, we propose a modern proof that the graph formed from any number of disjoint copies of a common graph is itself common. This novel proof leads to innovative partial results and opens other questions about the commonality of disjoint graphs. We then prove that the graph obtained from a pentagon by adding a chord is common, resolving a central open problem in the field of graph commonality.

232. On a Modification of the Collatz Problem

Miriam Ramirez California State University, Northridge
Melida Paz California State University, Northridge
Advisor(s): Dr. Werner Horn, California State University, Northridge

The Collatz problem is investigating the map

$$C(x) = \begin{cases} \frac{x}{2} & \text{if } x \equiv 0 \bmod 2 \\ 3x+1 & \text{if } x \equiv 1 \bmod 2 \end{cases}$$

It has remained an open problem to prove that for any positive integer the sequence $x, C(x), C^2(x), \ldots$ will end up in the limit cycle $(4; 2; 1)$. We investigate the map $T : \mathbb{N} \to \mathbb{N}$ defined by

$$T(x) = \begin{cases} \frac{x}{p} & \text{if } p|x \\ mx + r(x) & \text{if } p \not| x \end{cases},$$

where $r(x)$ is such that $p|(mx + r(x))$ and $0 < r(x) < p$. For $m < p$ we have the following:
(i) for any positive integer x the sequence $x, T(x), T^2, \ldots$ will end up in a limit cycle,
(ii) if p divides x then $T(x) \leq x - 1$ and if p does not divide x then there exists a positive integer n such that if $x \geq n$ then $T^2(x) \leq x - 1$,
(iii) there are at most $p - 1$ limit cycles $c_1, c_2, \ldots, c_{p-1}$ for all positive integers p and m,
(iv) $\{x \in \mathbb{N} : x \longrightarrow c_j\}$ is infinite for all limit cycles c_j.
We also investigate the special case where $m = p - 1$.

233. Equal Excircles and Their Foundation in the Equal Incircles Theorem

Rebecca Rapp Washington and Jefferson College
Advisor(s): Dr. Michael Woltermann, Washington and Jefferson College

We examine a generalization of an old Sangaku problem, now well known as the Equal Incircles Theorem, and extend this result to excircles. The Equal Incircles Theorem states that if a triangle is divided into subtriangles such that the incircles of these subtriangles have equal radii, then the incircles constructed by joining 2 adjacent subtriangles must also have equal radii. This result also holds for joining $3, 4, 5, \ldots$ adjacent subtriangles. We believe that we have found a new relationship to determine the radii of a triangle's excircles, which leads to a similar result for equal excircles.

234. ETF-Based Models for Liquidity Risk

Zoe Rehnberg Washington University in St. Louis
Jonathan Beall Mercer University
Kelli Dowd Utah State University
Rivers Jenkins Clemson University
Advisor(s): Dr. Marcel Blais, Worcester Polytechnic Institute

An exchange-traded fund (ETF) is a security that represents an underlying collection of assets and trades on an exchange. ETFs provide easy diversification for investors and a more liquid market for positions dependent on the underlying assets. Liquidity in ETF markets is an important issue for financial institutions like State Street Global Advisors (SSgA) who must manage risk and participate in the buying and selling of assets in these markets. In illiquid markets, large transactions can result in high trading costs or an inability to liquidate the necessary assets in the necessary time frame. In order to accurately assess the liquidity in the underlying basket, financial institutions rely on liquidity metrics that model and track changes in liquidity over time. One such metric was developed in 2014 by the WPI Financial Math REU team that exploits the difference between the ETF share price and the net asset value (NAV), or price per share of the basket of underlying assets, to quantify liquidity. Because these two values are based on the same securities, the difference between them can be attributed solely to a liquidity difference. In our work, we expand this model beyond simple considerations of ETF price and NAV to include the effects of market depth and time-to-liquidation.

235. Numerical Solutions of the Helmholtz Equation via the Modified Galerkin Method for the Shape of a Biconcave Disk.

Jill Resh Roger Williams University
Advisor(s): Dr. Yajni Warnapala, Roger Williams University

The objective of this research is to investigate numerical solutions of several boundary value problems for the Helmholtz equation for the shape of a Biconcave Disk. The boundary value problems this research mainly focuses on are the Neumann and Robin boundary problems. The Biconcave Disk is a closed, simply connected, bounded shape modified from a sphere where the two sides concave toward the center, mapped by a sine curve. There are some numerical issues in this type of analysis; any integration is affected by the wave number k, because of the oscillatory behavior of the fundamental solution of the Helmholtz equation. This project was funded by NASA RI Space Grant and the NASA EPSCoR Grant for testing of boundary conditions for the Biconcave Disk. This method has already been investigated for the sphere, ellipsoid, superellipsoid, and the oval of cassini. The primary purpose of this research is to extend those known results to the Biconcave Disk with calculating the possibility of this shape acquiring sufficient conditions to be part of a spacecraft that might one day land on planet Mars.

236. Edge Detection from Fourier Phase Data

Alexander Reynolds Arizona State University
Advisor(s): Dr. Anne Gelb, Arizona State University

Accurate feature detection in signals is necessary in wide-ranging applications from medical imaging to computer vision. Spectral data is often collected in such applications, where many methods are used to extract information about the signal. The *concentration factor* method uses a first order relationship between the Fourier coefficients and jumps of a signal to devise filters that generate approximations which concentrate at the singular support of the signal, resulting in a highly customizable edge detector. The method has recently been expanded upon to detect edges in a signal given noisy, intermittent, or non-uniform Fourier data. Typical feature detection algorithms rely on both the magnitude and phase of the collected Fourier data. However, the spectral phase carries particularly useful information about the features of a signal. Thus, the development of an edge detector using only phase data will be beneficial in applications where the magnitude information is not able to be collected or is otherwise corrupted. Recent numerical results have shown that concentration factors can be designed for these situations. An analysis of the method will lend insight to the accuracy of the phase-only edge detector and its robustness to noisy, non-uniform, or intermittent data.

237. Two-Step Diffusion Model of Epithelial Cell Migration

Ryan Reynolds Trinity University
Chrissy Nielsen Trinity University
Advisor(s): Dr. Jonathan King, Trinity University

Tight junctions are macromolecular structures within epithelial cells that regulate solute flux between cells and are associated with sheet formation. Zonula Occludens-1 (ZO-1) is an essential component of the tight junction scaffolding barrier proteins together to the actin cytoskeleton. We employed the classic polarized epithelial cell line, MDCK cells for wounding assays. These cells with endogenous and knock-down levels of ZO-1 were stably transfected with a fluorescent mApple-Histone-2B construct to determine real-time cell density measures. In order to determine an accurate mathematical model of cell movement, confluent MDCK monolayers were wounded and migration was visualized using real-time confocal microscopy. Previous literature suggests an initial stage of cell migration that can be modeled by diffusion, then a transition to simple transport in a second stage. Our project develops a two-step model to determine the rate of diffusion, velocity of transport, and a transition time from diffusion to transport. Through this model, we show how the ZO-1 protein affects each of these variables during cell migration.

238. Algebras associated with the Hasse graphs of polytopes

Austin Riedl University of Wisconsin - Eau Claire
Austin Holmes University of Wisconsin - Eau Claire
Mitchell Lemons University of Wisconsin - Eau Claire
Advisor(s): Dr. Colleen Duffy, University of Wisconsin - Eau Claire

The primary goal of our project is to determine the structures of the graded algebras, $A(\Gamma)$, that are associated to the Hasse graphs, Γ, of polytopes. In particular, we are studying the n-dimensional semi-hypercube and the icosahedron, whose symmetry groups are finite Coxeter groups. For each symmetry, we consider the Hasse subgraph consisting of fixed k-faces of the polytope under the action of the symmetry. From each Hasse subgraph we determine the graded dimension of the subalgebra of $A(\Gamma)$ by counting the directed paths between each pair of levels in the graph. For both the n-semi-hypercube and icosahedron we have determined the generating functions which give us the graded dimensions of the subalgebras, which in turn allow us to describe the algebras $A(\Gamma)$.

239. The moduli space of 1|3-dimensional complex associative algebras

Austin Riedl University of Wisconsin-Eau Claire
Hengzhou Liu University of Wisconsin-Eau Claire
Chris Magyar University of Wisconsin-Eau Claire
Dylan Magnani University of Wisconsin-Eau Claire
Advisor(s): Dr. Michael Penkava, University of Wisconsin-Eau Claire

In this poster, we study the moduli space of 1|3-dimensional complex associative algebras, in other words, the codifferentials on a 3|1-dimensional complex vector space. We construct the moduli space by considering extensions of lower dimensional algebras. We also construct miniversal deformations of these algebras. This gives a complete description

of how the moduli space is glued together via jump deformations. We also discuss a theorem we proved which enabled us to construct the moduli space more easily, namely that every nilpotent $1|n$-dimensional complex associative algebra with $n > 0$ has a $1|(n-1)$-dimensional ideal.

240. A Metapopulation Model with Density Dependent Growth

Nicholas Roberts Arizona State University
Emily Masten Amherst College
Advisor(s): Dr. Richard Rebarber, Brigitte Tenhumberg, University of Nebraska Lincoln (both)

We develop an ecological model for metapopulations with density dependent population growth rates. A metapopulation is a collection of sub-populations living in distinct, but interacting, habitable patches. Density dependent growth rates occur when population density impacts population growth negatively or positively. A strong Allee effect is a type of positive density dependence where there exists a density threshold below which the population will go extinct. Our metapopulation model represents each patch with a population growth function that includes both negative density dependence and strong Allee effects and the interpatch dynamics through migration probabilities and migration induced mortality. Analysis on a restricted two-patch case shows that bifurcations in the system's equilibria structure are richest with respect to the migration parameters. Specifically, varying patch connectivity via migration parameters can create source/sink patch dynamics. Additionally, comparing the basin of attraction for the zero equilibria (extinction) between the deterministic model and stochastic simulations shows the deterministic model routinely underestimates viable population, an important consideration in conservation applications.

241. Guidance and Path Planning of Parallel Robot with Feedback

Thomas Rochais University of Wyoming
Courtney Long University of Wyoming
Advisor(s): Dr. Farhad Jafari, University of Wyoming

Optimal path planning and design is a fundamental and deep question in many areas of science and engineering. Here, we explore a bumper-car feedback approach to path planning and design. We describe the kinematics of a model parallel robot having contact with the surface and develop a path planning strategy guided by the robot bumping into the walls. Using a simple Snells' law model for reflection from the wall, a model for the local curvature (derivable from the tangent space) of the wall is developed, and the robot is guided by feedback. Such a model would be important in guiding a rescue robot through a circuitous mine shaft, where visibility/sensor feedback is not an option. This design is coded in Matlab and simulations are run to show the resulting dynamics. The effect of robot geometries on the path design is investigated.

242. Lattice-points enumeration in polytopes: Study of the coefficients of the Ehrhart quasi-polynomial

Helene Rochais University of Wyoming
Advisor(s): Dr. Tyrrell McAllister, University of Wyoming

A very important problem in discrete geometry is counting points with integer-coordinates (also called "lattice-points") in polytopes. Geometrically, a polytope is the smallest convex set containing the vertices defining the polytope (which is just a polygon in two dimensions). Lattice-point enumeration has numerous applications in mathematics such as combinatorics and operations research. Our goal is to study the function that counts the lattice-points in a polytope and its integer dilates when the vertices of the polytope have rational coordinates. This function is a quasi-polynomial. That is to say, it is a polynomial with coefficients that are themselves periodic functions in the variable. This function is the Ehrhart quasi-polynomial of the polytope. For this research, I am focusing on the periods of the coefficients of the Ehrhart quasi-polynomial in order to see if they can take on any value for some polytope or if there exist restrictions on these periods. It has already been proven that no interesting restrictions exist for the 2-dimensional case and we have constructed a family of polytopes that proves that no interesting restrictions exist for the non-convex 3-dimensional case. We are also able to generalize some of our constructions to higher dimensions.

243. A Matroid Generalitzation of Sperner's Lemma

Andres Rodriguez Universidad de los Andes
Gabriel Andrade University of Massachusetts, Amherst
Alberto Ruiz University of Puerto Rico, Rio Piedras
Advisor(s): Dr. Francis Su, MSRI-UP

In a 1980 paper, Lovász generalized Sperner's lemma for matroids. He claimed that a triangulation of a d-simplex labeled with elements of a matroid M must contain at least one "basis simplex". We present a counterexample to Lovász's claim when the matroid contains loops and provide a necessary condition such that Lovász's generalization holds. Furthermore, we show that under some conditions on the matroids, there is an improved lower bound on the number of basis simplices. We present further work to sharpen this lower bound by looking at M's lattice of flats and by proving that there exists a group action on the simplex labeled by M with $\mathcal{S}_n$.

