On the Mathematics of Income Inequality: Splitting the Gini Index in Two

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Abstract. Income distribution is described by a two-parameter model for the Lorenz curve. This model interpolates between self-similar behavior at the low and high ends of the income spectrum, and naturally leads to two separate indices describing both ends individually. These new indices accurately capture realistic data on income distribution, and give a better picture of how income data is shifting over time.

1. INTRODUCTION. In early 2011, *Forbes* Magazine [**8**] reported that hedge fund manager John Paulson's income for 2010 was $5 billion (give or take $100 million). That's a staggering amount. Just consider that it takes no fewer than 50,000 professors with an average income of $100,000 (we are being generous) to equal that income. That's more than all the math professors in this country combined. Looking in the other direction of the ladder of wealth, it is not implausible to imagine that there would be 50,000 among the world's poor whose combined wealth is less than that of an average professor in the U.S. The world's economic inequality is stunning.

But is it static? Is it simply a given, something we cannot do anything to change? A closer look at income data reveals that the *degree* of inequality is not at all constant, but rather varies greatly from country to country, and even within one country over time. Consider, for example, the following data shown in Figure 1 on the share of national income (including capital gains, but excluding government transfers) in the U.S. received by the richest 1% over the period 1913–2010, obtained from the IRS by Alvaredo, Atkinson, Piketty, and Saez [**1**].

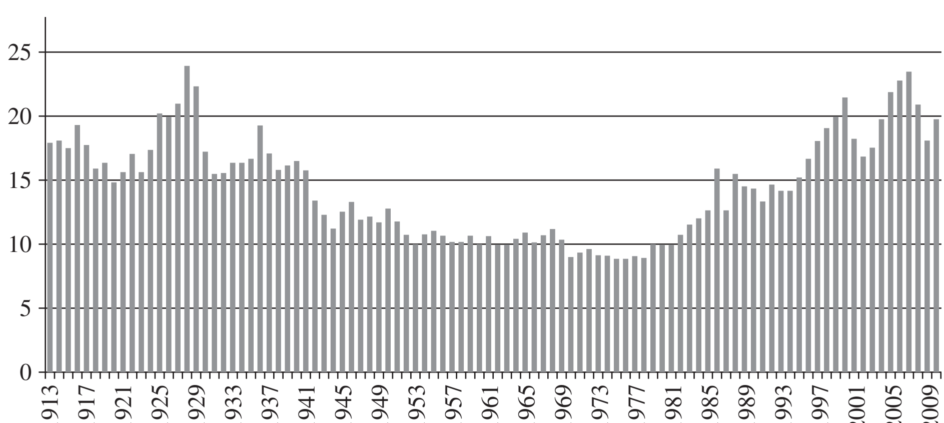

Figure 1. Share of total income (in percent) of top 1% earners in the U.S., 1913–2010.

By this measure, we can see a peak of inequality was reached at the height of the stock market frenzy of 1928, when the top 1% took a share of 23.5% of the national

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income, and again in 2007, right before the latest financial crisis. The low was reached in the 1970s, when this share was a "mere" 9%. This indicates a remarkable variation in measured inequality over a 100-year period, and provides grist for sociological inquiries: What causes the rising and falling of inequality? Is it change in the political leadership? Is it mainly economic factors? Or is it reflective of a fundamental change in what our society values?

To help figure this out, we first turn our attention to a more mathematical problem: As vivid an illustration as this 1%-index yields, it is not sufficiently representative of income inequality. By concentrating on the top 1%, this measure says little about the welfare of the middle class or the poor. So how could we more accurately portray income inequality so that it remains simple, yet comprehensive? The well-known Gini index is a first answer. But in this paper, we shall go beyond that index and introduce and promote the use of a two-parameter index, which is derived from a two-parameter family of functions used to model the Lorenz curve of incomes. This family was first proposed in [**11**], but our derivation, interpretation, and analysis is new.

2. THE LORENZ CURVE AND THE GINI INDEX. A long-known index that is more comprehensive than the 1%-measure is the **Gini index**, named after Italian statistician Corrado Gini (1884–1965) who defined it in 1912 [**6, 7**]. In a recent article in the MONTHLY, F. Farris [**5**] gave several nice illustrations of this index. Its definition is based on the **Lorenz curve**, introduced by American economist Max Otto Lorenz (1876–1959) in 1905 [**9**]. This curve plots the percentage $L(x)$ of the total income of a population that is cumulatively earned by the bottom $x\%$ of the population. In 2007 in the U.S., for example, the value was $L(.99) = .765$, since the top 1% took 23.5% of the total income, and the bottom 99% took the remaining 76.5%. Instead of using percentages, we use the corresponding proper fractions, so that the graph of $L(x)$ becomes a curve from the origin $(0, 0)$ to the point $(1, 1)$ in the unit square representing the two anchoring points; none of the people have none of the income, while all of the people have all of the income.

