The Doctrine of Fluxions
founded on
Sir Isaac Newton's Method
published by himself in his tract
upon the quadrature of curves
by
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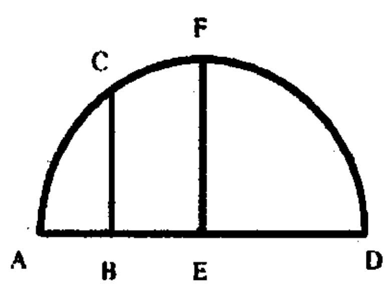
pages v-vii
[*On the difference between the Fluxionary method and Differentials:*] "[Differentials are] magnitudes as made up of an infinite Number of very small constituent Parts put together; whereas the Fluxionary Method teaches us to consider Magnitudes as generated by Motion. A Line is described, and in describing is generated, not by an Apposition of Points, or Differentials, but by the Motion, or Flux, of a Point; and that Velocity with which the generating Point moves, when the Line begins to arise, or in the first Moment of its Generation, or Formation, is called its Fluxion; so that to call a Differential a Fluxion, or a Fluxion a Differential, is an Abuse of Terms, and an Imposition upon the Reader. ...all the operations that are founded upon the method of Fluxions must be much more clear, accurate and convincing than those that are founded upon the Differential Calculus."

Section IV. [*page 331*]
Containing the Use of Fluxions in the Cubature of Solids, or in investigating methods to find their contents.

Example I.
Let it be required to find the Solidity of a Prism. ...

Example II.
Let it be required to find the Solid content of a Pyramid. ...

Example III. [*page 337*]
To find the Solid content of a Sphere.



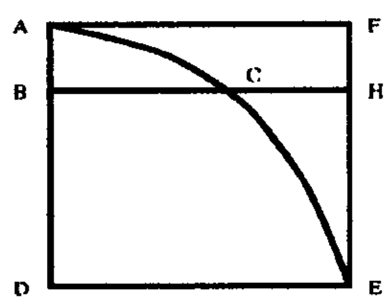
Let $AFDEA$ represent a Semi-circle, BC an Ordinate rightly applied, and imagine the Semi-circle to be turned about its Diameter AD as an Axis, 'till it return to the Place from whence it began to move; then will the Semi-circle, by this Motion, generate a Sphere, as will its Ordinate BC generate a Circle, whose Radius is BC.

Put $AB = x$, $BC = y$, $EF = r$, and the Circumference generated by the point $F = c$; then as $r : c :: y : \frac{cy}{r}$, the Circumference of the Circle generated by the Point C, whose Area will be $\frac{cyy}{2r}$, for $\frac{cy}{2r}$ multiplied by y, will be equal to $\frac{cyy}{2r}$; consequently $\frac{cyy\dot{x}}{2r}$ will be the Fluxion of the Solidity; but from the Nature of the Circle $yy = 2rx - xx$; whence $\frac{cyy}{2r}$ will be equal to $\frac{2crx\dot{x} - cxx\dot{x}}{2r}$ (by substituting $2rx - xx$ in the room of yy) $= cx\dot{x} - \frac{cxx\dot{x}}{2r}$, and consequently its Fluent $\frac{cxx}{2} - \frac{cxxx}{6r}$ will be the Solidity of the Portion of the Sphere generated by the Semi-segment ABC; but when AB flows into, or becomes $= AD$, x becomes equal to $2r$, and consequently we shall have $\frac{4crr}{2} - \frac{8crrr}{6r}$ (by substituting $2r$ in the room of x, in the Expression $\frac{cxx}{2} - \frac{cxxx}{6r}$)$= \frac{12crr - 8crr}{6} = \frac{4crr}{6} = \frac{2crr}{3} = \frac{2}{3}crr$, for the Value of the whole Sphere.

[*Note: The author also generates the volume by considering spherical surfaces. He relates volume to the cube of its diameter, as well as its circumscribing cylinder, and to the cone* 'which hath for its base the great circle of the sphere, and for its altitude the diameter of the same sphere.']

Example IV. [*p.340*]

Let it be required to find the Value of a Parabolic Conoid, generated by the Revolution of the Semi-Paraboloid $ACED$ about it Axis AD.



From any Point, as B, in the Axis AD, let the Ordinate BC be drawn parallel to the Base DE; then put $AB = x$,$BC = y$, and $DE = r$, also $AD = a$, and the Circumference described by the point $E = c$; whence to find the Circumference generated by the Ordinate BC, it will be as $r : c :: y : \frac{cy}{r}$ = to the Circumference described by the Point E; whence $\frac{cy}{2r} \times y = \frac{cyy}{2r}$, will be the Area of the Same Circle. This therefore being multiplied by $\dot{x}$, the Fluxion of the

Abcissa, will give $\frac{cyy\dot{x}}{2r} = \frac{cx\dot{x}}{2r}$ (by putting x in the room of yy, because, from the Nature of the Parabola, $1x = yy$) for the Fluxion of the Solid generated by the Space ABC, whose Fluent $\frac{cxx}{4r} = \frac{cyyx}{4r}$ (by putting yy in the room of x) equal to $\frac{carr}{4r} = \frac{1}{4}acr$ (by putting a in the room of x, and r in the room of y) will be the Value of the Solid generated by the whole Parabolic Space ADE; for it we imagine the Ordinate BC in a flowing State, and to move 'till it arrives at, or coincides with the Ordinate DE, the AB will become equal to AD, x will become equal to a, and y will become equal to r.