244. Minimal Complexity C-complexes for Colored Links

Grant Roth University of Wisconsin-Eau Claire
Advisor(s): Dr. Christopher Davis, University of Wisconsin-Eau Claire

A link is a collection of strings tangled up with their ends fused together. Given any link there is a two-dimensional surface it bounds. The genus of this surface gives a measure of the complexity of the link. In this project, we study the analogous measure of complexity given by a generalization of a surface called a C-complex. In order to show that this measure captures some information, we present an infinite family of links for which this new measure of complexity is arbitrarily high. It is an interesting question how the measures of complexity coming from classical surfaces and C-complexes compare.

245. Wavelet and Cosine Transform Based Eigenface Facial Recognition Algorithm

Hannah Rugger Western Connecticut State University
Advisor(s): Dr. Xiaodi Wang, Western Connecticut State University

Facial, fingerprint, and voice recognition software is becoming increasingly popular. But there are challenges in creating efficient and accurate recognition systems. Traditionally, the Eigenface Method, based on principle component analysis (PCA), is used to recognize faces and fingerprints. This method manipulates and compares vector representations of faces, which is more efficient than comparing the faces themselves. However, this method takes up a lot of memory space. A more efficient method would be using a special transformation to transform the image to corresponding domain. That will reduce memory space and computational time. In this research, we are going to use M-band wavelet transforms for different values of $M \geq 2$, and a combined wavelet transform and a cosine transform on top of an Eigenface Method and compare to the standard Eigenface Method for efficiency and accuracy.

246. Mathematically Modeling Motion of Cells in Porous Media

Ahaan Rungta MIT
Advisor(s): Mr. Andrew Rzeznik, MIT

Biological cells are known to float, or "swim", in the medium they inhabit. The motion of such swimming cells usually depends on the medium and other external factors such as constant forces created by fields. Many of these systems have been studied in past research. For example, researchers have discovered much about the swimming properties of single-celled organisms in an ideal free fluid with no disturbances. However, many questions are yet to be answered about how cells move in more realistic environments, such as porous media, which is what we study.

To simulate a porous structure for our cell's environment, we write code to simulate random walks for some simple lattice structures within boundaries. Then, to mimic factors that bias the walk, we add persistency to our code. For every scenario, we form frequency graphs which plot the number of times the particle/cell visited every point in our grid. These graphs give us some insights into what might be interesting to study in the future. Ideas on further research include generalizations, even randomizations, of lattices, analogs of diffusion coefficients in random flow, investigation of conjectured phase transitions, and the effects of boundaries on motion microorganisms, both analytically and experimentally.

247. Matrix Completions for the Commutativity and Transpose Equations

Jack Ryan North Central College
Elijah Cronk Ithaca College
Advisor(s): Dr. Geoffrey Buhl, California State University Channel Islands

A matrix completion problem attempts to determine if a partial matrix composed of specified and unspecified entries can be completed to satisfy some given property. This project focuses on determining which patterns of specified and unspecified entries for a partial matrix can be completed to solve the commutativity matrix equation $AX - XA = 0$ and the transpose equation $AX - X^T A = 0$. We approach this problem with two techniques: converting the matrix equations into linear equations and examining bases for the solution spaces of the matrix equations. We seek to determine whether a particular pattern can be written as a linear combination of the basis elements. If so, the pattern is admissible; otherwise, the pattern is inadmissible. This work classifies patterns as admissible or inadmissible for each of the matrix equations based on the ability or inability of their corresponding partial matrices to be completed to satisfy the equation for almost any matrix A. Our results present a partial characterization of admissible and inadmissible patterns for the commutativity and transpose equations.

248. On the Values of Running Sums and Stopping Times for Various Probability Distributions

Roshni Sahoo Boston University- PROMYS
Dhruv Medarametla Boston University- PROMYS
Vanshika Jain Boston University- PROMYS
Advisor(s): Dr. Jared Weinstein, Boston University

Given a probability distribution with random variables $X_1, X_2, \ldots$ and a predetermined threshold $x \geq 0$, let $N(x)$ be the stopping time, the least N for which the N-th running sum satisfies $S_N := X_1 + X_2 + \cdots + X_N \geq x$. We derive formulas for the expected stopping time and the expected running sum for the uniform distribution and variants of the exponential distribution. In our main result, we prove, for positive-valued distribution functions whose first three moments exist, that $\lim_{x\to\infty} \mathbb{E}[N(x)] - \frac{x}{\mu} - \frac{\phi}{2\mu^2} = 0$ and $\lim_{x\to\infty} \mathbb{E}[S_{N(x)}] - x - \frac{\phi}{2\mu} = 0$, where μ is the first moment and ϕ is the second moment.

249. Radio Labeling for Even and Odd Cycles

Jeffrey Salazar California State University, San Bernardino
Ibian Cervantes California State University, San Bernardino
Noe Nava California State University, San Bernardino
Advisor(s): Dr. Min-Lin Lo, California State University, San Bernardino

Let G be a connected graph. For any two vertices u and v, let $d(u, v)$ denote the distance between u and v in G. The maximum distance between any pair of vertices is called the diameter of G and denoted by $\text{diam}(G)$. A radio-labeling (or multi-level distance labeling) of a connected graph G is a function $f : V(G) \longrightarrow \{0, 1, 2, 3, \ldots\}$ with the property that $|f(u) - f(v)| \geqq \text{diam}(G) - d(u, v) + 1$ for every two distinct vertices u and v of G. The span of f is defined as $\max |f(u) - f(v)|$ for u, v in $V(G)$. The radio number of G is the minimum span over all radio-labeling of G. In this presentation we will discuss the progress we made towards finding the radio number for cycle graphs during a 2015 summer research program, which is a part of the NSF PRISM grant DMS-1035120 (Proactive Recruitment in Introductory Science and Mathematics).

250. Computing All Isolated Invariant Sets at a Fixed Resolution

Martin Salgado-Flores College of William & Mary
Advisor(s): Dr. Sarah Day, College of William & Mary

Conley Index Theory has inspired the development of rigorous computational methods to study dynamics. These methods construct *outer approximations*, a combinatorial representation of the system, which allows us to represent the system as a directed graph. Invariant sets appear as combinations of vertices and edges on the resulting digraph. Conley Index Theory relies on isolated invariant sets, which are maximal invariant sets that meet an isolation condition, to describe the dynamics of the system. In this work, we present a rigorous way of computing all isolated invariant sets given an outer approximation. We improve upon an existing algorithm that "grows" isolated invariant sets individually and requires an input size of 2^n, where n is the number of grid elements used for the outer approximation.

251. Recursion in the Jones Polynomial of Closed Twisted m-braids

Larissa Sambel Bates College
Advisor(s): Dr. Peter Wong, Bates College

The Jones polynomial $V(L)$ of an oriented link L is an important algebraic knot invariant. Recursions in the Jones polynomial can be used to explain similarities within link families and show relationships among different link families. We are interested in the Jones polynomials of links formed by closing twisted m-braids. Since any oriented link can be represented as the closure of some braid with n strands, a link $B_n(m)$ consists of a closed braid with n strands and m crossings. The Kaufmann bracket, and therefore the Jones polynomial, for these braided links can be computed using the skein relations for triple crossings as described by C. Adams. It was shown by Miles Isacke that the Jones polynomial for a certain subset of three stranded closed braids, $B_3(m)$, is given by $V(L) = ((-A)^{-3(3m)}(A^{4+3m} + A^{3m-4} + (-1)^m A^{-3m} + (-1)^{2m} A^{-3m}))_{t^{-1/2}} = A^{-2}$. We are extending these calculations to include other types of three stranded closed braids and four stranded closed braids. We hope to establish a relationship between the Jones polynomials of these links and to determine a standard way to transform three stranded closed braids into four stranded closed braids.

252. Combinatorics of Borel-Fixed Sets

Cecily Santiago Mount Holyoke College
Maya Urbschat Mount Holyoke College
Rose Dennis Mount Holyoke College
Advisor(s): Dr. Jessica Sidman, Mount Holyoke College

An ideal is an algebraic object that is closed under addition and multiplication in a ring. We focus on Borel-fixed ideals, a subset of monomial ideals whose behavior satisfies nice combinatorial properties. Let m be a monomial in n variables $x_1 > x_2 > ... > x_n$ in a set S. We say that S satisfies the Borel property if $m\frac{x_i}{x_j}$ is also in S for all $x_j|m$ and $x_i > x_j$. We refer to S as a Borel-fixed set. If the generators of an ideal I form a Borel-fixed set, then I is called a Borel-fixed ideal. Expanding our knowledge of Borel-fixed ideals and the Borel-fixed sets that generate them may translate to a better understanding of this important class of ideals. Our research investigates the structure of Borel-fixed sets in terms of partial ordering. We demonstrate that the partially ordered set of all monomials of degree d in n variables is a lattice, and is graded with rank function $r(m = x_1^{a_1}x_2^{a_2}...x_n^{a_n}) = \sum_{i=1}^{n}(i-1)a_i$. Additionally, we consider methods for counting the number of Borel-fixed sets of degree d in n variables, and show that there are $d+1$ Borel-fixed sets of degree d in two variables and $2^{d+1}-1$ Borel-fixed sets of degree d in three variables.

253. P-Positions in Modular Extensions to Nim

Karan Sarkar MIT PRIMES
Advisor(s): Dr. Tanya Khovanova, Massachusetts Institute of Technology

Nim is the basis of combinatorial game theory. It involves a set of heaps of tokens. There are two players who alternate taking away tokens from as set of piles of tokens. A player may take tokens from one pile only. The winner is the player who takes the last token. We investigate a generalization of Nim, which we call an m-Modular-Nim in which a player, in addition to the standard Nim moves, may also take away a multiple of m total tokens from any subset of piles. We found the winning strategy for this game with 2 piles and the winning strategy for any number of piles and odd m.

254. A Host-Parasite-Commensal Ecological Model Based on Field Studies in the Great Plains

Monika Satkauskas Creighton University
Advisor(s): Dr. Rebecca Gasper, Creighton University

Although mathematical models for ecosystem dynamics exist, most do not consider commensal relationships found in the niche. We offer a model for a freshwater snail-trematode-Chaetogaster ecosystem found at the Pine Ridge Indian Reservation. Trematode larvae infect the snails, while the Chaetogasters both prey on the trematode and have a commensal relationship with the snail. We analyze populations in terms of a nonlinear host-parasite-commensal ecosystem model using the Hartman-Grobman Theorem to find stable equilibria, and for more involved eigenvalue computations, we give intervals that guarantee asymptotic stability via the Routh-Hurwitz Theorem. Additionally, we plot the percent of snails (in- and out-of-transect) infected with Chaetogasters against the observed trematode infections, and a linear regression shows Chaetogaster prevalence correlates negatively with trematode prevalence.

255. A Study of Bar and Arc k-Visibility Graphs

Mehtaab Sawhney Commack High School
Advisor(s): Jonathan Weed, Massachusetts Institute of Technology

Bar Visibility Graphs were introduced by Duchet *et al.* and Schlag *et al.* to model Very Large Scale Integration (VSLI), the process where thousands of transistors are wired together on a chip. Dean *et al.*, Babbit *et al.* and Hutchison generalized Bar Visibility Graphs to Bar and Arc k-Visibility Graphs. The maximal number of edges in an Arc k-Visibility Graph with n vertices is improved to at most $(k+1)(3n-\frac{3k+6}{2})$ edges for $n > 4k+4$ and $\binom{n}{2}$ for $n \leq 4k+4$. Then the maximal edge bound given in Babbit *et al.* for SemiArc k-Visibility Graph is shown to be optimal, disproving a conjecture given in Babbit *et al.* Shifting focus, this research also progresses towards classifying these graphs. A question raised in Hartke *et al.* is resolved by proving that a family of Bar i-Visibility Graphs is never contained in a family of Bar j-Visibility Graphs for $i \neq j$. Finally the notion of random k-Visibility Graphs is introduced and the expected number of edges in a SemiBar k-Visibility Graph is calculated given a random distribution of endpoints.

256. Sums of Cubes in Quaternion Algebras

Blake Schildhauer McDaniel College
Advisor(s): Spencer Hamblen, McDaniel College

Generalization of Waring's Problem - that for every natural number k there exists an integer $g(k)$ such that every natural number can be written as the sum of at most $g(k)$ k-th powers - have been studied in a variety of contexts from algebraic number fields to non-commutative groups. We extend results on sums of cubes in the complex numbers to give bounds for $g(3)$ for certain quaternion algebras.