In a (Utopian) society, where everyone has the same income, the Lorenz curve would approach the $L(x) = x$ line. The other extreme is the case where one person receives all the income, in which case $L(x) = 0$ for all $x < 1$, and $L(1) = 1$. A realistic Lorenz curve lies between these extremes, as in Figure 2.

The more equal the distribution of income, the closer the curve is to the diagonal line. It is therefore natural that we define as a measure of inequality the *area* between the diagonal and the Lorenz curve, as a percentage of the entire area between the two extremes (which is 1/2). This number is the **Gini index**, and its formula is therefore

$$G = 2\int_0^1 x - L(x)\,dx.$$

As an example, if $L(x) = x^p$ with $p \geq 1$, we quickly calculate that $G = \frac{p-1}{p+1}$. So this index is a *summary measure*, giving weight to the income shares of the poor and the wealthy. Table 1 shows the latest numbers for different countries, compiled by the CIA [**4**].

We need to take these numbers with a grain of salt, as the precision of the economic data is not everywhere equally good. Also, other factors, such as the size of the basic economic units (e.g., household, family, or individuals) and what counts as income (e.g., capital gains, government support) are not always strictly comparable [**2**]. Finally, these data are from different years, and as we have mentioned, the values do change with time.

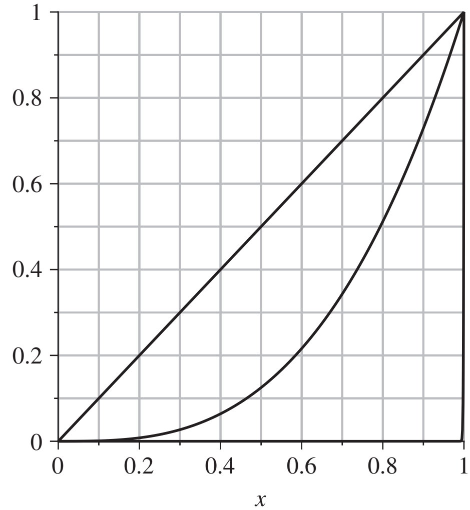

Figure 2. A typical Lorenz curve and the line of equality.

Table 1. Recent international Gini comparison data.

Country	**Gini index**
Sweden	.230
Germany	.270
Egypt	.344
Russia	.420
China	.480
Brazil	.519
South Africa	.650

3. HOW DOES THE U.S. STACK UP? It turns out that the U.S. Census Bureau [**13**] has compiled and published data on household income distribution, covering the years 1967–2010, allowing us to calculate the Gini index ourselves. Our only problem is that the data are published in quintiles, i.e., we only know the values of $L(x)$ for $x = 0$, .2, .4, .6, .8, and 1. For example, for 2009, the values given in [**13**] are shown in Table 2 and plotted in Figure 3.

Table 2. Quintile income data for the U.S. in 2009.

x	0	.2	.4	.6	.8	1
$L(x)$	0	.034	.120	.266	.498	1

How shall we calculate the Gini index from this? Numerical integration using the trapezoidal rule (connecting the dots by straight line segments) substantially underestimates the index, but yields a lower bound of .433. An upper bound of .517 can

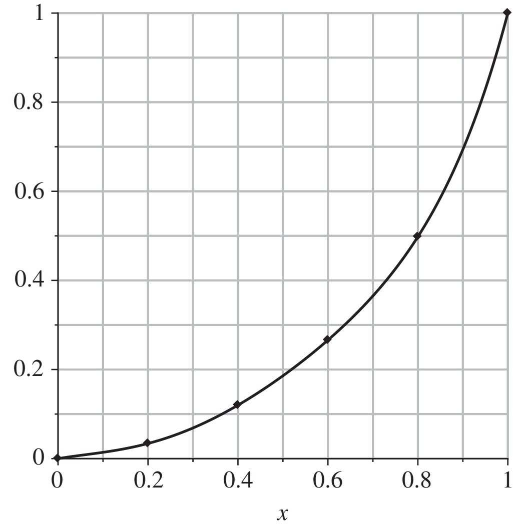

Figure 3. Quintile income data for the U.S. in 2009 plotted together with a fifth-degree polynomial fit.

be obtained by extrapolating the secant lines of the trapezoidal approximation to their intersection points, and adding the resulting triangular areas to that approximation.[1]

A more elegant approach is to approximate the data with a smooth curve, but what type of curve should we use? The first idea might be to use the lowest-power polynomial that fits well. If we use a fifth-degree polynomial, the fit looks very good, as shown in Figure 3.

