
E. H. Moore's Early Twentieth-Century Program for Reform in Mathematics Education

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1. INTRODUCTION. The purpose of this article is to examine briefly the nature and consequences of an early twentieth-century program to reorient the American mathematical curriculum, led by mathematician Eliakim Hastings Moore (1862–1932) of the University of Chicago. Moore's efforts, I conclude, were not ultimately very successful, and the reasons for this failure are worth pondering. Like William Mueller's recent article [16], which draws attention to the spirited debates regarding American mathematics education at the end of the nineteenth century, the present article may remind some readers of more recent educational controversies. I don't discourage such thinking, but caution that the lessons of history are not likely to be simple ones.

E. H. Moore is especially intriguing as a pedagogical promoter because of his high place in the history of American mathematics. He is a central source for much of twentieth century American mathematical research activity. As a gross measure of Moore's influence we may note the remarkable fact that more than twice as many Ph.D. mathematicians were produced by his University of Chicago mathematics department during his tenure than were produced by any other single institution in the United States. Indeed, the University of Chicago produced nearly 20% of all American mathematics Ph.D.s during the period [22, p. 366]. Moreover, among Moore's doctoral students at Chicago were such luminaries as George David Birkhoff (1884–1944), Oswald Veblen (1880–1960), Leonard Eugene Dickson (1874–1954), and Robert Lee Moore (1882–1974). This quartet, each one of whom became a president of the American Mathematical Society (AMS) as well as a member of the National Academy of Sciences, was especially fruitful in producing additional distinguished researchers, at Harvard, Princeton, the University of Chicago, and the University of Texas, respectively. For details of the research accomplishments of Moore and his progeny the reader may consult the excellent book by Karen Parshall and David Rowe on the formation of the American mathematical research community [18, pp. 323–426].

It is also clear that Moore was a formidable academic politician. Especially striking testimony on this point was provided by Oskar Bolza (1857–1942), a German mathematician who came to the United States seeking academic opportunities, and who joined Moore's department when the University of Chicago opened its doors in 1892. Moore was only thirty, five years Bolza's junior, and Bolza was dubious at first about this untried person as department head. But Bolza found himself witnessing not only the emergence of a powerful mathematics department at Chicago but of a newly vigorous American mathematical community as a whole. In his autobiography, written many years later after his return to Germany, Bolza had no doubt who was responsible for these developments. It was "E. H. Moore mit seinem aggressiven Enthusiasmus und seinem Organisationstalent" [3, p. 32], a characterization that readily leaps the language barrier.

Remarkably, amidst his vigorous activities on behalf of the research community, Moore found time to engage with educational reform in both the colleges and the secondary schools. His most notable pedagogical statement came in his retiring address as president of the AMS in December 1902, published in March 1903

[14, pp. 401–416]. This address created much comment initially, and has continued to stir interest in the years since. The National Council of Teachers of Mathematics (NCTM) reprinted Moore’s talk in its very first yearbook, published in 1926 [24, pp. 32–57]. A more recent reference to Moore’s address can be found in this MONTHLY for December 1997 [9, p. 955].

But what exactly did Moore stand for and what became of his ideas?

2. CONTEXT OF MOORE’S EDUCATIONAL PROPOSALS. E. H. Moore should first of all be situated within the context of the decline of the so-called classical curriculum of Greek, Latin, and mathematics at the end of the nineteenth century. This curriculum, which had dominated the colleges and many of the secondary schools, was frequently justified in terms of “mental discipline” and “faculty psychology”. The mind was held to be composed of various faculties, such as the powers of memory, reasoning, observation, and will. These faculties could be strengthened by mental exercise, much as muscles could be strengthened by physical exercise. Mathematics, it was claimed, was crucial for promoting certain key mental faculties [27, pp. 22–23], [2, pp. 78–85]. Thus we find an educator in 1893 opining on the virtue of studying arithmetic:

It is refreshing to know that there is one subject which [the student] must master for himself slowly, sometimes painfully, and always with much labor. If arithmetic stands in the way of drawing or music, of German or Latin, of book-keeping or civics, let us be thankful that it is an obstacle which will develop the sound mental muscle that is needed for the work further on [8, p. 135].