257. Identifying Spanning Trees of Graphs

Thomas Schuler Nebraska Wesleyan University
Advisor(s): Dr. Austin Mohr, Nebraska Wesleyan University

Given a tree T and a graph G both on the same number of vertices, how can we efficiently identify whether T is a spanning tree of G? This question is a special case of the well-studied subgraph isomorphism problem, which is known to be NP-Complete in general. The algorithm presented here makes use of the structure of spanning trees to minimize the extensive backtracking required by more general methods.

258. Beyond Planets

Felicia Sciortino Le Moyne College
Advisor(s): Dr. Jonathan Needleman, Le Moyne College

Set is a fun and somewhat addictive card game. With simple game play, the object is to make sets outs of three cards based upon the four characteristics of quantity, color, fill, and shape. Each set must have for each characteristic everything or nothing in common. The game can be viewed as a finite geometry. Every card describes a different point on the plane and each set creates a unique line. Based off of the College Mathematics Journal article "Sets, Planets, and Comets," (Baker, et al) a planet is any collection of four coplanar cards and is found by two different pairs of cards requiring the same card to complete them. This is equivalent to two distinct lines intersecting at a point. This poster explores when three lines converge at a single point. This generalization lead us to create special collections of six points that we call "airplanes," "spaceships," and "UFOs." Each one has a unique characteristic that sets them apart from one another. Various properties of these arrangements are studied, including a preliminary study of how many cards are needed to require the existence of each new definition.

259. Properties of the Roots of Tribonacci-type Polynomials

Huei Sears Michigan State University
Alexander Durbin Michigan State University
Advisor(s): Dr. Aklilu Zeleke, Michigan State University

Consider Tribonacci-type polynomials defined by the following recurrence relation $T_n(x) = \alpha(x) \cdot T_{n-1}(x) + \beta(x) \cdot T_{n-2}(x) + \gamma(x) \cdot T_{n-3}(x)$, where recurrence coefficients $\alpha(x)$, $\beta(x)$, and $\gamma(x)$ and initial conditions $T_0(x)$, $T_1(x)$, and $T_2(x)$ are arbitrary functions of x. On this poster, we present matrix representations of $T_n(x)$, namely $M_n(x)$, such

that $\det|M_n(x)| = T_{n-1}(x)$. Using this determinant representation, we discuss the nature of all roots of all polynomial sequences of this form using an alternative method of Gerŝchgorin's Circle Theorem, Laguerre's application of Samuelson's Inequality, and an application of Rouché's Theorem. Special cases of $T_n(x)$ shown include a recurrence with only real roots and a recurrence with only complex roots. We conclude with a presentation of ordinary generating functions for all polynomials mentioned.

260. Poster withdrawn.

261. On the least common $\tau_{(n)}$-factor

Susan Seijo-Mendez University of Puerto Rico-Mayaguez
Advisor(s): Dr. Reyes M Ortiz-Albino, University of Puerto Rico-Mayagez

The notion of a τ-factorization or τ-products in the general theory of (nonatomic) factorization was defined by Anderson and Frazier, in 2006. We are interested when τ is the equivalence relation modulo n over the integers, usually denoted by $\tau_{(n)}$. This type of factorizations were further studied by Hamon(2007), Ortiz (2008) and several Ortiz's students(2010-present). We study whether the least common $\tau_{(n)}$-product of two nonzero nonunit exist. In fact, the algebraic definition turns out to be too strong, hence we consider the order version of it. We called this version the minimum common $\tau_{(n)}$-product, considering that is the smallest $\tau_{(n)}$-product. We will present examples to show the non-existence of the least common $\tau_{(n)}$-product, and some characterization of how looks like the minimum common $\tau_{(n)}$-product for several n's.

262. The Broken Stick Problem in Higher Dimensions: From a Classic Puzzle to Modern Distance Geometry

Yuliya Semibratova University of Illinois at Urbana-Champaign
Yi Xuan University of Illinois at Urbana-Champaign
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Advisor(s): Dr. AJ Hildebrand, University of Illinois at Urbana-Champaign

If a stick is broken up at two random points, what is the probability that the three pieces will form a triangle? This question, called the broken stick problem, first appeared about 150 years ago in an examination at Cambridge University. It attracted the interest of 19th century French probabilists, and more recently was popularized by Martin Gardner. In this poster presentation, we consider the generalization of this problem to three (or more) dimensions. In particular, if a stick is broken up at five random points, what is the probability that the six pieces will form a tetrahedron? Questions of this type arise in the field of distance geometry, which has connections areas such as wireless sensor networks and molecular biology. This work was carried out at the Illinois Geometry Lab (IGL).

263. Data Based Behavior Modeling in DOTA2

Munira Shahir University of Maryland, Baltimore County
Advisor(s): Dr. James M. Keiser, Laboratory for Analytic Sciences, NCSU

In our project, we used data from the multiplayer online battle arena (MOBA) game Defense of the Ancients 2 (DOTA2) in order to develop a data driven behavioral model and predict a player's next behavior. Based off of the data gathered from dotabank.com, we developed a Classification and Regression Tree (CART) model that gave us the player's next location and action state. Utilizing inputs such as the character's levels, number of kills, last hits, and deaths of the character we predict the aforementioned player behaviors. Overall, we found that this model performed well on common player actions and gave good approximations of a player's locations.

264. Improving the accessibility and graphical realization of the USC Shoah Foundation archive

Megan Shearer University of Arizona
Hangjian Li UCLA
Advisor(s): Krishna, Bhogaonker

The RIPS 2015 USC Shoah Foundation (USC SF) group worked with the USC SF's *Visual History Archive* (VHA), which houses a vast repository of video testimonies of genocide survivors. As the archive grows, indexing and searching the database becomes more difficult. The RIPS USC SF project focuses on improving the search and visualization

aspects of the VHA. Searching the database for results that are relevant to a search query is a major part of the project. The team uses the mathematical theory of *topic modeling* as a backbone to the project. The existing theory of *Latent Dirichlet Allocation* (LDA) extends to a new theory, *multiple-corpora Latent Dirichlet Allocation* (mLDA), which is more useful for the Foundation's dataset. The new theory allows for the treatment of distinct types of corpora whilst still training a single topic model. Meaningful topics are extracted from the data using mLDA and provide a more accurate model than standard LDA. The team produces a search tool that compares the topic distributions of documents to a query. The search tool gives better search performance than existing word-level search algorithms. The team produces an interactive visualization that improves the ability to interact with VHA.

265. Large Acyclic and Outerplanar Subgraphs of Planar Graphs

Melissa Sherman-Bennett Bard College at Simon's Rock
Advisor(s): Dr. Glencora Borradaile, Oregon State University

Albertson and Berman have conjectured that every planar graph has an induced forest on half of its vertices; the current best result, due to Borodin, is an induced forest on two fifths of the vertices. We show that the Albertson-Berman conjecture holds, and is tight, for planar graphs of treewidth 3 (and, in fact, for any graph of treewidth at most 3). We also improve on Borodin's bound for 2-outerplanar graphs by finding a large outerplanar induced subgraph and invoking Hosono's result that outerplanar graphs have large induced forests. Finally, we discuss potential extensions of this approach to k-outerplanar graphs and the related problem of the vertex arboricity of 2-outerplanar graphs.

266. A Concrete Visualization of Lens Spaces

Casey Shiring University of Wisconsin - La Crosse
Advisor(s): Dr. Whitney George, University of Wisconsin - La Crosse

Although we cannot directly visualize four- and higher-dimensional spaces, we can manipulate how we view three-dimensional space to simulate higher-dimensional spaces. One important class of higher-dimensional topological spaces is called the class of *lens spaces*, which naturally arise in certain areas of theoretical physics, and are the first nontrivial collection of 3-manifolds to be completely classified up to homeomorphism. In this project, we survey visualization techniques for the lens spaces S^3, $S^2 \times S^1$ on a level of detail and breadth of perspective omitted in most textbook discussions of these spaces. A visualization of the $\mathbb{Z}/p$ construction of the lens space $L(p,q)$ is also presented. We present the constructions and visualizations of these spaces with computer graphics created using Mathematica.

267. Audio Watermarking using M-Band Wavelet Transformations

Adebowale Shoroye Western Connecticut State University
Alexandra Sabith Western Connecticut State University
Advisor(s): Dr. Xiaodi Wang, Western Connecticut State University

Wavelet transformations allow watermarking to be added to audio files with better security and robustness. In our study, we will look at watermarking an audio clip with a different audio message. This is accomplished by first reshaping our audio signals into a matrix, then applying a wavelet transformation to this matrix. Next, we apply a quantum signal transformation to our watermark and to our approximation portion of the wavelet transform matrix to convert them to pseudo quantum signals. Finally, we embed our pseudo quantum signal of the watermark into the corresponding pseudo quantum signal of the wavelet transform of our original audio signal. This method using watermarking will play a significant role in keeping audio files protected from theft by unintended parties.

268. The Territorial Raider Model with Strategic Movement and Multi-Group Interactions

Dennis Silva, Jr. Worcester Polytechnic Institute, Worcester MA
Nina Galanter Grinnell College, Grinnell IW
Advisor(s): Dr. Jan Rychtar, Jonathan T. Rowell, The University of North Carolina at Greensboro

We analyzed the territorial raider game, a graph based competition for resources, with strategic movement. First we investigated the game in which players are treated as individual organisms. Utilizing a machine learning algorithm, we discovered that the only strict Nash equilibrium strategy sets occur when all players raid one another and players do not compete with one another. This indicates equilibria are generated by derangement functions of a graph. Thus,

we found that a graph will permit a derangement if and only if it permits a strict Nash equilibrium. We then extended the game to the case where players are "hives" or "armies" which can divide themselves among multiple territories. We examined this division of armies both in the discrete and continuous cases. Our results include Nash equilibria for regular graphs and regular bipartite graphs in both of these cases. Our results suggest that while group entities defend in more cases than in the individual organism game, the portion of a group defending varies based on the degree of vertices and the advantage given to owners in protecting resources.

269. Modeling habitat use for the endangered James Spinymussel (Pleurobema collina): An approach for detecting rare cryptic organisms

Katie Sipes James Madison University
Dakota Kobler James Madison University
Advisor(s): Christine May, James Madison University

Freshwater mussels are keystone species in their ecosystems, and their filter feeding ameliorates water quality in downstream areas. Over 70% of freshwater mussel species worldwide are listed as vulnerable or more greatly threatened. The James Spinymussel (Pleurobema collina) is a species of top priority for conservation in Virginia. Due to limited research, cryptic appearance and behavior, and small population sizes, freshwater mussel conservation efforts have been hindered. A mark and recapture study has tracked approximately 40 James Spinymussels and 80 Notched Rainbow (Villosa constricta) mussels marked with Passive Integrated Transponder (PIT) tags at Swift Run in the summer of 2015. Multiple mussel recapture histories provide data about habitat use, stream-bed surface expression, and inform source-sink models. This integrated approach aims to further our understanding of rare cryptic organisms and analyze their fluctuations in the reach of the study.

270. Volume and Determinant Densities of Hyperbolic Links

Mia Smith Williams College
Gregory Kehne Williams College
Alexander Kastner Williams College
Xinyi Jiang Stanford University
Advisor(s): Dr. Colin Adams, Williams College

The hyperbolic volume of a knot is a powerful invariant for tabulating knots. To better understand hyperbolic volume, we investigate the *volume density* of a hyperbolic link, which is the ratio of hyperbolic volume to crossing number. We study its properties and those of a closely-related invariant called the *determinant density*. It is known that the sets of volume densities and determinant densities of links are dense in the interval $[0, v_{oct}]$. We show that for any $x \in [0, v_{oct}]$, we can construct sequences of alternating knots whose volume and determinant densities both converge to x. We also investigate the distributions of volume and determinant densities for hyperbolic rational links and prove that they are dense subsets of $[0, 2v_{tet}]$ and $[0, 2\pi \log \phi]$, respectively. These results are the work of our SMALL 2015 knot theory group.

271. Fast or Slow? The effects of a two-semester Calculus class versus a traditional one-semester class on retention of material.

Sydne Smith George Mason University
Alicia Suchicital George Mason University
Advisor(s): Ms. Catherine Sausville, Mary Nelson, George Mason University

Students often complain that they would perform better in a course if the pace were slowed down. In order to assess this, a comparison between a traditional one-semester Calculus I class, slower paced two-semester Calculus I classes, and an inquiry based learning one-semester Calculus I class was conducted. A short assessment covering Calculus topics up through the chain rule, which is the stopping point of the first semester of the two-semester Calculus class, was given to students in order to examine their knowledge of course material while also inquiring about their thoughts and confidence levels in each class. This poster will address the differences found in course pacing on students confidence, comprehension, and long-term retention of knowledge.