Because of the number of data points, and the number of coefficients in the polynomial, the curve goes exactly through each data point. The curve is

$$L(x) = 0.17083\,x - 0.70208\,x^2 + 4.70833\,x^3 - 6.82292\,x^4 + 3.64583\,x^5$$

and the Gini index with this curve evaluates to $G = 0.457$.

However, there are two problems with this approach.

1. The Gini index calculated and published for 2009 by the U.S. Census Bureau, presumably using their complete data, is $G = 0.468$, a substantial discrepancy. So while the model curve meets every data point, it does not do the right things in between the points. Indeed, one can get a better estimate of .475 for the Gini index without any curve fitting at all by averaging the upper and lower bounds calculated above!
2. The coefficients of the polynomial yield no information, because the model lacks any economic meaning. In other words, when we compare modeling equations over the years, the coefficients change substantially without giving us a clue what is really happening in the different segments of society.

4. THE PHENOMENON OF (RIGHT-SIDED) SELF-SIMILARITY. A hint of what to do comes from further data by Piketty and Saez [**1**]. Consider that in 2008, the top 10% received 48.23% of the total income (including capital gains). The top

[1]We thank an anonymous referee for this clever observation, together with the suggestion used below to average these to get a direct estimate of the Gini index.

1% received 20.95%, the top .1% had 10.40%, and the top .01% had 5.03%. There is a pattern: The degree of inequality *repeats* even as we restrict ourselves to richer and richer slivers of the population! Informally expressed, the top 10% of the whole population got roughly half of the total pie. The top 1% got again (almost) half of *that* pie. The top .1% received again half that pie, and so on. One could say that "money begets money." We shall call this phenomenon a (right-sided) self-similarity. It is also known as the *Pareto principle*. Strikingly, with small deviations, it is present in the Piketty-Saez data in all years from 1913–2010, even as the degree of inequality varies. For example, in Figure 4 we plot the ratio of income shares of the top .1% to the income shares of the top 1% (with asterisks), and the ratio of income shares of the top 1% to the income shares of the top 10% (with squares). It is remarkable how similar the ratios are over nearly 100 years.

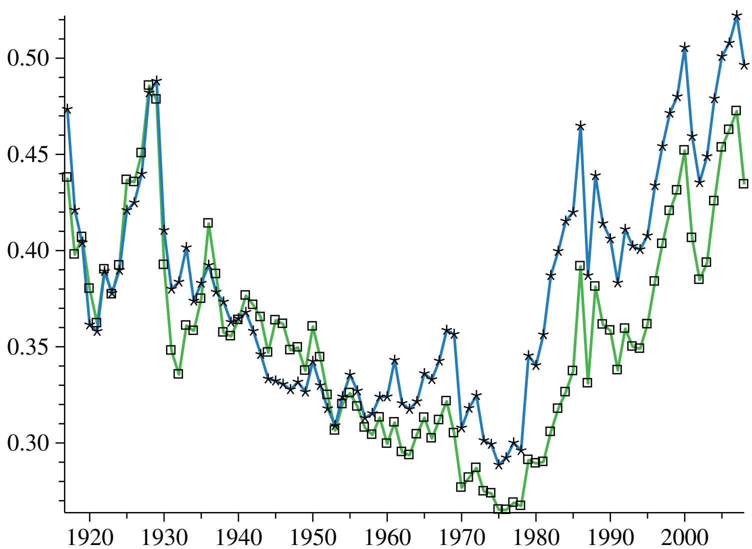

Figure 4. Ratios of income shares of the top .1% to the top 1% (asterisks) and of the top 1% to the top 10% (squares) for the years 1913–2007.

What equation for the Lorenz function would model such a phenomenon? Let us assume that the richest segment R of the population receives a portion P of the total income. Assume furthermore that this ratio repeats itself within that segment, and so on. Then the Lorenz function must have the following property: $L(1 - R^k) = 1 - P^k$ for any k. Solve the input $x = 1 - R^k$ of this unknown function $L(x) = 1 - P^k$ for k in terms of x and then substitute the result into the right-hand side of $L(x)$, also using the fact that $a^{\ln(b)} = b^{\ln(a)}$, to find that $L(x) = 1 - (1 - x)^{\frac{\ln(P)}{\ln(R)}}$. We'll abbreviate this to $L(x) = 1 - (1 - x)^q$, where $0 < q \leq 1$ for this to be a Lorenz curve. This model is known as the *Pareto Distribution*. Figure 5 shows a family of functions of this form, with q ranging over the reciprocals of 1 through 10. Note that $q = 1$ corresponds to complete equality and $q \mapsto 0$ to complete inequality.