[*Note: The author also compares the volume to the volumes of a circumscribed Cylinder and an inscribed Cone. He also extends the results to $x = y^m$*]

Example V. [*p.342*]

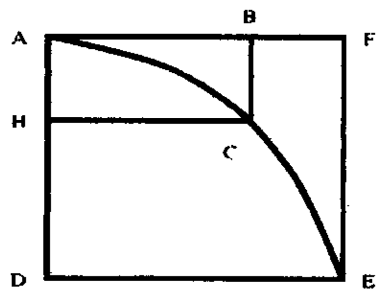
Let it be required to find the Value of the Solid formed by the Rotation of the Parabolic Space ADE, about the Line FE, parallel and equal to the Axis AD. (see the preceding Figure.)

Let ACE represent a Semi-Parabola, AD its Axis, BC an Ordinate rightly applied, which being produced to cut FE in the Point H, imagine the Semi-Parabola to revolve about the line FE, as an Axis, then will the line BH generate a Circle. Putting therefore $FE = AD = a$, $AB = x$, $BC = y$, AF or $DE = r$, c for the Circumference of the Circle described by the Point D, we shall have $CH = r - y$, and $\frac{cr - cy}{r}$ for the Circumference of the Circle generated by the Point C; for as $r : c :: r - y : \frac{cr - cy}{r}$; whence $\frac{r - y}{2} \times \frac{cr - cy}{r} = \frac{crr - 2cry + cyy}{2r}$, will be the Area of the Circle described by the line CH. This therefore being taken from $\frac{cr}{2}$, the Area of the Circle described by the line BH, will give $\frac{2cry - cyy}{2r}$ for the Area of the Annulus described by the ordinate BC; and this being multiplied by $\dot{x}$ will give $\frac{2cry\dot{x} - cyy\dot{x}}{2r}$ for the Fluxion of the Solid generated by the Space ABC equal to $\frac{2rcx^{\frac{1}{2}}\dot{x} - cx\dot{x}}{2r}$ by putting $x^{\frac{1}{2}}$ in the room of y, and x in the room of yy, to which they are respectively equal, from the Nature of the Parabola, whose Fluent $\frac{2crx^{\frac{1}{2}}x}{3r} - \frac{cxx}{4r} = \frac{2cryx}{3r} - \frac{cxyy}{4r}$ (by substituting y in the room of $x^{\frac{1}{2}}$, and yy in the room of x) $= \frac{8cryx - 3cxyy}{12r}$, will give the Value of the Solid generated by the same Space ABC. Now if we consider the Ordinate BC in a flowing state, and to move 'till it arrive at, or coincide with the ordinate DE, then AB will become equal to AD, x will become equal to a, y will become equal to r and $\frac{8cryx - 3cxyy}{12r}$ will become $\frac{8crar - 3carr}{12r} = \frac{8acr - 3acr}{12} = \frac{5acr}{12}$ the Value of the Solid generated by the whole Space ADE. Whence it is evident, that the Solid thus formed is to its circumscribed Cylinder as 5 is to 6; for the Value of the Cylinder is $\frac{acr}{2}$, and $\frac{5}{12}acr$ is to $\frac{acr}{2}$ as $\frac{5}{12}$ to $\frac{1}{2}$ or as $\frac{5}{12}$ to

$\frac{6}{12}$ or as 5 to 6.

Example VI.

Let it be required to find the Value of the Solid formed by the Rotation of the Parabolic Space $ACED$, about the line AF, as an Axis, touching the Parabola in the Vertical Point A.



From any Point H, in the Axis AD, draw the Ordinate HC, parallel to the base DE, it is evident, that while the Curvilinear Space AED is revolved about the Line AF as an Axis, that this Line HC will describe a Cylindrical Surface. Put therefore $AD = r$, $DE = AF = a$, $AB = HC = x$, $BC = AH = y$, then $\dot{y}$ will be the Fluxion of the Ordinate $BC = AH$, and let the Circumference of the Circle formed by the Point D be put equal to c; whence ac will be the Cylindric Surface formed by the line DE, whence to find the Cylindric Surface formed by the Line HC, it will be as $DE \times DA = ar : ac$, or as r to $c :: HC \times HA = xy : \frac{cxy}{r}$; whence $\frac{cxy\dot{y}}{r}$, will be the Fluxion of the Solid Generated by the Space AHC; and because, from the Nature of the Parabola, $xx = y$; whence $x = y^{\frac{1}{2}}$, if we substitute $y^{\frac{1}{2}}$, in the former Equation, in the room of x, to which it is equal, we shall have $\frac{cyy^{\frac{1}{2}}\dot{y}}{r} = \frac{cy^{\frac{3}{2}}\dot{y}}{r}$ for the Fluxion of the same Solid, whose Fluent $\frac{2cy^{\frac{1}{2}}y^2}{5r} = \frac{2cxyy}{5r}$, by putting x in the room of $y^{\frac{1}{2}}$, will give the Value of the Solid itself.

Now if we conceive the Ordinate HC in a flowing state, and to move according to the Direction AD, 'till it arrives at, or coincides with the Ordinate DE, then AH will become equal to AD, y will become equal to r, x will become equal to a; consequently $\frac{2}{5}\frac{cxyy}{r}$ will become $\frac{2}{5}\frac{carr}{r} = \frac{2}{5}acr$, for the Value of the Solid generated by the whole Parabolic Space ADE.