272. Some Properties of Generating Matrices for Generalized Pell Sequences

Elijah Soria Saint Mary's College of California
Advisor(s): Dr. Kristen Beck, Saint Mary's College of California

The muse for this research project are a couple of generalizations of the Pell sequence, named the k-Pell sequence and Generalized k-Pell sequence, which are defined respectively as $P_{k,n} = 2P_{k,n-1} + kP_{k,n-2},\ P_{k,0} = 0, P_{k,1} = 1$ and $G_{k,n} = 2G_{k,n-1} + kG_{k,n-2},\ G_{k,0} = G_{k,1} = a \in \mathbb{Z}^+$. The methods used to come up with new theorems for these sequences were to use $n \times n$ tridiagonal generating matrices whose determinants are equal to the n^{th} term in each respective sequence. Using these generating matrices, it was possible to make connections between certain characteristics of these matrices with the determinant, creating an accessible way to work with the sequences in question.

273. Locating Numbers and Sets for Disconnected Graphs

Jonathon Spaw Millikin University
Advisor(s): Dr. Joe Stickles, Millikin University

The locating code for a vertex v in a finite graph $G(V, E)$ is a vector representing distances of v with respect to vertices in an ordered subset W of $V(G)$. We call W a locating set if each vertex in the graph has a distinct locating code and we call the number of vertices in the minimal locating set the locating number. The concepts of locating sets and numbers were developed by Pirzada et. al. in order to study graphical representations of commutative rings. Previous work involving locating sets has depended on the assumption that $G(V, E)$ is connected. We prove conditions for locating sets of disconnected graphs and present a closed form for computing the locating number of any undirected graph. Furthermore, we give characterization theorems for graphs of particular locating numbers, which generalize past results by Pirzada et. al. Finally, we examine an algorithm we have developed in Sage for computing locating numbers and sets for a graph.

274. Classifying all Knots of 17 Crossings

Dylan Spence University of Delaware
Advisor(s): Dr. Stavros Garoufalidis, Georgia Institute of Technology

Knots — closed curves in three dimensional space that do not intersect themselves — appear in diverse fields such as biology, dynamical systems, and high energy physics, and are of utmost importance to many areas of geometry and topology. Information about the various properties of knots could lead to insight in a variety of fields. A knot invariant is a mathematical object that can differentiate between two knots that are not the same. They can take many forms, but the most interesting examples are the quantum invariants — invariants derived or calculated from methods in quantum field theory. In 1989, physicist Edward Witten connected a knot theory construction called the Jones Polynomial to topological quantum field theory, earning himself the Fields medal. There are several important conjectures in both physics and mathematics as to the behavior of the Jones polynomial for knots of crossing number 17 or larger. In our work, we implement the latest methods in computational topology and big data to compute and analyze a table of knots and their related invariants up to 17 crossings. This work extends the current table of known knots from Hoste, Thislewaite, and Weeks, and work is in progress to extend this further to higher crossings.

275. Model Validation and Student Life Tables for Post-Secondary Enrollment in the United States

David Spencer University of North Carolina at Chapel Hill
Advisor(s): Dr. Arial Cintron-Arias, East Tennessee State University

The Life Table is common in both demography and actuarial science. It is an age structured table containing insightful information (e.g. survival and expectancy) about members of a population. In this project we translate demographic concepts into the context of undergraduate student enrollment. We consider several candidate models of school progression, expressed in matrix form, and validate these models against statewide undergraduate enrollment data obtained from the Integrated Postsecondary Education Data System. Least squares methods are implemented to obtain parameter estimates, while Akaike Information Criterion is employed as a statistic for model selection.

Upon selecting the best suited model we then use bootstrapping sampling to obtain model parameter uncertainty bounds. The latter give large standard errors for the estimated Life Table functions, thus only point estimates of these quantities are reported. Moreover, we report on estimates of student life expectancy, probability of school departure, average number of years in each school group (e.g., sophomore), for fifty states and the District of Columbia.

276. Detecting gene-gene interactions that underlie cancer using the R package algstat

Melissa Stadt University of Washington
Iliana De La Cruz St. Mary's University
Taylor Spino North Central College
Catherine Sullivan Siena College
Advisor(s): Dr. Luis Garcia-Puente, Sam Houston State University

Interactions between single nucleotide polymorphisms (SNPs) and complex diseases have been an important topic throughout epidemiological studies. Previous genome-wide-association studies have mostly focused on gene variables at a single locus. In our project, we perform a focused candidate gene study to test the interaction of multiple SNPs with the risk of different types of cancer. Using the R package algstat, developed by Kahle, Garcia-Puente, and Yoshida, we developed an algorithm which can test for independence between several variables and the disease. We applied our methods to the study of gene-gene interaction on cancer data obtained from the European case-control study Gen-Air.

We were able to find strong evidence to reject independence of many triplet combinations of SNPs with the disease. These results are relevant to the general field of epidemiology due to the strong association found between the variables and the disease. Outside of the study of SNP-cancer association, this algorithm can be easily adjusted to perform general gene interaction studies using arbitrary log-linear statistical models.

277. Context-Directed Reversals of Signed Permutations

Caleb Stanford Brown University
Haley Schilling University of Massachusetts Amherst
Jackson Ramsey Baylor University
Hannah Li Pomona College
Advisor(s): Dr. Marion Scheepers, Boise State University

This project examines the context-directed reversal (cdr) sorting operation on signed permutations, and its generalization, the gcdr operation (sometimes known as local complement) on vertex-signed graphs. The cdr operation is a restriction of the well-studied standard reversal move on signed permutations. While all signed permutations are sortable by reversals, not all signed permutations are sortable by cdr moves. We first find and characterize the strongest possible invariant on signed permutations under cdr. Using the invariant, we characterize precisely which signed permutations are sortable by cdr moves. Building on previous work which characterizes a signed permutation by its overlap graph, we then generalize both of these results to vertex-signed graphs and gcdr. Finally, we give some enumeration results on the number of graphs that are sortable by gcdr moves.

278. Towards a Stability Theory for Feynman's Operational Calculus in the Purely Discrete Setting: A Direct Approach

Samantha Stanley Creighton University
Advisor(s): Dr. Lance Nielsen, Creighton University

Given the monomial $P^{m_1,m_2}(z_1, z_2) = z_1^{m_1} z_2^{m_2}$, we use Feynman's operational calculus to compute the function $P^{m_1,m_2}_{\eta_1,\eta_2}(A, B)$ of the not necessarily commuting operators $A, B \in \mathcal{L}(X)$, where η_1, η_2 are purely discrete, finitely supported probability measures on $[0, 1]$. Selecting sequences $\{\eta_{1,k}\}_{k=1}^{\infty}$, $\{\eta_{2,k}\}_{k=1}^{\infty}$ for which $\eta_{j,k} \rightharpoonup \eta_j$, $j = 1, 2$, as $k \to \infty$, we wish to prove that $P^{m_1,m_2}_{\eta_{1,k},\eta_{2,k}}(A, B) \to P^{m_1,m_2}_{\eta_1,\eta_2}(A, B)$ as $k \to \infty$ in norm on $\mathcal{L}(X)$. Therefore, for a function of two complex variables, $\lim_{k\to\infty} f_{\eta_{1,k},\eta_{2,k}}(A, B) = f_{\eta_1,\eta_2}(A, B)$. For our research, we are working to make a connection between discrete and continuous time-ordering measures. To do so, we select sequences $\{\eta_{1,k}\}_{k=1}^{\infty}$, $\{\eta_{2,k}\}_{k=1}^{\infty}$ for which $\eta_{j,k} \rightharpoonup \mu_j$, $j = 1, 2$, as $k \to \infty$, where μ_1 and μ_2 are continuous probability measures on $[0, 1]$. We wish to prove that $\lim_{k\to\infty} f_{\eta_{1,k},\eta_{2,k}}(A, B) = f_{\mu_1,\mu_2}(A, B)$.

279. On Applications of Generalized Functions in the Discontinuous Beam Bending Differential Equations

Austin States Virginia Military Institute
Bradley Lipscomb
Advisor(s): Dr. Dimplekumar Chalishajar, Virginia Military Institute

This paper discusses the mathematical modeling for the mechanics of solid using the distribution theory of Schwarz to the beam bending differential equations. This problem is solved by the use of generalized functions, among which the well known Dirac delta function. The governing differential equation is an Euler-Bernoulli beams with jump discontinuities on displacements and rotations. Also, the governing differential equations of a Timoshenko beam with jump discontinuities in slope, deflection, flexural stiffness, and shear stiffness are obtained in the space of generalized functions. The operator of one of the governing differential equations changes so that for both equations the Divac delta function and its first distributional derivative appear in the new force terms as we present the same in an Euler-Bernoulli beam. Examples are provided to illustrate the abstract theory. This research is useful to Mechanical Engineering, Ocean Engineering, Civil Engineering, and Aerospace Engineering.

280. An Analogue of the Median Voter Theorem for Approval Voting

Miles Stevens Morehouse College
Kyle Duke
Ethan Bush
Advisor(s): Dr. Francis Edward Su, MSRI-UP

The Median Voter Theorem is a well-known result in social choice theory for majority-rule elections. We develop an analogue in the context of approval voting. We consider voters to have preference sets that are intervals on a line, called approval sets, and the approval winner is a point on the line that is contained in the most approval sets. We define median voter by considering the left and right end points of each voter's approval sets. We consider the case where approval sets are equal length. We show that if the pairwise agreement proportion is at least 3/4, then the median voter interval will contain the approval winner. We also prove that under an alternate geometric condition, the median voter interval will contain the approval winner, and investigate variants of this result.

281. A Stochastic Ricker Model for Population Growth

Gavin Stewart Colorado State University - Fort Collins
Advisor(s): Dr. Simon Tavener, Colorado State University

Working from standard assumptions about the production of offspring and mortality of individuals, we derive a discrete stochastic Ricker population model. The model contains variables for the number of individuals of each genotype in time, and future generations are related to past generations by assumptions of Mendelian genetics. We apply this model to the problem of determining when newly introduced genotypes can persist in a population.

282. Pattern Avoidance in Reverse Double Lists

Alexander Stoll Clemson University
Monica Maus Minnesota State University Moorhead
Marika Diepenbroek University of North Dakota
Advisor(s): Dr. Lara Pudwell, Valparaiso University

Pattern avoidance is a branch of combinatorics that arose in 1968 when Donald Knuth began studying stack sorting. One central problem in pattern avoidance is finding the number of permutations of length n that avoid a specific pattern ρ. We expanded this problem to reverse double lists, or lists built by combining a permutation with its reverse. We computed the number of reverse double lists of each length that avoid patterns of up to length four and then conjectured and proved formulas to explain these sequences. In this poster, we will specifically focus on avoiding the pattern 1342.

283. Classificiation of Skew Sturmian Sequences

Sara Stover Mercer University
Jason Saied Lafeyette College
Advisor(s): Dr. Benjamin Itza-Ortiz, Universidad Autnoma del Estado de Hidalgo

Sturmian sequences are a class of bi-infinite sequences definied by Morse and Hedlund in 1940. In our research, we examined a less-studied subclass of these called skew Sturmian sequences and attempted to understand the shift spaces they generate. We classified these shift spaces up to conjugacy, an important equivalence relation in symbolic dynamics.

284. Computation in the completion of the free group algebra

Hang Lu Su McGill University
Advisor(s): Dr. Tim Hsu, San Jose State University

It is known (e.g., due to independent results of Malcev and B.H. Neumann) that $\mathbf{Q}[F_n]$, the (rational) group algebra of the free group of rank n, can be embedded in a division algebra D. We consider the problem of making this embedding algorithmic. More precisely, if D_0 is the smallest sub-division algebra of D containing $\mathbf{Q}[F_n]$, we consider the problem of representing elements of D_0 by a finite data structure with which the algebra operations (addition, multiplication, inversion) can be calculated algorithmically. We present a solution to this problem on a particular subalgebra of D_0, using noncommutative formal power series with recurrence relations, and discuss how to generalize this idea to all of D_0. We also discuss applications of our results to problems in algebraic topology.

285. Fringe pairs in generalized MSTD sets

Hong Suh Pomona College
Megumi Asada Williams College
Advisor(s): Dr. Steven J. Miller, Williams College

A *More Sums Than Differences* (MSTD) set is a set A for which $|A+A| > |A-A|$. Martin and O'Bryant proved that the proportion of MSTD sets in $\{0, 1, \ldots, n\}$ is bounded below by a positive number as n goes to infinity. Iyer, Lazarev, Miller and Zhang introduced the notion of a *generalized MSTD set*, a set A for which $|sA - dA| > |\sigma A - \delta A|$ for a prescribed $s+d = \sigma+\delta$. We offer efficient constructions of k-generational MSTD sets, sets A where $A, A+A, \ldots, kA$ are all MSTD. We also offer an alternative proof that the proportion of sets A for which $|sA-dA|-|\sigma A-\delta A| = x$ is positive, for any $x \in \mathbb{Z}$. We prove that for any $\epsilon > 0$, $\Pr(1-\epsilon < \log|sA-dA|/\log|\sigma A-\delta A| < 1+\epsilon)$ goes to 1 as the size of A goes to infinity and we give a set A which has the current highest value of $\log|A+A|/\log|A-A|$. We also study decompositions of intervals $\{0, 1, \ldots, n\}$ into MSTD sets and prove that a positive proportion of decompositions into two sets have the property that both sets are MSTD.