For this income distribution, the inequality also repeats in the sense that the Gini index is *right invariant*. In other words, for any cutoff point r between 0 and 1, the Gini index among the richest r% is the same as in the whole population, as illustrated in Figure 6.

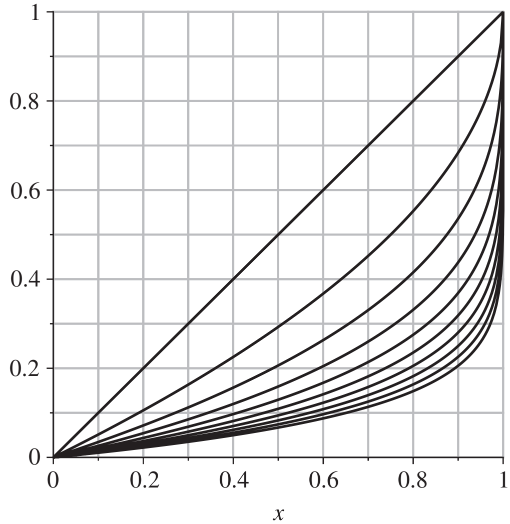

Figure 5. A family of Lorenz curves associated with the Pareto distribution.

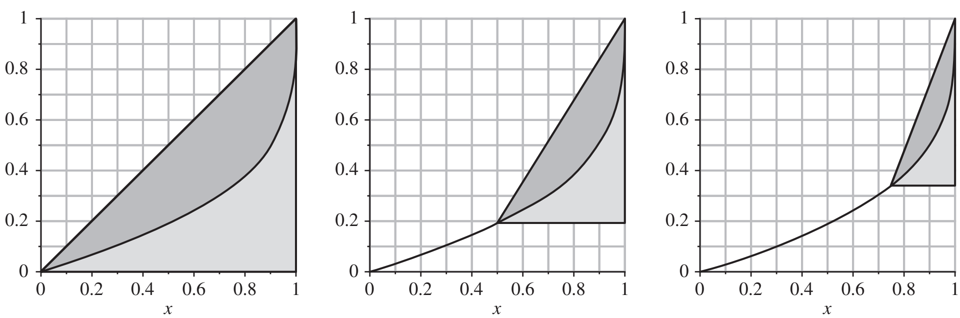

Figure 6. Restricting the Gini index calculation to the top income earners at successively higher income share cutoffs always yields the same result for the Pareto distribution.

To make this statement more precise, let us define for any Lorenz curve $L(x)$ the *right-sided Gini function Gr* as the area between the Lorenz curve from a point $x = r$ to $x = 1$ and the idealized line of equal distribution among that segment, again as a percentage of the maximum area of a completely unequal distribution among that segment. So

$$Gr(r) = \frac{\displaystyle\int_r^1 \left(\frac{1 - L(r)}{1 - r}(x - 1) + 1\right) - L(x)\, dx}{\frac{1}{2}(1 - r)(1 - L(r))}.$$

In the case that $L(x) = 1 - (1 - x)^q$ we calculate that

$$Gr(r) = \frac{\displaystyle\int_r^1 \left(\frac{1-(1-(1-r)^q)}{1-r}(x - 1) + 1\right) - (1-(1-x)^q)\, dx}{\frac{1}{2}(1 - r)(1 - (1 - (1 - r)^q))}\, dx = \frac{1-q}{1+q}, \qquad (1)$$

which is independent of r and equal to the Gini index for the whole population. So with the Pareto distribution the Gini index is right invariant.

Sociologically speaking, this self-similarity phenomenon explains why rich people might not "feel" so rich. Making \$1 million a year places a person easily in the top 1% of earners in the U.S., yet this income pales in comparison to the neighbor who makes \$1 billion.

How good is this function in modeling a realistic Lorenz curve? The 2008 data of Piketty and Saez for the top U.S. incomes are well modeled with $R = 0.01$ and $P = 0.213$ (hence $q = 0.336$), especially for the top 5% as illustrated in Figure 7. However, when we try to extend this model to the Census Bureau data for the *entire* population, Figure 8 shows that it does not really work.

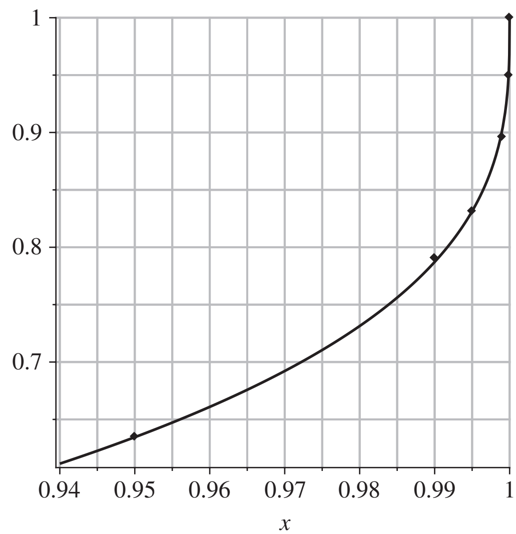

Figure 7. The Pareto distribution fit to top U.S. incomes in 2008.