More advanced mathematics was also valued as a mental discipline. This is epitomized in a statement made by A. Lawrence Lowell (1856–1943), who claimed to remember with special fondness his undergraduate studies at Harvard with mathematician Benjamin Peirce (1809–1880) in the 1870s, before Lowell moved on to become a lawyer and eventually president of Harvard:

To those of us who have not pursued the study of mathematics since college days the substance of what he taught us has faded away, but the methods of thought, the attitude of mind and the mode of approach have remained precious possessions [29, pp. 40–41].

Pleasing as such sentiments may have been to many mathematicians in the late nineteenth century, there were some who were growing skeptical. They recognized that the mental discipline thesis could be a barrier to introducing more advanced topics into the school curriculum. Why should students study algebra when they could exercise their mental muscles simply by grinding away at arithmetic year after year? Furthermore, a number of mathematicians began to join other educators in asserting that true mental discipline could come only in a subject in which the student was genuinely interested. Thus the Mathematics Conference of the celebrated Committee of Ten of 1893, chaired by Simon Newcomb (1835–1909) and including Henry Burchard Fine (1858–1928) and Florian Cajori (1859–1930), made the following statement:

The opinion is widely prevalent that even if the subjects are totally forgotten, a valuable mental discipline is acquired by the efforts made to master them. While the Conference admits that, considered in itself this discipline has a certain value, it feels that such a discipline is greatly inferior to that which may be gained by a different class of exercises, and bears the same relation to a really improving discipline that lifting exercises in an ill-ventilated room bear to games in the open air. The movements of a race horse afford a better model of improving exercise than those of the ox in a tread-mill [2, pp. 133–34].

But by emphasizing such distinctions of disciplinary value, mathematicians were opening an avenue for fundamental attacks on their own subject. If algebra was better discipline than arithmetic, perhaps physics or biology, for example, would be better still. Indeed, a steady buildup of arguments for the superior disciplinary value of the natural sciences had been going on since mid-century in both Britain and America, with mathematics often subject to some degree of attack in the process. Some scientists saw mathematics as characterized by numbing drill, in stark contrast to the invigoration of scientific study, wherein the student stood face to face with nature, in the field or in the laboratory. David Starr Jordan (1851–1931), biologist and first president of Stanford University, described students coming to science “by escape from Latin and calculus with the eagerness of colts brought from the barn to a spring pasture.” [7, p. 358]

3. MOORE’S 1902 ADDRESS. It was into this environment that E. H. Moore ventured in his 1902 address as retiring president of the AMS, and it is thus not surprising to find him proposing to capitalize on the rising prestige of science and engineering by tying mathematical instruction closely to those endeavors, both in the secondary schools and in the early college courses. Moore had come to the conclusion that the training of future scientists and engineers had become a crucial justification for teaching mathematics, and that it was destined to become more so in the future. He championed the ideas of English engineering educator John Perry (1850–1920), who strongly urged that the rigorous axiomatic development of mathematics be de-emphasized in favor of teaching it as an inductive science. For example, beginning geometry students should test propositions from Euclid by careful measurements using graph paper. Perry’s influence can be found in Moore’s insistence that mathematical rigor was a relative rather than an absolute concept: “Sufficient unto the day is the precision thereof”, declared Moore. Both Moore and Perry sought to bring students very rapidly to the employment of powerful mathematical concepts, *without* insisting that all supporting technical details be mastered [14, p. 411].