286. Statistical Model of Carbon Nanotubes for Nanocomp Technologies, Inc.

Tiffany Sunderland Winona State University
Advisor(s): Dr. Matthew Willyard, Penn State

Carbon nanotubes are cylindrical structures made from carbon atoms that are only billionths of a meter in diameter. Though they have only been widely studied since 1991, carbon nanotubes have already been incorporated into a variety of industries for the improved strength, conductivity, and resilience they offer over other materials. At a submicroscopic level, these nanotubes can be classified into three distinct types which are differentiated by their amounts of curvature and bundling. However, it is unknown what specific qualities of the carbon nanotubes are related to a given type. Our goal is to establish relationships visual characteristics and their resulting varieties of nanotubes. Using scanning electron microscope and transmission electron microscope images provided by our sponsor, Nanocomp Technologies, Inc., we used supervised learning techniques such as random forests and logistic regression to automate the classification of the images into the three categories.

287. Modeling the Interactions Between Acacia Trees and Rothschild Giraffes

Kristina Sundy University of Central Oklahoma
Advisor(s): Brittany Bannish, University of Central Oklahoma

Rothschild giraffes are on the verge of extinction, so many are kept in conservatory enclosures. While in enclosures, giraffes face threats and hardship, including their food supply. This project provides a solution to this problem. We create mathematical models of the relationship between acacia trees and the Rothschild giraffe. We analyze the biology of each of these species and how they affect each other in a conservation enclosure. While in enclosures, giraffes have a limited quantity of acacia trees available, resulting in over-browsing and debarking of acacia trees. Acacia trees adapted to the amount of browsing done by giraffes by increasing tannin production to keep the giraffes from over-browsing. We develop a predator-prey model to study the interaction between giraffes and acacia trees as well as a model of how the tannin levels of the acacia tree affect the health of the giraffe. We use these models to help conservationists with the health and wellness of the Rothschild giraffe species.

288. Optimizations and Findings Regarding Constant-Communication Oblivious RAM

Krishna Suraj Lexington High School
Akiva Gordon Weston High School
Advisor(s): Mr. Ling Ren, Massachusetts Institute of Technology

In today's world, ever-increasing amounts of data are being stored online in external, untrusted servers — the "cloud". Considering the importance of such data to users, businesses, and organizations, it is necessary to protect data from mishandling and exploitation on those servers. While current encryption schemes can protect the actual data on a server from being read, a user's access pattern to files on a server is vulnerable. These metadata can be analyzed by malicious servers, and potentially used to gain information on the actual data. Oblivious RAM (ORAM) is a cryptographic primitive that hides a user's access pattern metadata from untrusted servers. Our goal in this paper is to optimize the latest tree-based ORAM algorithm, Constant Communication ORAM (C-ORAM) using a simulation of the eviction process. We provide an experimental analysis of C-ORAM using our implementation written in Python 2.7. Using data from our experimental analysis, we determine runtime as well as optimal bucket size and eviction frequency based on multiple tests. We also provide an intuitive security explanation as well as an optimization for the oblivious merging operation in C-ORAM. Our results enable us to optimize the C-ORAM algorithm.

289. Rational combinations of Betti diagrams of complete intersections

Alexander Sutherland Oberlin College
Michael Annunziata Wake Forest University
Cole Hawkins Amherst College
Advisor(s): Dr. Courtney R. Gibbons, Hamilton College

One interesting numerical invariant of a module M is the Betti diagram of M. We consider the cone generated by Betti diagrams of complete intersection modules over a polynomial ring. Betti diagrams of complete intersections are well understood, and so a decomposition of a Betti diagram β into such diagrams will quickly yield numerical information about β. Our main theorem is as follows:
Theorem *Each ray spanning the Betti diagram of a complete intersection is extremal in the subcone* $B_{\mathbb{Q}}^{c.i.}(R)$.
Using this result, we provide a rudimentary algorithm to determine membership in the subcone, as well as a promising direction for future work.

290. Dispersal-Induced Global Extinction

Margaret Swift The College of William & Mary
Advisor(s): Dr. Leah Shaw, The College of William & mary

Centuries of overfishing and the gradual destruction of habitat have led to the rapid devastation of the Chesapeake Bay's oyster supply. We study asymmetric dispersal between two coupled patches (oyster reefs) under the Allee effect. This effect is displayed by many species, and is one under which initial populations below a certain threshold decline, while those above the threshold can persist. We extend a previous ordinary differential equation model with symmetric dispersal rates between patches (Kang & Lanchier, *Bull. Math. Biol.* 2011), and explore the steady state bifurcation structure while varying the dispersal rates and Allee threshold. We also show analytically that there are no periodic orbits. At high Allee thresholds, we find large parameter ranges in which the extinction state is the only fixed point.

Previous symmetric models did not uncover this behavior, and it raises concerns for environmental restoration of other species that may exhibit the Allee effect and asymmetric dispersal, such as in estuarine and marine systems.

291. Rainbow Coloring Graphs

Jackson Swindells La Salle University
Advisor(s): Dr. Janet Fierson, La Salle University

Consider a graph G and a number of colors k. If k is sufficiently large, it is possible to color the edges of G in such a way that, for every pair of vertices, a path exists consisting of edges with distinct colors. Any such coloring is considered a rainbow coloring. We can then create a new graph, the coloring graph of G, by creating a single vertex for every distinct rainbow coloring of G. We add an edge connecting each pair of vertices whose corresponding rainbow colorings differ by only one edge. By examining particular graphs and their coloring graphs, it is possible to find rules that can be proven to hold true for entire families of graphs. The methods of finding and proving these rules, as well as the rules themselves, will be discussed.

292. Intercardinal Adjacencies: A New Landscape Metric

Nicholas Taliceo Westfield State University
Advisor(s): Dr. Julian Fleron, Westfield State University

Geographic Information Systems (GIS) have become crucial to daily life. They are particularly useful in scientific areas, including landscape ecology where they can measure the density of land relative to water using *landscape indices*. Current landscape indices like FRAGSTATS' *aggregation index* only measure cardinal adjacencies - edge to edge adjacencies in the N/S/E/W directions. Essential to this method is knowledge of the optimal configurations, a result established by Haray and Harboth in "Extremal Animals" (1976). GIS experts have asked us to help them develop a method of measuring adjacencies which include diagonal, or corner to corner, adjacencies as well. We call these *intercardinal adjacencies*. An essential component of this new index is the determination of optimal configurations - which polyominoes of each size have the greatest number of intercardinal adjacencies. We have discovered a family of *archetypes* for the optimal configurations and have found a closed-term expression for the number of intercardinal adjacencies. We are working now to prove that these families are indeed optimal. Once completed we will work with GIS experts on integrating this new method, called the *intercardinal metric*, into GIS systems.

293. A Multiscale Approach to the Boltzmann-BGK Equation

Emma Talis Marist College
Jae-Jae Young Iowa State University
Alan Medinger Lewis & Clark College
Advisor(s): Dr. James Rossmanith, Iowa State University

We consider the Boltzmann equation with the Bhatnagar-Gross-Krook (BGK) collision operator, a kinetic model for the dynamics of particles of a gas. Solving this system is difficult because it is a high-dimensional PDE. The dimension of this equation can be decreased by considering a fluid approximation, but this model assumes thermodynamic equilibrium. We aim to develop an asymptotic-preserving (AP), multiscale method to solve the Boltzmann-BGK equation by combining the specificity and physical accuracy of a kinetic solver with the efficiency and numerical accuracy of a fluid solver. Specially, we develop a kinetic solver based on a semi-Lagrangian discontinuous Galerkin method and a uid solver based on a high-order Runge-Kutta discontinuous Galerkin method. We present our preliminary multiscale solver.

294. Dramatically composite numbers and generalizations

Sarah Tammen The University of Georgia
Advisor(s): Dr. Paul Pollack, The University of Georgia

We study natural numbers that are nearly prime in the sense that if one tests primality by trial division, then they are shown to be composite late in the process. We define a natural number n to be *dramatically composite* if $\mathrm{lpf}(n)$ (i.e., the least prime factor of n) is the largest prime less than or equal to $\sqrt{n}$; such a number is deemed composite only in the last trial division. More generally, given a natural number $k \geq 2$, we define n to be *dramatically k-composite* if $\mathrm{lpf}(n)$ is the largest prime not exceeding $\sqrt[k]{n}$. Using Landau's estimates for counting functions of k-semiprimes,

we show that the reciprocal sum of the dramatically k-composite numbers converges for any k. Finally, one can fix $\alpha \in (0,1)$ and consider n for which $\alpha \sqrt[k]{n} \leq \mathrm{lpf}(n) \leq \sqrt[k]{n}$; this gives a variable lower bound for $\mathrm{lpf}(n)$ that grows with n. We show that the numbers n satisfying this property also have a convergent reciprocal sum, and we present work towards an asymptotic estimate for their counting function.

295. Global solutions to the generalized Magnetohydrodynamic system of equations

Alex Tarter Creighton University
Advisor(s): Dr. Nathan Pennington, Creighton University

The Magnetohydrodynamic (MHD) system of equations governs the magnetic and velocity fields in electrically conducting fluids (like plasmas and liquid metals) and is given by

$$\begin{aligned} &\partial_t u(u\cdot\nabla)u - \nu\triangle u = -\nabla p + (b\cdot\nabla)b,\\ &\partial_t b + (u\cdot\nabla)b - \eta\triangle b = (b\cdot\nabla)u,\\ &\div u = \div b = 0,\\ &u(0,x) = u_0(x),\ \ b(0,x) = b_0(x), x\in\mathbb{R}^n,\\ &u, b : \mathbb{R}\times\mathbb{R}^n \to \mathbb{R}^n, \end{aligned} \tag{2}$$

where u is the velocity field and b is the magnetic field.
The generalized Magnetohydrodynamic (gMHD) system, which allows for more complicated interactions between particles, follows from the MHD by replacing the Laplacians in the MHD equation with more general Fourier multipliers. In this project, we extend a previously constructed low-regularity unique local solution to the gMHD system to a global solution.

296. A Bijective Proof of a Combinatorial Problem for the Symmetric Group

Jamel Thomas California State University, Monterey Bay
Advisor(s): Dr. Lipika Deka, California State University, Monterey Bay

In this project we study an important research problem proposed by Professor Richard Stanley, who is considered the father of Algebraic Combinatorics. The problem involves special types of permutations in Sn known as n-cycles, where n is a positive integer. An n-cycle is a permutation of length n when permutations are written using cycle notation. The problem states that for n, a positive odd integer, and two randomly picked n-cycles, σ, ν in S_n, the probability that the number 1 and 2 appear inside the same cycle of the product $\sigma\nu$ is $1/2$. This result is known to be true, but there is no combinatorial proof. A combinatorial argument, or combinatorial proof, is an argument that involves counting a set with particular properties using more than one methods. In our project We are trying to find a combinatorial proof of this open problem. First We verify the result numerically using a C++ program we created and then give a combinatorial proof in special values of n. This problem is an important result in understanding structure of the symmetric group S_n, which is one of the most important group in Algebraic Combinatorics. We hope to generalize the proof to any positive integer n.

297. Study of Mean Reaction Time of Bimolecular Reactions in Two Dimensional RDME Systems

Erin Tierney Palm Beach Atlantic University
Advisor(s): Dr. Yang Cao, Virginia Tech University

Biochemical species that diffuse in cells and react together are known as reaction-diffusion systems. As seen in Alan Turing's original work, certain biological systems remain in a state of equilibrium; however, these may become unstable in the presence of diffusion. This sensation is known as the Turing Pattern. Stochastic modeling of reaction-diffusion kinetics has produced various models, including the reaction-diffusion master equation (RDME). Recent developments related to the RDME have shown that when voxel size decreases infinitely in two or three dimensional domains, some bimolecular reactions are eventually lost. Results from a study conducted by Stefen Hellander, a post-doctoral student at the University of California, suggest a new formula to determine the mean time of a bimolecular reaction between two particles when firing for the first time. The purpose of this research is to test Hellander's formula for accuracy. We found that Hellander's formula does not match with the numerical experiment and then proposed an alternative formula to better match the mean of the biomolecular reaction's firing time in the RDME System. Through numerical experiments, we were able to show that our formula matches with the experimental data far better than Hellander's.