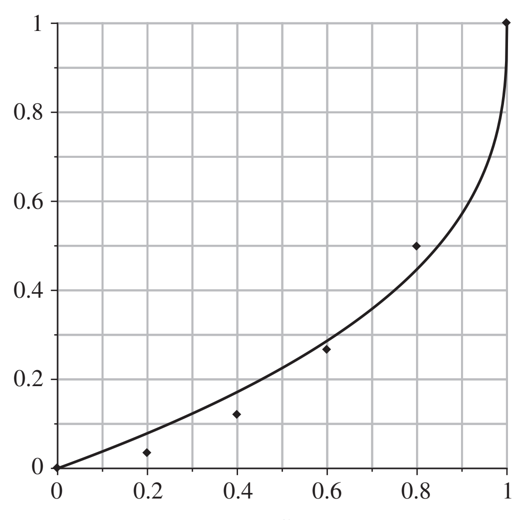

Figure 8. The same Pareto distribution fit extended to all U.S. incomes in 2008, not very satisfactory.

The right-sided self-similarity holds only in the upper echelons of earners. The economical dynamics in the lower echelons is different.

5. LEFT-SIDED SELF-SIMILARITY. Let's return to the Census data in Table 2. A loose reading suggests a *left-sided* self-similarity; the bottom 40% receives about one fourth of what the bottom 80% receives. The bottom 20% receives about one fourth of what the bottom 40% receives, and so on. Even among the poor, there appears to be a gradient of inequality that reflects the inequality in a larger segment of the society.

Mirroring the argument for right-sided similarity, the Lorenz curve $L(x)$ with strict left-sided self-similarity would obey $L(R^k) = P^k$, where the poorest segment R of the population receives the proportion P of the total income. Solving $x = R^k$ for k and substituting on the right side, we find that such a Lorenz curve would have to have the form $L(x) = x^{\frac{\ln(P)}{\ln(R)}} = x^p$, a simple power function, where the power must satisfy $p > 1$ for it to be below the line of strict equality. However, strict self-similarity is not warranted by the data. But a modified model $L(x) = cx^p$ works well among the bottom 60%. For example, for 2008 we have the least-square-fit curve $L(x) = 0.71135\, x^{1.9288}$. See Figure 9. (This curve cannot be used on the entire interval since it falls short of the value 1 at $x = 1$.)

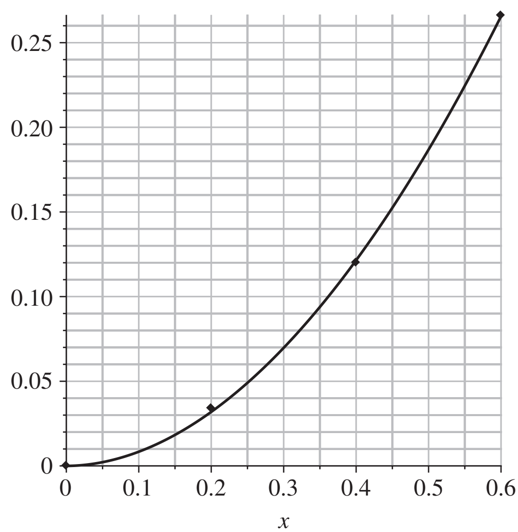

Figure 9. A power law Lorenz curve fit to the lower income levels of 2008 data.

This power law equation still exhibits the property of the left-invariance of the Gini index—for any segment between 0 and some $s < 1$, the Gini index over that segment is the same. Figure 10 illustrates the case $c = 1$.

The proof again is easy. We define the *left-sided Gini function* as

$$Gl(s) = \frac{\displaystyle\int_0^s \frac{L(s)}{s}x - L(x)\,dx}{\frac{1}{2}sL(s)}.$$

If $L(x) = cx^p$, we calculate that

$$Gl(s) = \frac{\displaystyle\int_0^s \frac{cs^p x}{s} - cx^p\,dx}{\frac{1}{2}(s)(c)(s^p)} = \frac{p-1}{p+1} = G \tag{2}$$