Moore decried various “chasms” he observed in American mathematical activity: between pure and applied mathematics; between research mathematicians and school teachers; between the public’s foggy perception of mathematics and “the very high position in general esteem and appreciative interest which it assuredly deserves.” [14, pp. 405–408]

The key to resolving these problems, according to Moore, was reformation of mathematical pedagogy at all levels, and the centerpiece of his plan was what he called the “laboratory method”. In Moore’s vision teachers would work with students individually or in small cooperative groups, depending on the needs and strengths of the students. Beginning with “matters of thoroughly concrete character”, they would proceed by means of graphs, models, colored chalks and elementary demonstrations of physical phenomena, “to develop on the part of every student the true spirit of research, and an appreciation, practical as well as theoretic, of the fundamental methods of science.” He further proposed “to organize the algebra, geometry, and physics of the secondary school into a thoroughly coherent four years’ course,” thus doing away with the standard partition of these subjects into “water-tight compartments”. Moore also explained how problems in practical mathematics could be introduced to students so as to awaken interest in abstract concepts. Thus problems involving measurement of physical quantities could become exercises in the study of approximation and allowable error bounds, eventually leading to the theory of limits and irrational numbers; study of the areas under specially selected curves could yield the notion of uniform convergence. Important results should be demonstrated in at least two different ways. In those

cases where formal proof was appropriate “much of the proof should be secured by the research work of the students themselves.” To accommodate the rich level of activity hoped for in his mathematical laboratory, Moore proposed that two consecutive class periods be allocated to it [14, pp. 409–413]. Finally, the students in Moore’s laboratory were to be much engaged in manual activity, a feature he stated most explicitly in his private correspondence: “Provision should be made for the actual construction by the students of many of the simpler drawings, models and mechanisms.” [13]

Another aspect of Moore’s program is notable: he sought to uplift the professional standing of mathematics teachers in the secondary schools. In particular he called on the AMS to expand its membership by actively recruiting secondary school teachers to join its ranks [14, p. 414].

4. CONSEQUENCES. Moore enjoyed several advantages as an educational spokesman: he was widely considered to be one of the preeminent mathematicians of the country; further, he was associated with an institution, the University of Chicago, of considerable educational influence; and finally, as Bolza noted, he was a skilled academic politician. Moore promoted his ideas strenuously for about three years. He arranged to teach laboratory method classes at the University of Chicago, and he taught the introductory calculus course himself using this method. He encouraged secondary school teachers to adopt similar methods. He and colleagues from the Chicago mathematics department wrote articles directed at the secondary schools [23, pp. 301–325]. In 1906, for example, Moore wrote a paper in which he elaborated on the use of graph paper and the function concept in secondary school instruction, and proposed that nomography be emphasized [15]. At meetings of school teachers of mathematics Moore urged these teachers to form their own associations. These associations would become perhaps Moore’s most substantial educational legacy. The founding of the NCTM in 1920 has clear roots in Moore’s efforts earlier in the century [23, pp. 364–367].

But enthusiasm for the substance of Moore’s program soon began to wane, at Chicago and elsewhere, and mathematics education did not develop as he had envisioned. Moore’s laboratory method did not become the dominant method of instruction; a “thoroughly coherent four years’ course” of algebra, geometry, and physics did not become the norm in the secondary schools; mathematical instruction in the colleges and universities did not undergo a major reorientation to better serve the needs of science and engineering applications. Nor did the AMS ever recruit large numbers of high school teachers as Moore had urged [23, pp. 326–339].

Moore’s special efforts at the University of Chicago also soon faded away. For two or three years the whole mathematics department pulled together to reform instructional methods and to promote the laboratory method in particular. But then the reaction set in. The laboratory method was found to be too labor intensive, and too complicated to administer. In addition, sharp lines of stratification began to assert themselves among the Chicago mathematical educators. Those who would carry on the agitating and organizing and broad reform efforts were those who had elected to specialize in education, such as Herbert Ellsworth Slaughter (1861–1937) and Jacob William Albert Young (1865–1948). Those like Moore and Dickson who had research ambitions faded into the background, educationally speaking [23, pp. 320–321].

Meanwhile, the secondary school environment was undergoing an upheaval. Between 1890 and 1900 the number of students going to public high schools increased more than two and half times. Another doubling of enrollment occurred between 1900 and 1912, and yet another between 1912 and 1920. With these huge increases in enrollment came increasing pressure to tie education of students more closely to their

future employment. With the rise of this so-called vocational education movement the key distinction was no longer between classical and modern educational subjects but between academic and practical, and the academic subjects were put on the defensive. Mathematics had successfully crossed from classical to modern, but had a much more difficult time shaking the disparaging label of “academic” [10, pp. 170–173; p. 236; p. 244].