298. Sociodemographic factors influencing household energy efficiency in the United States

ML Tlachac University of Wisconsin- Eau Claire
Advisor(s): Dr. Ian Lange, Colorado School of Mines

Knowing which households are more energy efficient is increasingly important as the effects of local pollution and global warning are becoming more pronounced, since this allows for better targeting of marketing and policies. Using the Residential Energy Consumption survey (RECs) 2009 and the American Housing Survey (AHS) 2011, multiple variables were isolated to determine household energy efficiency: having Energy Star appliances, buying Energy Star appliances, saving heat and air conditioning during the night, installing energy efficient lightbulbs, and performing energy efficient jobs on homes. An analysis of Energy Star appliance availability was also conducted. OLS regressions were run on the energy efficient variables using the sociodemographic factors available in each study, and some patterns emerged. Householders who live with a partner/spouse, have achieved higher levels of education, own their home, and have a higher household income are likely to be more energy efficient. Less likely are those who rent their homes, and live in East North Central and West North Central census divisions. The impact of race is much more varied than previous studies suggest, though those who are not born as US citizens are less likely to be energy efficient.

299. Statistical Analysis of Crime in Eau Claire, WI

ML Tlachac University of Wisconsin- Eau Claire
Matthew Tlachac
Advisor(s): Dr. Abra Brisbin, University of Wisconsin- Eau Claire

When moving off campus or to a new city, safety from crime is a primary concern. Crime data from Eau Claire, WI was collected from crimemapping.com for critical times within a semester, and we identified the latitude and longitude of each crime. This was done to facilitate an analysis of the spatial distribution of crimes within the city, which would allow for a safer choice of residence. Based on Ripley's K-function, the crimes exhibit evidence of clustering such that a homogeneous Poisson distribution does not fit. We will present results identifying regions and times of higher crime density.

300. The Banquet Seating Problem

Alexis Jane Torre University of Arizona
Michelle Rosado Perez University of Puerto Rico - Mayaguez
Ashley Scruse Clark Atlanta University
Advisor(s): Dr. Francis Su, MSRI-UP

Suppose you want to seat $n = mk$ people around k tables with m people at each table. Each person gives you a list of j people next to whom they would enjoy sitting. What is the smallest j for which you can always make a seating arrangement that would seat each person next to one of the people on their list? In this paper we show that j must be strictly more than half of n,the total number of people. Our key tool is a particular 'blue-green-red' lemma that helps us construct 'worst-case scenario' seating arrangements. We consider cases with two tables and more than two tables.

301. Identifying the Core Curriculum for Undergraduate Students

Jessica Tran University of California, Los Angeles
Ritvik Kharkar University of California, Los Angeles
Advisor(s): M. Puck Rombach, Charlie Z. Marshak, University of California, Los Angeles

In this work, we develop statistical tools to identify core courses at the university level. Traditionally, professors and administrators label courses as "core" when the courses contain foundational material. Such courses are often required to complete a major, and, in some cases, allocated additional educational resources. Here, we take a data-centric approach characterizing core courses as those having greatest impact on student's overall major GPA. We propose three statistical tests for identifying these impactful courses. The first of these tests fashions a metric out of standard correlation measures. The other two utilize regression. We apply these methods on student data coming from the UCLA department of mathematics to extract core math courses and then compare these courses to the prescribed UCLA coursework.

302. Frequency domain analysis of the diblock copolymer equation

James Trichilo George Mason University
Advisor(s): Dr. Tom Wanner, Evelyn Sander, George Mason University

Since the launch of the Materials Genome Initiative, an emphasis has been placed on the discovery, understanding, and manufacture of advanced materials. Diblock copolymers are an important material in the 21st century, and are used in adhesives, asphalt additives, and chemical reactions. We analyze the diblock copolymer equation, a 4th order parabolic PDE, and numerically simulate it through a Galerkin spectral method. It is observed that in many instances, there is a direct relationship between the largest Fourier coefficient and the number of transition layers. Through parallel computing, a large simulation was implemented in which thousands of different parameter combinations were accounted for, revealing fascinating changes in the bifurcation diagram of the equation are revealed.

303. Boggle Logic Puzzles With Repeated Letters

Kirsten Trotta Le Moyne College
Advisor(s): Dr. Jonathan Needleman, Le Moybe College

Boggle Logic Puzzles involve playing the popular Hasbro game Boggle in reverse. In a Boggle Logic Puzzle a person is given a list of words and must recreate the unique Boggle board from this list. This problem can be represented as a graph theory problem by viewing each letter in the board as a node and each consecutive pair of letters in each word as an edge. Then the problem becomes whether or not the $n \times n$ sized graph created by a list of words is a subgraph of the $n \times n$ King's graph. Research has already been done on this problem where all letters are unique. My research builds upon this to look at cases where there are repeated letters by studying which boggle boards with repeated letters are suitable for creating Boggle Logic Puzzles. That is, I determine which boards with repeated letters can be recreated from a list of words.

304. In-111 Uptake in SPECT Images of Murine Tumors with Varying Antigen Expression: A Topological Approach

Sarah Tymochko College of the Holy Cross
Brian Toner College of the Holy Cross
Advisor(s): Dr. David Damiano, College of the Holy Cross

Using computational topology and methods previously developed by Damiano and McGuirl, we analyzed the antibody uptake of single-photon emission computed tomography (SPECT) images of murine tumors. Study data consisted of images of three groups of mice, with each injected with a different xenograft cancer cell line. The cell lines were distinguished by different levels of antigen expression. Images utilized an indium-111 labeled targeting antibody. Using Morse Theory and dimension zero persistence homology, we analyzed the points of maximum uptake in an attempt to distinguish between uptake behaviors of the different groups and find heterogeneity of uptake within the groups. Statistical analyses indicate that through focusing on areas of high and low uptake, we could efficiently distinguish uptake behavior of the antibody in the three different tumor cell lines.

305. Topological Analysis of Biological Aggregations

M Ulmer Macalester College
Advisor(s): Dr. Lori Ziegelmeier, Macalester College

We suggest new ways to analyze models of biological aggregations using tools from topological data analysis. Our data consists of numerical simulation output from the classic Vicsek model of alignment and two models to approximate the motion collected from experimental data of pea aphids. Using this position or position-velocity data, we analyze certain topological invariants, such as the number of structures connected by points within some proximity parameter, for each snapshot in simulation time. We display the number of topological features as a contour plot of both proximity parameter and simulation time. Since these invariants give a sense of the 'structure' of a dataset, comparing contour plots offers a qualitative view of the ways in which models differ from one another or from experimental data. We apply this method to two case studies. In the first, we examine two proposed models of aphid motion and showed that the one in which motion was determined by distance to nearest neighbor better approximated experimental data than did a random-motion model. In the second, we ran simulations of the Vicsek model under different parameters, and then showed that corresponding contour plots exhibited strong clustering within simulation runs of similar parameter regimes.

306. Poster withdrawn.

307. Shrinking Core Model of a Lithium-Ion Battery

Erich VanBergen George Fox University
Advisor(s): Dr. Corban Harwood, George Fox University

As technology develops, more powerful batteries with longer life-spans are needed to support the energy-load. We developed a mathematical model system to predict the optimal formulation of non-conductive porosity additives, active material powder, and conductive additives that would yield the best improvement in power and capacity performance in a lithium-ion battery. We programmed the governing partial differential equation and linear systems of our model system in Matlab using parameter values common to the research literature to simulate the concentration of Li+ ions, nodal current, and voltage in a lithium-ion battery. The Crank-Nicolson numerical method was chosen to ensure stability and special care was taken to ensure oscillation-free solutions. We compared simulated voltage discharge curves to the literature to validate our model. Our future work is to develop our own set of data using a different chemical composition to provide a safer and more effective battery.

308. A reinterpretation of Macaulay's multivariate resultant and resolvent from a modern commutative algebraic perspective

John Vastola University of Central Florida
Advisor(s): Dr. Joseph Brennan, University of Central Florida

The resultant, an elimination theoretic tool that determines (given some reasonable assumptions) if a system of polynomial equations has a solution, has recently been demonstrated to be a viable alternative to Grobner basis methods in solving such systems; in particular, as Sturmfels and others have observed, a resultant approach is sometimes more efficient, and may succeed where Grobner basis methods fail. In light of this, we have reexamined Macaulay's work on the resultant and resolvent, and have reinterpreted his results in terms of modern commutative algebraic ideas. Additionally, we have filled in some important gaps in his proofs, reproved other results more succinctly using modern methods, and proved that constructions like Macaulay's multivariate resultant are well-defined. Finally, we offer some novel results related to the multivariate resultant, and discuss issues of algorithmic efficiency in its application to solving systems of polynomial equations.

309. Comparison of Classification Methods for Open Chain Knots

Emily Vecchia University of Saint Thomas
Erin Brine-Doyle University of Saint Thomas
Advisor(s): Dr. Eric Rawdon, University of Saint Thomas

The recent discovery of open chain knotting in physical systems such as polymers, proteins, and DNA has motivated studies aimed at classifying knotting in open chains. Knotting is trapped in closed knots making it possible to study the knotting in closed curves mathematically. By using different schemes to close the endpoints of an open chain knot, one can define the knotting of open chains in a number of different ways. The purpose of this project is to analyze and compare five different methods used to classify knots in open chains. These methods are known as the Infinite Method, the Random Equilateral Arc Method, the Straight Line Method, the Center of Mass Method, and the Convex Hull Method. The knotting in open chains is classified with these five methods and the results are compared and analyzed statistically.

310. Signatures of the Contravariant Form on Representations of the Hecke Algebra and Rational Cherednik Algebra associated to $G(r, 1, n)$

Girishvar Venkat University of Texas Online High School
Saarik Kalia Massachusetts Institute of Technology
Advisor(s): Mr. Siddharth Venkatesh, Massachusetts Institute of Technology

The Hecke algebra and rational Cherednik algebra of the group $G(r, 1, n)$ are non-commutative algebras that are deformations of certain classical algebras associated to the group. These algebras have numerous applications in representation theory, number theory, algebraic geometry and integrable systems in quantum physics. If the deformation parameters is generic, these irreducible representations, called Specht modules for the Hecke algebra and Verma modules for the Cherednik algebra, are in bijection with the irreducible representations of $G(r, 1, n)$. However, while every

irreducible representation of $G(r, 1, n)$ is unitary, the Hermitian contravariant form on the Specht modules and Verma modules may only be non-degenerate. Thus, the signature of this form provides a great deal of information about the representations of the algebras that cannot be seen by looking at the group representations.
We compute the signature of arbitrary Specht modules of the Hecke algebra and the signature character of the polynomial representation of the Cherednik algebra for generic parameter values. Additionally, we use the signatures of the Specht modules to give explicit formulas of the parameter values for which these modules are unitary.

311. Average Intensity of the Distribution of Complex Zeros of Random Polynomials

Richard Vradenburgh Tennessee Tech University
Advisor(s): Dr. Andrew Ledoan, University of Tennessee at Chattanooga

The presenter will demonstrate how we establish a formula for the average intensity of the distribution of complex zeros of the random polynomial $\eta_0\omega_0 + \eta_1\omega_1 z + \eta_2\omega_2 z^2 + \cdots + \eta_{n-1}\omega_{n-1}z^{n-1}$, with $z \in \mathbb{C}$, for any sequence of real constants ω_j and standard normal independent random variables η_j. Furthermore, the presenter will show how we obtain the limiting behavior of this intensity function as n tends to infinity for the following sequences ω_j:

$$\omega_j = \sqrt{\frac{\binom{n-1}{j}}{j+1}}, \qquad \omega_j = \sqrt{j}, \qquad \text{and} \qquad \omega_j = \sqrt{\frac{1}{j!}}.$$

This research was conducted as part of a 2015 REU program at the University of Tennessee at Chattanooga under the supervision of Dr. Andrew Ledoan. Funding for this research was provided by an REU grant from the National Science Foundation.

312. Geometric Realization of Sparse Neural Codes

Aleina Wachtel Harvey Mudd College
Robert Jeffs Harvey Mudd College
Advisor(s): Dr. Mohamed Omar, Harvey Mudd College

Understanding how a neural code stores information is one of the most pressing problems in neuroscience. It is vital to study these neural codes from a mathematical perspective to gain a better understanding of the brain. The combinatorial information in a neural code can allow us to determine if the code reflects the firing behavior of neurons with convex receptive fields. Thus, we seek to determine which neural codes can be realized as convex open sets in $\mathbb{R}^2$. We restrict to codes showing sparse behavior, specifically codes in which no more than 2 neurons fire simultaneously, to reduce the intractability of determining which neural codes can be so realized. Using geometric properties of neural realizations, we find necessary and sufficient conditions for realizability of 2-sparse codes based on the associated co-firing graph. This yields a complete characterization of which 2-sparse codes can be realized in $\mathbb{R}^3$. We also exhibit several classes of codes that are always realizable in $\mathbb{R}^2$, and give a class of graphs never realizable in $\mathbb{R}^2$. With a characterization of 2-sparse neural codes, we can better comprehend the manner in which biologically viable codes are spatially represented in the brain.