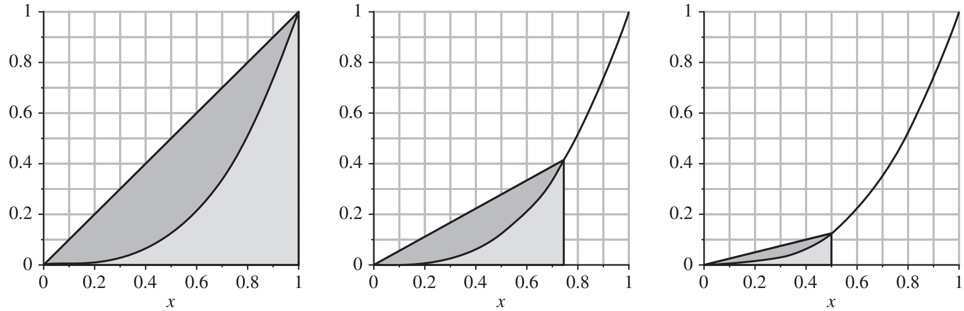

Figure 10. A power law Lorenz curve has the same Gini index when restricted to the lowest income share for different income cutoffs.

which is independent of s, and equal to the Gini index G of the entire population. So if the Lorenz function is a power law function, the Gini index is left invariant.

Note that setting $p = 1/q$ transforms this Gini index into that of the Pareto distribution, corresponding to the fact that reflection across the line $y = 1 - x$ transforms the power function $y = x^{1/q}$ into the corresponding Pareto distribution $1 - (1 - x)^q$.

6. THE HYBRID MODEL. So we have two models that work well in their respective segments of the population: the Pareto distribution among the highest incomes, and the power law function among the lowest incomes. How can we find a model equation that marries the two regimes? Motivated by the fact that the *product* of any two (nonnegative) Lorenz curves is again a Lorenz curve (any nondecreasing nonnegative concave-up function on the interval [0, 1] between the endpoints (0, 0) and (1, 1) has the right properties, and any product of two such curves has the same properties), we can combine the high-income fit of the self-similar model with the low-income fit by power law functions to get a hybrid model that interpolates between the two endpoint behaviors. The result works exceptionally well over the entire population:

$$L(x) = x^p(1 - (1 - x)^q).$$

We might ask what happened to the extra parameter c. It turns out, perhaps surprisingly, that q plays the role of c, so that a third parameter c is unnecessary. As a rule, the fewer parameters, the more natural the model, so we'll work with just two parameters p and q.

In Proposition 1, we show that this model has correct asymptotic behavior in the two limiting regimes.

Proposition 1.

- *As* $x \mapsto 0$, $L(x) \mapsto x^p(qx) = qx^{p+1}$, *a power law function. Moreover, the left-sided Gini function* $Gl(s)$ *approaches the value of the Gini index for this power law function, i.e.,*

$$\lim_{s \mapsto 0} Gl(s) = \frac{(p+1) - 1}{(p+1) + 1} = \frac{p}{p+2}.$$

- *As $x \mapsto 1$, $L(x) \mapsto 1-(1-x)^q$, the Pareto distribution. Moreover, the right-sided Gini function $Gr(r)$ approaches the value of the Gini index for the Pareto distribution, i.e.,*

$$\lim_{r \mapsto 1} Gr(r) = \frac{1-q}{1+q}.$$

Proof. The asympotics just rephrase that

$$\lim_{x\mapsto 0} \frac{L(x)}{q} x^{p+1} = \lim_{x\mapsto 0} \frac{1-(1-x)^q}{qx} = \lim_{x\mapsto 0} \frac{q(1-x)^{q-1}}{q} = 1$$

and

$$\lim_{x\mapsto 1} \frac{L(x)}{1-(1-x)^q} = \lim_{x\mapsto 1} x^p = 1.$$

As for the limit of $Gl(s)$, the calculation requires two successive applications of L'Hospital's Rule:

$$\begin{aligned}
\lim_{s\mapsto 0} Gl(s) &= \lim_{s\mapsto 0} \frac{\int_0^s \frac{s^p(1-(1-s)^q)}{s} x - x^p(1-(1-x)^q)\,dx}{\frac{1}{2}(s)(s^p)(1-(1-s)^q)} \\
&= \lim_{s\mapsto 0} \frac{((p-1)s^{p-2}(1-(1-s)^q) + q(1-s)^{q-1}s^{p-1})s^2}{s^p((p+1)(1-(1-s)^q) + sq(1-s)^{q-1})} \\
&= \frac{(p-1)q+q}{(p+1)q+q} = \frac{p}{p+2}.
\end{aligned}$$

The calculation of $\lim_{r\mapsto 1} Gr(r)$ is similar, and left as an exercise. ■

This model works exceedingly well. For example, for 2009, the least-square-fit curve is $L(x) = x^{0.79869}(1-(1-x)^{0.56077})$, shown in Figure 11.