Moreover, the notion of mental discipline now came under full-fledged attack from a range of psychologists and educators. School subjects were asked to justify themselves as supporting “socially efficient” goals. Efficiency-minded educators combed the curriculum for useless topics. In one notable study, some 4,000 sixth, seventh, and eighth grade students were asked to follow their parents around for two weeks, collecting mathematical “problems” solved by the adults in the course of business and household tasks. Most adults were found to use little except the most elementary arithmetic processes; proof, according to this line of thought, that much mathematics could be jettisoned from the curriculum. [28, p. 52]

Mathematical educators eventually became greatly alarmed by these attacks on their subject. One response was a further round of teacher association building around the time of the First World War, culminating in the founding of the NCTM in 1920. Even the AMS was roused to form a special committee in 1914, charged with determining “whether any action should be taken by the Society in regard to the movement to displace mathematics in the schools.” [1] This committee presented a one and one-half page report of very modest suggestions, beginning with a Freudian typographical error that set the tone for the resulting inaction. In the very first sentence it was proclaimed that the question at issue was “whether any action is desirable on the part of the Society in the matter of the movement against mathematicians in the schools.” [21] Mathematicians did not want to be in the schools; ergo, there was no problem requiring action.

At this same time, 1914–1915, there was a renewal of Moore’s earlier call that the AMS engage in efforts to improve the teaching of mathematics. The leading agitator was Moore’s Chicago colleague and former student Herbert Slaught; Moore was clearly supportive, but stayed behind the scenes. Specifically, Slaught proposed that the AMS rescue the financially distressed *Mathematical Monthly*, and at the same time form a special membership class of “Associate” for those primarily interested in educational issues. The AMS rejected these proposals, but at the same time suggested that those with interests similar to those of Slaught ought to form their own society. Slaught took the hint, resulting in the foundation of the Mathematical Association of America (MAA) with the MONTHLY as its official journal. [12, pp. 19–20]

One of the first actions taken by the MAA was to form a National Committee on Mathematical Requirements (NCMR) to study the secondary school curriculum. E. H. Moore was one of the original five members, and it proved to be the last major educational initiative in which he engaged. The evidence suggests that he let others do the bulk of the work, especially his brother-in-law, John Wesley Young (1879–1932) of Dartmouth College. The report of this committee appeared in 1923 in the form of a 600 page book entitled *The Reorganization of Mathematics in Secondary Education* [20]. This report has been hailed as a landmark [4], [6, pp. 208–209]. But if one looks at it in relation to the ambitions of the segment of the mathematical community that had sought to follow the vision of E. H. Moore’s 1902 address, one must come to a rather modest assessment. The NCMR directly followed Moore’s lead in promoting the pedagogical benefits of graphic representation and the function concept, and in both cases some genuine success can be discerned. But much of Moore’s original program was considerably muted. In general the NCMR took a far more defensive posture, aware of the attacks on mathematics education that had erupted over the previous 20 years. The

high hopes for the “laboratory method” seem to have largely dissipated, nor was there any mention of the desirability of aligning mathematics education with engineering. Belatedly realizing that few of the students surging into the high schools were going to become scientists and engineers, the NCMR modified Moore’s original call to justify mathematics as a tool of science, and instead tried to reinvigorate the mental discipline thesis.

But for all its efforts the NCMR did not succeed in vanquishing those attacking mathematics in the schools. High schools increasingly dropped mathematics as a graduation requirement. Even required ninth-grade mathematics, a point at which the NCMR had chosen to make a stand, was under challenge. Percentages of students taking algebra and geometry in grades nine through twelve of the public schools declined steadily into the 1930s [26, p. 195]. The period 1915–1940 has been called a “twenty-five year depression” in school mathematics [5, p. 24].

5. CONCLUSIONS. What went wrong with Moore’s program? Many reasons might be cited, from the death of Moore’s strong supporter, University of Chicago president William Rainey Harper in 1906, to Moore’s increasing preoccupation with his own pure mathematical research program in what he called General Analysis [11, pp. 134–135]. Certainly the decentralized American educational system provided a formidable barrier to even the most enthusiastic and resourceful of reformers.