313. Poncelet Curves and Finite Blaschke Products

Nathan Wagner Bucknell University
Advisor(s): Dr. Pamela Gorkin, Bucknell University

Poncelet's closure theorem is a well-known geometric result concerning polygons that are inscribed in an ellipse and circumscribe a smaller ellipse contained in the larger one. Ellipses that can be so circumscribed are called Poncelet ellipses. On the other hand, Blaschke products are important analytic functions of a complex variable, because they are precisely the functions that are analytic on an open set containing the closed unit disk, map the unit disk to itself and the unit circle to itself. It turns out there is a deep connection between these functions and Poncelet ellipses. By examining properties of matrices representing the compression of the shift operator on the model space corresponding to a particular Blaschke product, we connect the compression of the shift, numerical range, Blaschke products, and Poncelet ellipses. This allows us to provide a new proof of a recent result due to Fujimura concerning necessary and sufficient conditions for a degree-4 Blaschke product to have an ellipse as its associated Poncelet curve.

314. Huge WENO Simulations of the Fermi Bubbles Emitted by Our Galaxy

Benji Wagner Arizona State University
Advisor(s): Dr. Carl Gardner, Arizona State University

In 2010, two gamma-ray /x-ray bubbles were detected in the center of the Milky Way Galaxy. These bubbles extend symmetrically $\approx 30,000$ lightyears above and below the Galactic Center, with a width of $\approx 27,000$ lightyears. These bubbles emit gamma-rays at energies between $1 \lesssim E_\gamma \lesssim 100$ gigaelectronvolts, have approximately uniform surface brightness, and are expanding at $\approx 30,000 \frac{km}{s}$. We believe that these Fermi Bubbles are the result of an astrophysical jet pulse that occurred millions of years ago. We present realistic simulations of the equations of gas dynamics with radiative cooling using the WENO method of the Fermi Bubbles to better understand the mechanism that powers the bubbles.

315. Knotting Changes in Glueball Decay

John Wallace University of St. Thomas
Advisor(s): Dr. Eric Rawdon, University of St. Thomas

Historically, knot theory has been applied to many of the sciences. Knots were the focus of some of the early theories of matter and recently have been applied to the study of proteins and DNA. Previously, we have explored knots as a model for glueballs - hypothetical subatomic particles that are predicted to exist by the Standard Model. Research suggests that glueballs take on a tight knotted structure. Due to the quantum realm that glueballs live in, they are subject to the stochastic nature of quantum mechanics and therefore may decay through a procession of quantum events. This project explores how a tight knot, or glueball, evolves when a change resulting from a quantum event is introduced into its structure. We are looking at two types of quantum events: quantum tunneling, where a crossing is flipped, and quantum reconnection, where a knot is split to form a link. We begin by analyzing critical locations of the knot where these quantum events may occur, and then we execute each quantum event at every location to create a 'fingerprint' that describes the behavior of this tight knot when it undergoes a quantum event. As a result of our work, we now have a collection of these fingerprints for several hundred tight prime and composite knots, which describe how glueballs decay.

316. Clustering with Prototypes for Specialized Knowledge Groups in a Tech Economy

Sirui Wang Cornell University
Advisor(s): Dr. Victor Nee, Jacob Bien, Cornell University

The technology sector of New York City has rapidly grown in the past decade, drawing upon well-established firms in the area, as well as new talent from across the world. We study the structure of this emerging community by analyzing social media data from a large online group organized around tech-based startups. We apply hierarchical clustering using Ward's minimum variance criterion to partition the group members' interests or involvements. Each cluster is then identified with its prototype defined using the minimax radius of each partition of the data. Inspection of the prototypes and feature composition of the partitions suggests distinctive sub-communities within the larger economy. We find that these sub-communities are frequently organized around recreational and lifestyle interests in addition to technical skills and business-professional networking.

317. Equal Compositions of Rational Functions

Felix Wang The Roxbury Latin School
Matthew Lipman Boston University Academy
Advisor(s): Michael Zieve, Thao Do, University of Michigan, MIT

We study the following questions:

1. What are all solutions to $f \circ \hat{f} = g \circ \hat{g}$ in complex rational functions $f, g \in \mathbb{C}(X)$ and meromorphic functions $\hat{f}, \hat{g}$ on the complex plane?
2. For which rational functions $f(X)$ and $g(X)$ with coefficients in an algebraic number field K does the equation $f(a) = g(b)$ have infinitely many solutions with $a, b \in K$?

We resolve both questions in the case that the numerator of $f(X) - g(Y)$ is an irreducible polynomial in $\mathbb{C}[X, Y]$ of sufficiently large degree. Our result comprises a vast generalization of previous works by Ritt, Avanzi-Zannier, Bilu-Tichy and others, which are restricted to the case that $f(X)$ and $g(X)$ are polynomials (rather than rational functions) which moreover satisfy $\gcd(\deg f, \deg g) \leq 2$.

318. Three-player Hackenbush Game

Kewen Wang Penn State Erie
Advisor(s): Dr. Joseph Previte, Penn State Erie

Hackenbush is a two-player combinatorial mathematical game where each player removes pieces until a player has no more possible moves, and this player is defined to be the loser of the game. The game is completely understood, in that a numerical value can be placed on any instance of the game which determines the winner or loser of the game (assuming optimal play). In this research project, the researcher will analyze a three-player Hackenbush game. This game has a much richer theory than the two-player game. Several instances of the game will be shown to illustrate these differences. In particular, we will show that the additive structure of the two-player game does not hold for the three-player game. Also, several conditions which guarantee a particular winner will be stated and proven.

319. A Combinatorial Proof for a Fibonomial Coefficient Description

Jordan Weaver
Advisor(s): Dr. Kendra Killpatrick, Pepperdine University

The fibonomial coefficients $\binom{n}{k}_{\mathcal{F}}$, defined by $\binom{n}{k}_{\mathcal{F}} = \frac{F_{2n}!}{F_n!F_n!}$ where F_n is the well-known Fibonacci number, are an analogue of the well-known binomial coefficients. Similar to the binomial coefficients, they satisfy a recurrence given by $\binom{n}{k}_{\mathcal{F}} = F_{k-1}\binom{n-1}{k}_{\mathcal{F}} + F_{n-k+1}\binom{n-1}{k-1}_{\mathcal{F}}$. In 2010, Sagan and Savage gave a combinatorial interpretation of the fibonomial coefficients in terms of certain Fibonacci tilings of a k by $n-k$ rectangle. Their proof for this interpretation involved showing the set of tilings satisfied the fibonomial recurrence relation. In my talk, I will give a combinatorial proof for the Sagan and Savage interpretation of the fibonomial coefficients.

320. Graph Reduction Methods for Multiscale Dictionary Design

Elle Weeks Macalester College
Andre Archer Macalester College
Stefan Faridani Macalester College
Yan Jin Macalester College
Advisor(s): Dr. David Shuman, Andrew Beveridge, Macalester College

Some data, such as the GDP of a country over time, exist as signals on the real number line. Other data, such as the number of photos each individual posts on a social network, reside on more complex domains that can be represented with weighted graphs. In this project, we design techniques to compress, denoise, and process signals on graphs. To accomplish this, we adapt classical signal processing methods to the graph domain. We explore graph reduction methods that preserve spectral properties of a graph and incorporate these methods into the filter bank framework to generate multiscale transforms. Specifically, we describe a graph reduction method that optimizes the correspondence of the graph Laplacian eigenvectors at different resolution levels.

321. Stochastic Epidemic Models with Computer Simulations

Valerie Welty Georgia Southern University
Advisor(s): Dr. Patricia Humphrey, Georgia Southern University

Discrete stochastic models are a useful component of understanding and studying epidemics of infectious diseases. However, there exists a trade-off between capturing the true nature of the epidemic and model simplicity. Models similar to the standard SEIR could more accurately describe an epidemic of a disease such as Measles, where infected individuals are infectious to others before they present advanced symptoms of the disease, and the population has varying levels of immunity due to vaccination. We examine new possible models for measles that may be applied to other diseases with similar progressions and vaccination patterns. An applet was created in JavaScript to produce realizations of a stochastic model of a measles epidemic resulting from possible parameter values, given a wide variety of assumptions. This applet may be more realistic and thus accurate than the other epidemic applets currently available, due to the model used and the stochastic processes involved. Numerical results obtained from simulations of such an applet may be used as theoretical data to study the impact of vaccination on herd immunity and disease transmission.

322. Lazy Cops and Robbers on Chess and Product Graphs

Mikayla Werzanski Emmanuel College
Nikolas Townsend Emmanuel College
Advisor(s): Dr. Brendan W. Sullivan, Emmanuel College

The pursuit-evasion game of Cops and Robbers on graphs has been studied extensively since its introduction in the 1980s. A recently-proposed variant, "Lazy Cops and Robbers", allows just one cop to move per round. We studied several classes of graphs, looking for possible differences between the Ordinary and Lazy versions; in both scenarios we seek the *cop number* (denoted c or c_L, respectively), the minimum number of cops needed to guarantee victory. We have proven sharp upper bounds for c_L of graphs obtained by Cartesian and strong products, and we have established algorithmic results for specific graphs, e.g., $c_L(C_n \square P_m) = 2$. We also considered chess graphs and found a relationship between c_L and their domination numbers. Furthermore, we have characterized the cop number of Queens graphs, $c(Q_n)$, for all but finitely many n. Finally, we propose conjectures and open problems related to product graphs and domination numbers.

323. Parameter Estimation in Continuous Time Markov Chain Models using Extended Kalman Filtering

Rebekah White East Tennessee State University
Ella Burnham St. Catherine University
Andrew Pensoneault SUNY Geneseo

Advisor(s): Dr. Michele Joyner, East Tennessee State University

The Extended Kalman Filter (EKF) was applied to estimate parameters for a Continuous Time Markov Chain (CTMC) model. The EKF allows one to approximate nonlinear systems locally as linear systems and then use both the data and model to potentially gain a more accurate prediction for the system than with the data or model alone. This method also takes into account the covariance in the data measurements, as well as the covariance in the model output. By treating the parameters of the stochastic model as time dependent variables, we can use EKF as a parameter estimation technique. We implemented this technique on a SIS-epidemic model with the parameters being the rate of transmission of a disease and recovery rate. Given data from a CTMC model, we compared applying EKF for parameter estimation to an alternative approach involving minimizing an ordinary least squares cost function using a deterministic approximation of the CTMC model. We found that although the EKF approach to parameter estimation provided more accurate parameter estimates for this model than the alternative approach, our methodology was still not accurate enough to be used for parameter estimation in CTMC models.

324. Breaking CAPTCHAs Using Wavelets

Kathryn Wifvat University of St. Thomas
Advisor(s): Dr. Patrick Van Fleet, University of St. Thomas

CAPTCHA is an acronym for "Completely Automated Public Turing test to tell Computers and Humans Apart". It is a type of challenge-response test in which distorted letters and numbers must be accurately identified and is helpful in determining whether a user on a website is human or not. Many websites have this in place, because they do not want their system to be broken into by a robot. Grand Valley State University students Julia Olsen and Jesse Windle and their advisor Edward Aboufadel in their paper "Breaking the Holiday Inn Priority Club CAPTCHA" (The College Mathematics Journal, Vol. 36, No. 2, March 2005) figured out how to break the CAPTCHA codes used by the Holiday Inn Priority Club. Their algorithm made use of the Haar wavelet. Our goal is to build upon their work using wavelets, geometry, and statistics to break CAPTCHAs that are slightly more sophisticated than the ones used by the Holiday Inn. In this paper, we will discuss our algorithm and give examples to show that it can accurately break certain CAPTCHAs with a high level of success.

325. New Results for Lambert series, q-Digamma function, q-Pochhammer symbol, and Jacobi theta functions

Blake Wilkerson Rhodes College
Advisor(s): Dr. Shubho Banerjee, Rhodes College

We study Lambert series, $L(q)$, with coefficients of the type $a_n = n^s$ for all $s \in \mathbb{C}$ near $q = 1$. The singularity present at $q = 1$ makes the analysis of these series difficult in this limit. As $q \to 1$, the number of terms of $L(q)$

required to achieve a given accuracy increases dramatically. We obtain the complete asymptotic expansion of $L(q)$ near $q = 1$, which displays very rapid convergence, with only two terms providing accuracy of 1 part in 10 million for $0.1 < q < 1$. Our analysis of the Lambert series yields the asymptotic forms for several related functions: the q-gamma and q-polygamma functions, the q-Pochhammer symbol, and the Jacobi theta functions. Some typical results include $\Gamma_e(\frac{1}{2}) \simeq \sqrt{\pi(e-1)}\, e^{-\frac{1}{16}}$, $(\frac{1}{\sqrt{e}}, \frac{1}{e})_\infty \simeq \sqrt{2}\, e^{-\frac{\pi^2}{6}-\frac{1}{48}}$, and $\vartheta_3(\frac{\pi}{2}, e^{-\frac{1}{\pi}}) \simeq 2\pi e^{-\frac{\pi^3}{4}}$.