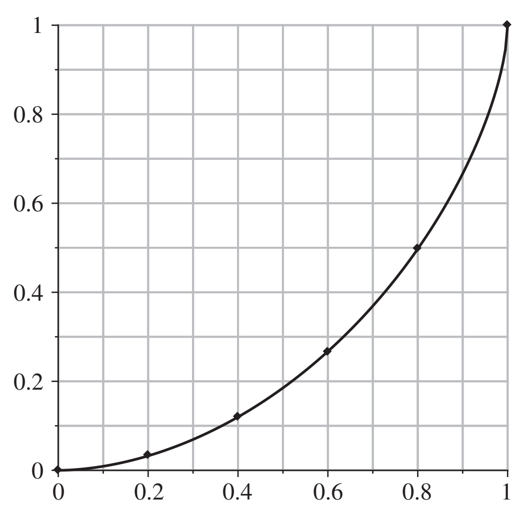

Figure 11. The hybrid model fit to the 2009 U.S. quintile income data.

The Gini index with this model is calculated to be **.468**, which is *identical* to the value of the index published by the Census Bureau. Recall that the quintic model had estimated it to be **.457**. Thus the hybrid model is a great fit achieved with only two degrees of freedom (instead of 5). This goodness-of-fit holds for all the Census Bureau data from 1967 to 2010 with the estimated Gini index (connected line) consistently within .003 of the published values (diamond points), as shown in Figure 12.

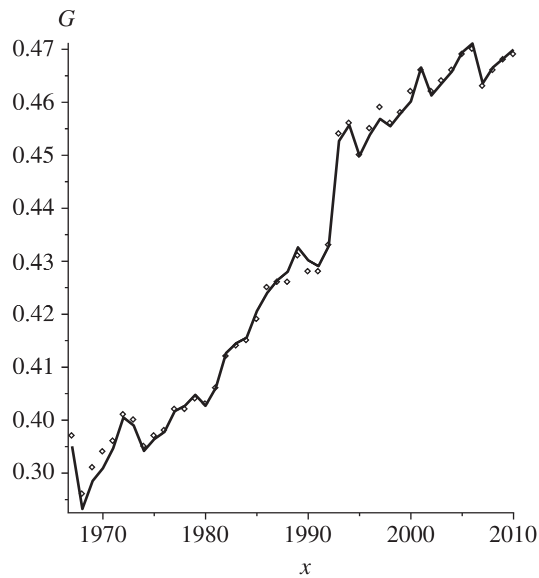

Figure 12. The hybrid model fit (connected line) to the Gini index data (diamond points) from 1967 to 2010.

Notice the near-steady increase of the index from .386 in 1968 to .469 in 2010. Notice also that as of 2010, the Gini index of .469 places the U.S. between Russia and China in terms of income inequality in Table 1.

7. SPLITTING THE INDEX IN TWO.

This hybrid model has several advantages.

- The fit of the data is excellent across a long range of longitudinal data.
- It can distinguish between intersecting Lorenz curves with identical Gini index.
- It requires only two parameters.
- In contrast with the parameters of typical income fitting models, these two parameters have a direct economic interpretation that yields interesting information about the development of inequality that is not apparent from the Gini index alone.

To better harness this economic information of the two parameters p and q, we transform them into Gini-like indices G_0 and G_1, where $G_0 = \frac{p}{p+2}$ and $G_1 = \frac{1-q}{1+q}$. G_0 can be thought of as the "low-end" Gini index, while G_1 is the "high-end" Gini index. More precisely, the above proposition showed that G_0 and G_1 are the *limits* of the left-sided (respectively right-sided) Gini function as s goes to 0 (respectively as r goes to 1).

Proposition 2. *The Gini index for this hybrid model is an analytic function of the two separate Gini indices.*

Proof. Inverting the two fractional linear relationships gives

$$p = \frac{2G_0}{1 - G_0}, \qquad q = \frac{1 - G_1}{1 + G_1},$$

while the Gini index integral may be evaluated exactly in terms of the Gamma function:

$$G = 2\int_0^1 x - x^p(1 - (1 - x)^q)\,dx = 1 - \frac{2}{(p+1)} + 2\frac{\Gamma(1+q)\Gamma(p+1)}{\Gamma(2+p+q)}.$$

Composing G with the expressions for p and q yields G as an explicit function of G_0 and G_1.