Some critics were distressed by Moore’s apparent abandonment of rigor and formal drill. William Fogg Osgood (1864–1943) of Harvard, speaking as the retiring AMS president in 1906, implicitly criticized Moore’s approach:

Let us not for a moment fail to recognize the fact that, whatever changes it may be desirable to make in the suggestive instruction of the course, the process by which the youth actually acquires the ideas of the calculus is to a large extent and essentially through formal work of substantial character [17, pp. 449–450].

There was also an unavoidable incongruity in the spectacle of a mathematician such as Moore arguing on behalf of intuition and applications. Moore was the purest of pure mathematicians; it is very difficult to construe any of his research as being applied mathematics. Even as he was preaching the value of developing geometric intuition for instructional purposes, the exploration of the axiomatic foundations of geometry was being characterized by a fellow American mathematician as follows: “Geometric intuition has no place in this order of ideas which regards geometry as a mere division of pure logic.” This observer named the international leaders of this anti-intuitional approach; in America it was E. H. Moore [19, p. 158].

But two other factors deserve emphasis as well. Moore’s pedagogical program was in conflict both with the environment enveloping the secondary schools and with the professionalization project of American college and university mathematicians.

With regard to the school environment, Moore, like many other educators, largely failed to foresee the consequences of changing demographics. In the face of the surge of students into the schools, calls for educational efficiency that had emerged during the last half of the nineteenth century became much more insistent and attractive. The efficiency advocates claimed to offer means to control the flood of students by carefully circumscribing requirements in terms of time and effort. In contrast Moore’s “mathematical laboratory”, which called for such extravagances as performing all demonstrations in two different ways and for blurring of subject-matter boundaries, could well be seen as a prescription for waste and confusion. Moreover, at the very time that Moore was proposing to justify mathematics education primarily as an aid to science and engineering, the population of high school students was exploding with students, most of

whom were not aiming to become scientists or engineers. Why should such students be required to take substantial amounts of mathematics? This was an awkward question for Moore, who had almost entirely jettisoned mental discipline and related concepts from his vocabulary. In the dawning era when school subjects were being judged on “social efficiency”, mathematics was in a precarious position for which Moore and his allies were unprepared [10, pp. 278–280].

Secondly, the community of research mathematicians, which Moore’s own efforts did so much to invigorate, was increasingly reluctant to involve itself with educational matters. This was evident immediately after his 1902 address. The researchers had been shocked to find their president stooping to discuss education. They were disappointed, as one observer put it, “that a man who stood among the few recognized leaders in higher mathematics in this country should lose the opportunity offered to consider the great problems of the science *per se*” [25, p. 135]. Instead of following Moore’s call to seek closer engagement with the schools, the AMS sought to protect the sanctity of its research mission by relegating discussion of educational issues to the MAA.

Central to Moore’s basic argument for enticing research mathematicians into the educational arena was the contention that the health of the research community was ultimately dependent on the health of mathematics in the schools. But consider how difficult it would have been to defend this position during his lifetime. As noted, school mathematics experienced hard times during the first decades of the 20th century. Meanwhile the AMS was growing, college mathematics departments were expanding, and Moore’s doctoral students in particular were colonizing departments of mathematics across the country. In short, the connection argued for by Moore, that the research community depended on school mathematics, was not obvious in the early twentieth century. I leave as an open question whether it has become more obvious since.

A strong claim can be made for Moore as a primary initiator of the use of graph paper in the American mathematical curriculum, although nomography never came to have the importance that Moore envisioned. It is also likely that he paved the way for making the function concept more central to instruction, although widespread adoption would prove slow. But Moore’s broader aims for reorienting mathematical instruction were not achieved, and with the luxury of hindsight it seems fair to say that they were undermined by his failure to analyze sufficiently both the internal workings of his own profession and the import of wider social developments. Today’s mathematical education reformers might do well to take heed of the difficulties experienced even by such a one as “E. H. Moore mit seinem aggressiven Enthusiasmus und seinem Organisationstalent”.

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