326. On Canonical Algebraic Curvature Tensors via Chain Complexes

David Williams Tennnessee Tech University

Advisor(s): Dr. Corey Dunn, California State University at San Bernardino

The presenter lays the framework for the investigation of canonical algebraic curvature tensors via chain complexes. Two examples of complexes with associated linear dependence equations are illustrated, one from previous work and one newly developed. The presenter demonstrates the utility of precomposition in reducing the number of terms in a linear dependence equation. Finally, the presenter develops a new result which relates the structure of a chain complex to the commutativity of the underlying operators. This research was completed through the 2015 California State University, San Bernardino Math REU, with support from the National Science Foundation, Grant DMS-1461286.

327. Polytopes of the Kimura 3-parameter Phylogenetic Model

Hayley Williams Macalester College

Advisor(s): Dr. Joseph Rusinko, Winthrop University

Group-based phylogenetic models describe evolutionary relationships among taxa on a phylogenetic tree. A group action on the set of edges of the tree gives rise to a toric variety, and properties of the variety are encoded in the combinatorial structure of an associated convex polytope. For the Kimura 3-parameter phylogenetic model ($G = \mathbb{Z}_2 \times \mathbb{Z}_2$) on an m-claw tree, a complete description of all faces of the associated polytope is unknown. We offer a method by which to enumerate the faces of the Kimura-3 polytope.

328. The death-Birth Process on Graphs with Weakly Advantageous Mutants

James Withers Emmanuel College

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Advisor(s): Dr. Ben Allen, Yulia Dementieva, Christine Sample, Emmanuel College

The spatial structure of a population can promote or hinder the fixation of advantageous mutants. These effects can be modeled using the death-Birth process on graphs. Recent numerical results suggest that the fixation probability of an advantageous mutant is maximized for complete graphs. We investigate this question under the assumption of weak selection, and reduce it to a problem of meeting times of random walks. We also prove that fixation probabilities on isothermal graphs and hitting-time reversible graphs are equivalent under weak selection to fixation probabilities on complete graphs.

329. Game Theory Auction Strategies and the Application to Construction Bidding

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Advisor(s): Dr. Adam Fox, Western New England University

Chapter 16 from the fourth edition of "Games of Strategy" written by Dixit, Skeath and Reiley was used to begin studying the strategy behind the auction. The term "auction" is in reference to any transaction where the final price for sale is determined by way of competitive bidding. Though there are multiple types of auctions, the focus is on a blind Dutch auction style. The bidding auctions that take place in a construction company follow this form when the companies are competing to be hired for a specific job. This article focuses on determining the best strategy for any such auction and how to bid on a job while increasing the chances of getting the contract or increasing those odds.

330. Marching to the Tune of Data Analytics: A Rank Aggregation Problem

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Drum Corps International (DCI) is the non-profit governing body for junior drum and bugle corps, for members ages 21 and younger. The DCI competitive summer tour, consisting of competitions throughout the United States and Canada, finishes in August with the DCI World Championships. Bands are assigned scores by judges based on their performance in several different categories, including general effect, visual, and music. Performance order for the World Class Preliminaries is generally based on the average scores from three of the contests on tour. The top 20 bands in Preliminaries advance to Semifinals, and the top 12 there will advance to Finals. We have used data analytics to measure the performance of the bands during the competitive summer tour and quantify the factors that determine the competition outcome between the bands. We have developed our own ranking system using tools from Linear Algebra to predict the scores and results of the DCI finals. Our method includes adaptations of the Colley and Oracle methods as well as predictive analytic methods using the scores in each of the categories. We also address the problem of determining how early in the summer tour one can make an accurate prediction of the DCI final competition.

331. Detecting non-randomness: A suite of statistical tests to detect fake coin flips and identify patterns in real-world data

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Advisor(s): Dr. AJ Hildebrand, University of Illinois at Urbana-Champaign

Can one distinguish a head/tail sequence obtained by flipping a coin from a similar "made up" sequence of heads and tails, or a win/loss sequence of a basketball team from the scoring sequence in a basketball game? To the naked eye, such sequences all appear similarly "random", and seem indistinguishable from one another. Yet, statistical tests can reveal unique features that make it possible to correctly classify the sequence, with a high level of confidence. We present preliminary results of an undergraduate research project at the University of Illinois aimed at answering questions of this type. We describe a suite of statistical tests we have developed for this purpose, and an interactive online "fake coin toss detector" that is based on these tests. We also report on the results of these tests when applied to a variety of real-world sequences from sports, finance, and everyday life. This is joint work with Tong Li and Xusheng Zhang.

332. Graph packing with constraints on edges

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Advisor(s): Dr. Gexin Yu, College of William and Mary

List packing is a general way to describe important graph theory problems such as list coloring. Previous studies have shown sufficient conditions for two graphs to pack, but only a few addressed packing with constraints (i.e., list packing). Therefore, it is needed to extend the findings from previous research by studying list packing (adding a G_3 that introduces "forbidden" ways to map G_1 onto G_2 and finding a condition such that there is a list packing). We show that given $\varepsilon > 0$, there is some $N \in \mathbb{N}$ such that for any $n > N$, if $d_3(v) \leq n-1$ where $v \in V_1 \cup V_2$ and $e_1e_2 + \frac{n-1}{2} \cdot e_3 < (2-\varepsilon)\binom{n}{2}$, then the triple (G_1, G_2, G_3) pack with some exceptions. This extends the results from Kostocha and Yu [2].

333. Unipancyclic Matroids as Graphs and Binary Matrices

Corrine Yap Sarah Lawrence College
Alana Huszar The College of New Jersey
Advisor(s): Dr. Colin Starr, Willamette University

Based on the concept of uniquely pancyclic graphs, a unipancyclic (UPC) matroid of rank r consists of exactly one circuit of lengths 3 through $r+1$. We analyze UPC matroids represented as graphs and as binary matrices. We use Markström's results, determining the existence of exactly four non-isomorphic graphic UPC matroids (on up to 56 vertices), to find transversal representations for three of five known UPC matroids. We also apply the concept of matroid connectivity to UPC matroids, ultimately proving that all UPC matroids have a connectivity of 2.

334. Double Interval Circular Societies

Sarah Yoseph Loyola Marymount University
Edwin Baeza Purdue University
Nikaya Smith University of North Carolina Chapel Hill
Advisor(s): Dr. Francis Su, Harvey Mudd College

Klawe, Nyman, Scott and Su considered linear societies where approval sets are double intervals, which are disjoint unions of two intervals. In our work, we consider double intervals on circular societies where the approval sets are arcs of equal length. Our research focuses on the minimum fraction of voters that will agree with each other in societies where the maximum number of intersecting sets is as small possible. We construct bounds that relate the diameter of these intervals to the number of double intervals, which we denote by n. This motivates the question of determining the minimal agreement proportion for pairwise intersecting approval sets, and we conjecture that as n increases, the agreement proportion approaches $1/3$.

335. A Critical Analysis of Random Response Techniques

Emanuel Zanzerkia Bridgewater State University
Advisor(s): Dr. Kevin Rion, Bridgewater State University

Processes known as random response techniques have been introduced to allow interviewers the ability to extract information they need from survey participants, while preserving the privacy of the respondents by introducing randomness into the surveying process. In this research we analyze two such procedures, a biased coin technique and S.L. Warners two question technique. For each of these methods, we provide estimators for the population proportion and introduce measures for the level of privacy protection afforded by these estimators. We then compare the performance of the estimators at given privacy levels and sample sizes for unknown population proportions. Finally, we determine the conditions for maximum accuracy and privacy protection.

336. Spin canonical rings of log stacky curves

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Aaron Landesman Harvard University
Peter Ruhm Stanford University
Advisor(s): Dr. David Zureick-Brown, Emory University

Consider modular forms arising from a finite-area quotient of the upper-half plane by a Fuchsian group. By the classical results of Kodaira–Spencer, this ring of modular forms may be viewed as the log spin canonical ring of a stacky curve. In this paper, we tightly bound the degrees of minimal generators and relations of log spin canonical rings. As a consequence, we obtain a tight bound on the degrees of minimal generators and relations for rings of modular forms of arbitrary integral weight.

337. Degrees of Minimal Solutions to a Linear Diophantine Equation in Polynomials

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Advisor(s): Dr. Keith, Conrad

Our research is concerned with investigating a measure of the degrees of minimal solutions to a linear Diophantine equation in $\mathbb{F}[x]$, where $\mathbb{F}$ is a field. For nonconstant relatively prime f and g in $\mathbb{F}[x]$, define $D(f,g)$ to be the degree of fu or gv where $u, v \in \mathbb{F}[x]$ satisfy $fu + gv = 1$ with v such that $\deg(u) < \deg(g)$, or equivalently $\deg(v) < \deg(f)$. Our primary result counts the number of pairs of polynomials (f,g) with fixed degrees and a fixed value of $D(f,g)$ value over finite fields. Specifically, over a finite field of order q, the number of monic pairs of polynomials (f,g) such that $\deg(f) = r$, $\deg(g) = s$, and $D(f,g) = n$ is equal to $(q-1)q^n$ if $\max(r,s) = n$, is equal to $(q-1)^2 q^{n-1}$ if $r, s < n < r + s$, and is 0 otherwise. We also discuss an interpretation of $D(f,g)$ beyond its original definition: it is related to the precision with which fraction f/g can be approximated by other rational functions in $\mathbb{F}[x]$.

338. On the Minimal Period for the Catenary Degree in Numerical Monoids

Albert Zheng University of California Berkeley
Advisor(s): Mr. Christopher Miller, University of California Berkeley

A numerical monoid is an additive subset of the natural numbers generated by a given list of integers. Such a subset forms a non-unique factorization domain, with irreducibles as the generators. The catenary degree of an element in a numerical monoid is a combinatorial constant that measures the spread of the element's factorizations. It has been shown previously that the catenary degree is eventually periodic, with period dividing the LCM of the generators. We show that this bound on the period is sharp, and that tighter bounds exist in special cases.

339. Magic Squares of Squares of Dimension Three Modulo Prime Numbers

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Advisor(s): Dr. Aihua Li, Montclair State University

In this research project, we investigate 3 by 3 Magic Squares of Squares (MSS) modulo an prime p. Our interest is finding what primes produce MSS with 9 distinct elements, called degree 9 MSS. We show that there are infinitely many primes p such that there exists MSS with degree 9 modulo p.

340. A Canonical Classification of Coordinate Systems in R^3

Arianna Zikos Bridgewater State University
Advisor(s): Dr. Vignon Oussa, Bridgewater State University

A coordinate system is a system that uses one or several numbers to uniquely represent the position of an object living on a manifold. Once a coordinate system is fixed on a given space, any motion can be described by a set of suitable equations. In order to describe a specific motion in simple ways, it is important to make a careful choice of a coordinate system which is compatible with the geometry of the motion. To the best of our knowledge, there is no procedure available in the literature which addresses this problem in a fully satisfactory manner. In this work, we provide a complete classification of all coordinate systems arising from one-parameter groups acting on a three-dimensional vector space. First, we show that all classical coordinate systems occur from some one-parameter matrix group acting linearly on a Euclidean space. Secondly, by exploiting well-known techniques such as the spectral decomposition of linear operators, matrix exponentiation, and a precise layering procedure, an exhaustive list (up to fixed cross-sections) of all coordinates arising from linear actions of one-parameter groups in dimension three is obtained.

341. Portfolio Optimization with Gaussian Mixture Models

Emily Zucker North Carolina State University
Advisor(s): Dr. Tao Pang, North Carolina State University

Gaussian mixture distributions are often used to model the distribution of data when a single normal distribution will not provide a suitable fit. A normal distribution only matches the first and second moments of the data, but a Gaussian mixture can fit higher moments of data as well. We consider several existing methods for optimizing a Gaussian mixture, both in the univariate and multivariate case and develop a computationally efficient method for univariate quantile regression. Also, we develop a method which allows Gaussian mixture distributions into the Black-Litterman model, a well known financial model used to combine the views of an investor with a neutral starting point. The primary benefit of this revised model is that it allows for the preservation of the higher co-moments of asset returns.

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