Note that this allows a reparametrization of the hybrid model Lorenz curve by parameters confined to the unit square, with real geometric significance lacked by the exponents p and q:

$$L(x) = x^{\frac{2G_0}{1-G_0}}\left(1 - (1 - x)^{\frac{1-G_1}{1+G_1}}\right), \qquad \text{for } 0 \le G_i \le 1\,.$$

Here, each separate Gini index reduces to the Gini index when the other vanishes, describing the limiting power law model ($G_1 = 0$) and limiting Pareto model ($G_0 = 0$), whose intersection gives the line of perfect equality ($G_0 = 0 = G_1$). See Figure 13. The limiting case of perfect inequality corresponds to the boundary $G_0 \to 1$ or $G_1 = 1$. The plot of $G(G_0, G_1)$ is confined to the unit cube. We propose to call this hybrid expression for the Lorenz curve the *Pareto-power-law interpolation Lorenz curve*. ∎

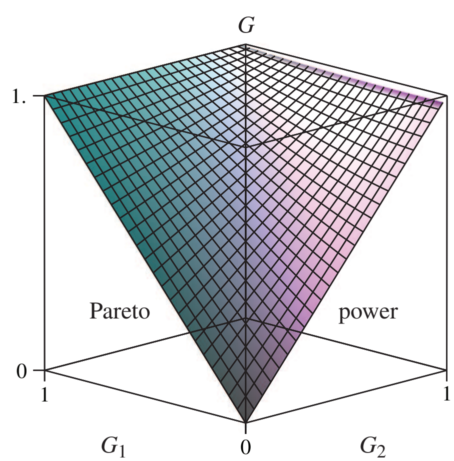

Figure 13. The Gini index plotted as a function of the two-parameter hybrid model, interpolating between the limiting power law ($G_1 = 0$) and Pareto ($G_0 = 0$) distribution Gini index functions.

Let us tabulate the historical development of these two indices in Figure 14: circles indicate G_0, asterisks G_1. We see that between 1967 and 2010, income inequality evolved very differently at the two opposite poles of society. This was not apparent in the Gini index alone. It sheds interesting light on research by Atkinson, Piketty, and Saez [**2**] who have shown that most of the economic gains of the last 30 years have gone to the upper income brackets. In that process, the income gradient in this bracket has increased substantially, with the gains ever more skewed to the highest incomes of society. The low-income portion of the population largely did not participate in this process, with only a small increase in the income gradient over the last decade, mirroring the development of poverty rates over this period [**14**]. Household median incomes (inflation-adjusted) increased only by an average of .45% per year over this time period [**15**], while gains in the upper income brackets were the greater the higher the bracket [**1**].

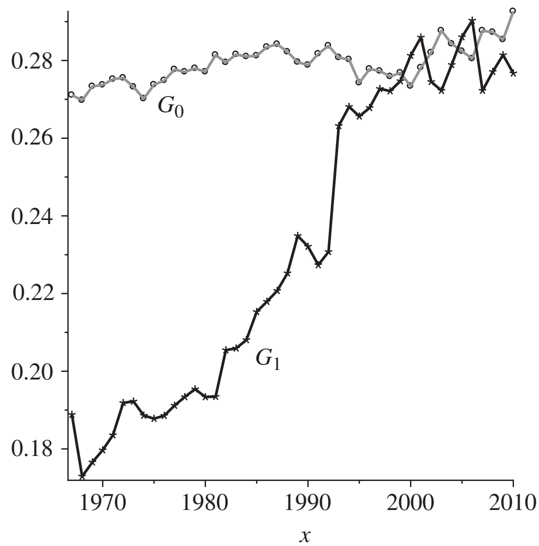

Figure 14. The historical values of the two Gini indices.

While the income bar graph for the top 1% earners at the start of our discussion appears to correlate well with the index G_1 over the last four decades, this index tells us even more about the increasing spread of incomes within the top 1%. The dynamics resembles that of a gold rush taking place at the front of a pack of people, pulling apart the fastest runners, while the slow majority hardly changes pace, as if not knowing that there is gold to be had.

What exactly was the gold? There were at least two major speculative bubbles (dot-com, real estate) in this time period, but it was chiefly the gains of the economy through higher productivity, which kept increasing steadily [**3**]. There was a rush to grab a larger share of those gains, producing some exceedingly high incomes. As for what exactly brought on this rush, economic explanations might vary, but factors that certainly played an important role include [**10, 12**]:

- government policies such as deregulation and privatization;
- the emergence of stock options for compensation of executives;
- financial derivatives (e.g., credit default swaps) that allowed banks and hedge funds to steadily increase risk, leverage, and maximum payouts;
- low interest rates that made credit (and leverage) cheap;
- lowered marginal tax rates, that did little to slow down the top earners.

The frequently-cited factor of globalization seems less persuasive, as that is not really a new phenomenon and was present even when inequality decreased (as in the 1950s). Also, some countries have recently decreased their inequality (e.g., Brazil) despite the fact that they too face globalization. As for the question of whether fundamental values have changed in our society, and whether they will change again, we'll leave that to our readers to decide.

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